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WHEELER'S GRADED ARITHMETICS

BY

FREDERICK H. SOMERVILLE

Formerly of the Lawrenceville School and the
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AND

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Indianapolis

ADVANCED BOOK

CHICAGO

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PREFACE

THE leading courses in arithmetic for the Seventh and Eighth grades apply elementary principles to actual business problems, and they recommend that good teaching shall combine a certain amount of general information with a good working knowledge of business forms. In recent years these courses have recognized the necessity for coördinating practical arithmetic with the problems of home economics, while manual training, domestic science, and agriculture have been included as imperative essentials. Manifestly, therefore, a well-planned text designed for the last two years of the elementary schools must meet all of the older requirements, and, at the same time, provide adequate material for the broadening influences that seek to vitalize modern teaching. The authors feel that this third book of the series meets both the old and the new demands.

While it is true that the methods of all business and manufacturing firms develop certain details of form that are not found in school textbooks, it is also true that these details are merely variations which extend beyond the fundamentals that the school must thoroughly teach. The keen business man does not expect young boys and girls to have a knowledge of business practice; but he has the right to expect that they shall be habitually careful, systematic, and accurate. Consequently, the textbook and the teacher must unswervingly concentrate on the development of these qualities in order to render the best service for both the pupil and the future employer.

This book covers a wide range of modern arithmetical applications. Briefly, but in all essential details, it treats General Banking, Savings Banks, Federal Reserve Banks, Liberty Loan Investments, Mortgages, Building and Loan Associations, General

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Business Forms, Insurance, and Taxation. It also devotes unusual space to the Problems of the Home; giving extended treatment to the "Budget Plan"; to the relative expense of owning or renting property; to problems of furnishing and maintaining a home; and to the teaching of practical Thrift and Economy. Furthermore, it covers the ordinary problems of Manual Training, and, under Additional Topics for Study and Reference, provides certain work that, at the discretion of the teacher, may be treated as essential where a prescribed course demands it.

The book is purposely limited to recognized essentials, and it intentionally avoids certain tendencies and theories which, while attractive as experiments, have not yet established themselves as a part of practical arithmetic. The authors have endeavored to give a practical foundation, to provide simple methods, and to confine concrete problems to those which actually occur in average home and business life; and they believe that all obsolete topics have been eliminated, that the book gives an interesting transformation from abstract to applied arithmetic, and that it will be a definite preparation for the everyday problems of the child's life.

The authors gratefully acknowledge their indebtedness to the teachers who have given helpful criticisms, and to the business men who have aided them by the contribution of accurate information and expert suggestions.

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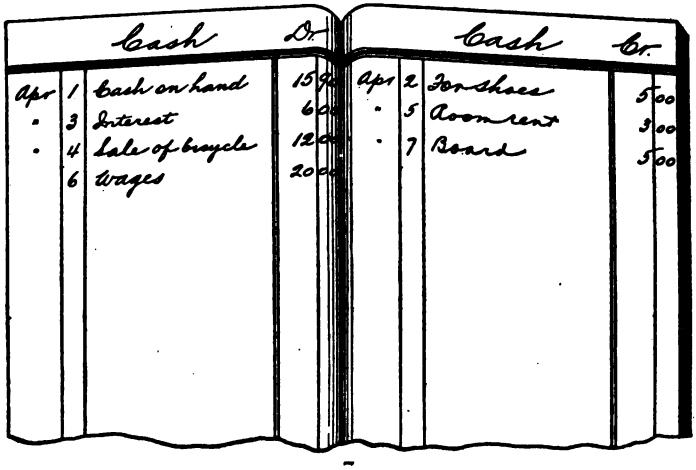
SEVENTH GRADE

BUSINESS FORMS

I. The Cash Account. A Cash Account is a record of the money received and of the money paid out by an individual or a firm.

Usually two pages are used for a cash account. On the left page the amounts of cash received are written, and on the right page the different amounts of each paid out are written. A cash account is balanced when the difference between the cash received and the cash paid out equals the cash on hand or in the bank. Some cash accounts are balanced at the end of each month, others at the end of each week, but most business firms balance their cash accounts daily.

The side of the cash account upon which the cash received is recorded is called the **Debit Side.** The side upon which the cash paid out is recorded is called the **Credit Side.**



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The usual method of keeping a personal cash account is shown in the illustration.

Explanation: On the debit side are entered the amount of cash on hand at the beginning of the month and every item of cash received thereafter.

1. The balance left over from the preceding week.
2. An interest check received for the use of a loan.
3. The receipts from the sale of a bicycle.
4. The weekly wages of the man keeping the account.

On the credit side there are entered his expenses for the week.

To balance the account:

1. Add the amounts on the debit side.
2. Add the amounts on the credit side.
3. Subtract the second sum from the first sum.

The difference is the **balance**, or the **Cash on Hand.**

WRITTEN APPLICATIONS

Prepare a sheet ruled to represent two pages like those in the illustration, and on this sheet write cash accounts for the conditions given below.

1. John Fox works for his father on the farm. He receives from his father a weekly wage of $5.00, and it is agreed that he is to have one half of the amount received from the sale of young stock, and that he is to pay one fourth of the expense for seed. The items to be entered for the first week of John's cash account are: May 1, cash on hand, $7.50; May 3, received wages, $5.00; May 5, share from sale of calf, $3.50; May 9, butternuts sold, $1.15; May 2, paid for shoes, $2.50; May 3, paid for seed corn, $2.50; May 8, paid for magazine, $.25.

2. Mr. Charles Hunt is a clerk in a city store and lives in a near-by suburban town. His cash account for one week includes the

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following items: August 5, Cash on hand, $42.75; August 6, received commissions for one week, $4.10; August 6, paid for commutation ticket, $5.90; August 8, paid for board, $4.50; August 9, paid for 1 dozen collars, $1.50, 1 pair rubbers, $1.00, book, $1.12, club dues, $10.00; August 10, received one week's salary, $21.00; August 11, paid room rent, $2.50, paid laundry bill, $1.18.

3. Tom French is an amateur photographer, and the following are his receipts and expenses for making four dozen photographs of the school football team. Nov. 1, 3 plates at 5¢ each; developer, 5¢; hyposulphite, 10¢; 4 dozen mounts at 15¢ a dozen; 5 dozen sheets of printing paper at 35¢ a dozen; received on November 20, 10¢ each for 48 photographs. Find the amount of his profit.

4. Robert Coleman has 20 hens, and for the month of January his receipts and expenses were as follows: January 2, paid for 3 bushels of corn at $.90 per bushel; received 45¢ a dozen for 2 dozen eggs; January 8, paid 5¢ a pound for 25 pounds of bone meal; January 9, received 48¢ a dozen for 2 dozen eggs; January 15, paid 10¢ a pound for 3 pounds of grit; January 16, received 50¢ a dozen for 4 dozen eggs; January 23, paid $.75 for whitewashing henhouse; January 30, received 47¢ a dozen for 4 dozen eggs. Find the amount of his profit for the month.

II. **Bills.** A Bill is a written statement of goods sold, giving the date of the sale, the quantity sold, the prices per unit of quantity, and the total amount of the sale when the different items are included.

When a bill is paid the person receiving the payment writes "Paid" or "Received Payment" at the bottom of the sheet, and signs his name. This is called **receipting a bill.** If an employee signs a receipt for his firm, he writes the firm's name and then writes his own name or initials underneath.

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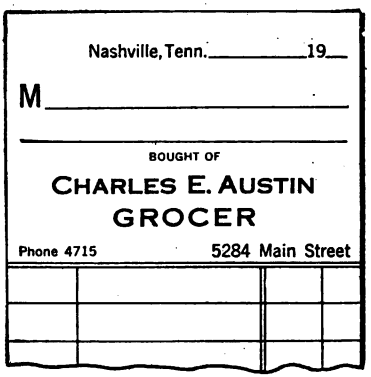
The Grocer's Memorandum. This is one of the simplest forms of a bill. The illustration shows a form of memorandum used by many grocers. This is usually sent to a customer with the delivery of her order. By this means she is kept informed of each daily transaction, and the bill also serves as a check upon the order of groceries delivered.


Nashville, Tenn. _____ 19__

M _____

BOUGHT OF

CHARLES E. AUSTIN

GROCER

Phone 4715 5284 Main Street

WRITTEN APPLICATIONS

Prepare memoranda for each of the following, using your own name as the purchaser and the name of your local grocer as the seller.

1. On August 4 you purchased groceries as follows: 5 pounds of sugar for $.38; 5 pounds of lard for $.70; 6 bars of soap for $.30; 1 quart of molasses for $.25; 2 pounds of rice for $.36; 1 sack of flour for $1.40; 1 can of soup for $.12.

2. On August 10 you bought the following: 12-pound sack flour, $.60; 5 pounds rice, $.35; 4 bars soap, $.30; 1 bushel potatoes, $1.25; 1 pound cocoa, $.35; 1 quart olive oil, $.80; 3 pkgs. corn flakes, $.30.

3. On September 4 your purchases were 2 pounds beefsteak at $.35; 3 pkgs. oatmeal at $.10 each; 1 bottle bluing at $.15; 1 bottle vanilla at $.45; 3 pounds rice at $.18; 4 pounds starch at $.10; and 1 bushel potatoes at $.95.

4. The American Hotel orders the following from the grocer on July 10: 15 pounds chops at $.30; 10 pounds cornmeal at $.08; 100 bars soap at $.04; 5 dozen Dutch Cleanser at $.95 per dozen; 20 pounds cocoa at $.27 per pound; 100 pounds coffee at $.28 per pound; and 1 dozen pkgs. table salt at $.20.

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The simple form of bill given below is in common use. Several other forms are used, but all of them are similar to this one.

WRITTEN APPLICATIONS

Make a neat copy of the billhead illustrated above, and make out bills for the sales in the exercises given below. In each case receipt the bill, signing the firm name and your own initials.

1. On July 10, 1918, James Robinson purchased from the Pillsbury Furniture Company 6 dining-room chairs at $5.50; 1 buffet at $75.00; and 1 Axminster rug at $42.50. Make out the bill as directed above.

2. On the 12th of October, 1918, the Sterlington Hotel purchased of the Pillsbury Furniture Company the following: 24 brass beds at $16.25; 24 bureaus at $21.50; 24 chiffoniers at $15.75; and 24 Brussels rugs at $19.40. On the following day they also purchased 145 yards of Axminster carpet at $1.10 a yard. Make out a bill of these items and receipt it.

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3. Mr. Charles E. Wilson bought furniture from the Pillsbury Company as follows: 1 dining-room table, $45.00; 8 dining-room chairs at $7 each; 1 buffet, $87.50; 1 serving table, $35; 1 rug, $52.50; 1 library 3-piece suite, $115; 1 library table, $45; 1 mahogany rocker, $21; 1 wicker rocker, $13.50; 1 Axminster rug, $67.50; 1 Wilton velvet rug, $87.50; and 1 hall runner 6 yards long at $2.25 per yard. Make out a bill and receipt it for the firm.

4. The Hub Clothing Co. bought the following from Rogers, Richardson & Co.: 50 men's overcoats at $17.50; 150 men's suits at $21.50; 200 boy's suits at $6.75; 150 young men's suits at $18.75; and 300 knit sweaters at $4.20. Make out a bill head and fill in these items. Show that it was paid 30 days after it was dated, and that the firm name was signed by the cashier, J. E. Harris.

5. Finley, Houston & Co., of Cleveland, Ohio, are wholesale dealers in women's clothing. Make out a bill for the items given below, and show that the goods were bought by Miller and Fullerton of Columbus, Ohio, on March 1, 1919, and that the bill was paid March 10. 50 doz. suede gloves at $15.50; 300 pr. silk hose at $1.25; 125 Georgette crêpe blouses at $3.25; 60 serge capes at $13.75; 48 Foulard dresses at $21.50; and 200 gingham house dresses at $3.75.

6. Make out a bill in which you are the purchaser of the following goods: 10 tennis nets at $4.50 each; 6 tennis rackets at $4.75 each; 5 golf clubs at $3.25 each; 10 baseball bats at $.75 each; 5 doz. baseballs at $9.00 per doz.; 4 catcher's gloves at $4.25 each; 10 fielder's gloves at $2.35 each; 3 catcher's pads at $4.75 each; and 2 catcher's masks at $2.75 each. Show that the bill was paid 10 days after its date, and indicate its receipt.

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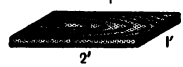
PRACTICAL MEASUREMENTS

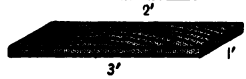
SURFACE MEASURE

I. The Builder's Applications

(*a*) **Lumber Measure.** The unit of lumber measure is the Board Foot, a board 1 foot long, 1 foot wide, and 1 inch thick.

1 board foot. $1' \times 1' \times 1'' = 1$ board foot.

2 board feet.  $2' \times 1' \times 1'' = 2$ board feet.

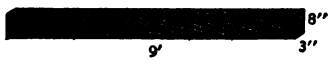
3 board feet.  $3' \times 1' \times 1'' = 3$ board feet.

In each case:

The area of one surface in square feet multiplied by the thickness in inches equals the number of board feet in the piece.

Length 4 ft.
Width $1\frac{1}{4}$ ft.
Thickness 2 in. $4' \times 1\frac{1}{4}' \times 2'' = 10$ board feet.

Length 6 ft.
Width $\frac{2}{3}$ ft.
Thickness 8 in. $6' \times \frac{2}{3}' \times 8'' = 36$ board feet.



Length 9 ft.
Width $\frac{1}{4}$ ft.
Thickness 8 in. $9' \times \frac{1}{4}' \times 8'' = 18$ board feet.

Lumber less than 1 inch in thickness is measured as if it were 1 inch thick.

Lumber more than 1 inch in thickness is measured by its actual thickness.

Lumber is usually bought and sold by the thousand board feet, and the Roman numeral " M " is used for thousand.

" $36 per M" means $36 per one thousand board feet.
" $36 per M" is the same as $.036 per board foot.

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BLACKBOARD PRACTICE

Find the number of board feet in each of the following pieces, the thickness in each case being 1 inch:

1. $12' \times 12''$.
2. $14' \times 12''$.
3. $12' \times 10''$.
4. $14' \times 10''$.
5. $12' \times 9''$.
6. $14' \times 10''$.
7. $12' \times 8''$.
8. $14' \times 9''$.
9. $10' \times 8''$.
10. $10' \times 3''$.
11. $12' \times 3''$.
12. $12' \times 4''$.
13. $12' \times 6''$.
14. $14' \times 3''$.
15. $14' \times 6''$.
16. $14' \times 8''$.
17. $8' \times 15''$.
18. $8' \times 16''$.
19. $8' \times 15''$.
20. $10' \times 15''$.
21. $10' \times 16''$.
22. $10' \times 18''$.
23. $12' \times 15''$.
24. $12' \times 18''$.

Find the number of board feet in each of the following, the length, the width, and the thickness being respectively:

25. $10' \times 12'' \times 2''$.
26. $10' \times 12'' \times 1\frac{1}{2}''$.
27. $10' \times 12'' \times 2\frac{1}{2}''$.
28. $12' \times 12'' \times 3''$.
29. $12' \times 10'' \times 1\frac{1}{2}''$.
30. $12' \times 10'' \times 2''$.
31. $12' \times 8'' \times 2\frac{1}{2}''$.
32. $12' \times 8'' \times 3''$.
33. $14' \times 6'' \times 2''$.
34. $14' \times 6'' \times 3''$.
35. $14' \times 4'' \times 2''$.
36. $14' \times 6'' \times 2\frac{1}{2}''$.

Find the number of board feet in:

37. 10 planks, each 10 ft. long, 10 in. wide, and 2 in. thick.
38. 12 planks, each 14 ft. long, 10 in. wide, and $2\frac{1}{4}$ in. thick.
39. 20 planks, each 12 ft. long, 10 in. wide, and $2\frac{1}{2}$ in. thick.
40. 16 planks, each 12 ft. long, 6 in. wide, and 4 in. thick.
41. 28 planks, each 14 ft. long, 2 in. wide, and 4 in. thick.
42. 40 boards, each 12 ft. long, 10 in. wide, and $\frac{7}{8}$ in. thick.
43. 60 boards, each 12 ft. long, 12 in. wide, and $1\frac{1}{8}$ in. thick.
44. Find, to the nearest cent, the cost of the following bill of lumber at \$30 per M. 8 sills, $10'' \times 12'' \times 16'$, 12 posts, $6'' \times 6'' \times 20'$, 2 beams, $8'' \times 6'' \times 12'$, 24 joists, $2'' \times 8'' \times 10'$.

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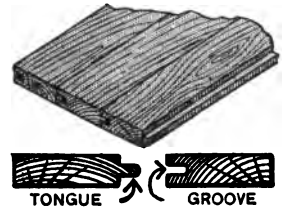
45. Find, to the nearest cent, the cost of the following bill of lumber at \$32.50 per M. 10 sills, $10' \times 12'' \times 14''$, 4 beams, $6'' \times 8'' \times 12'$, 48 joists, $3'' \times 8'' \times 12'$, 120 boards, $1'' \times 12'' \times 12'$.

46. In 1890 a builder could buy No. 1 spruce lumber at \$18 per M. At the present time the same lumber costs \$40 per M. If he uses spruce lumber, how much more will a builder pay now than in 1890 for the lumber bill in example 45?

(*b*) **Flooring.** Most of the lumber used for flooring is what is known as "matched lumber," the tongue on the edge of one board fitting tightly into the groove on the edge of the next board. The loss in area due to matching is usually made up by increasing the estimate of the amount of surface to be covered. For flooring, 3 inches or less, this increase is $\frac{1}{4}$; and for wider flooring, $\frac{1}{5}$.



Illustrations:

1. How many feet of flooring will be needed for a room 16′ by 12′ if the width of the flooring is $2\frac{1}{2}$ inches?

The area of the floor = (16 × 12) sq. ft. = 192 sq. ft.

Allowing for an increase of one fourth in the area because of the narrow flooring used:

192 sq. ft. + $\frac{1}{4}$ of 192 sq. ft. = (192 + 48) sq. ft. = 240 sq. ft.

Therefore, the number of square feet of flooring required = 240. *Result.*

2. At \$65 per M, how much will 4″ flooring cost for a room 15′ by 12′?

The area of the floor = (15 × 12) sq. ft. = 180 sq. ft.

Allowing for an increase of one fifth in the area because of the wide flooring used:

180 sq. ft. + $\frac{1}{5}$ of 180 sq. ft. = (180 + 36) sq. ft. = 216 sq. ft.

To find the cost: 216 sq. ft. = .216 M board feet.

Then .216 × \$65 = \$14.04, the cost. *Result.*

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BLACKBOARD PRACTICE

Find the number of feet of $2\frac{1}{2}''$ flooring needed for a room:

1. $14' \times 12'$.	**4.** $15' \times 12'\ 6''$.	**7.** $12'\ 6'' \times 10'\ 4''$.
2. $15' \times 12'$.	**5.** $16' \times 12'\ 8''$.	**8.** $16'\ 4'' \times 12'\ 6''$.
3. $16' \times 16'$.	**6.** $16' \times 14'\ 6''$.	**9.** $16'\ 8'' \times 14'\ 9''$.

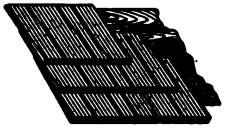
Find the cost of $4''$ flooring at \$60 per M for a room:

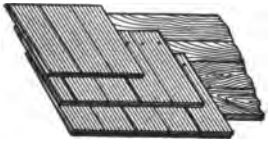
10. $15' \times 12'$.	**13.** $12'\ 6'' \times 9'$.	**16.** $12'\ 6'' \times 10'\ 6''$.
11. $16' \times 14'$.	**14.** $14'\ 8'' \times 12'$.	**17.** $14'\ 8'' \times 12'\ 4''$.
12. $18' \times 10'$.	**15.** $16' \times 14'\ 4''$.	**18.** $16'\ 6'' \times 15'\ 3''$.

(*c*) **Roofing.** The unit of measure for roofing is the **square**, or 100 square feet. The materials in common use for roofing are wood shingles and roofing slate.

Wood shingles average 16 inches in length and 4 inches in width.

Roofing slate is cut uniformly 16 inches in length and 10 inches in width.





Both are laid in such a manner as to overlap and completely cover the roof, and the amount of exposed surface of the shingle or slate determines the number that will be required. An exposed length of $4''$ is called "4 inches to the weather."

If shingles are laid with

$4''$ exposed, 1 shingle covers 16 sq. in., or $\frac{1}{9}$ sq. ft., and 900 cover 1 square.

$4\frac{1}{2}''$ exposed. 1 shingle covers 18 sq. in., or $\frac{1}{8}$ sq. ft., and 800 cover 1 square.

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If slates are laid with

4″ exposed, 1 slate covers 40 sq. in., or $\frac{5}{18}$ sq. ft., and 360 cover 1 square.

4½″ exposed, 1 slate covers 45 sq. in., or $\frac{5}{18}$ sq. ft., and 320 cover 1 square.

In allowing for the waste in shingles it is customary to figure 1000 to a square.

No allowance for waste is made in using slate.

Shingles are sold in bunches of 250 each, or 4 bunches per thousand.

Slates are sold by the square.

Roofing Tin is usually sold in sheets 20″ by 28″, in boxes containing 112 sheets.

In laying a tin roof the sheets of tin overlap at the seams. A box of 112 sheets is usually estimated to cover 360 square feet.

BLACKBOARD PRACTICE

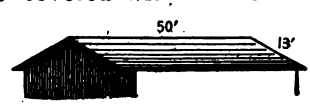
Making no allowance for waste, and laying the shingles 4″ to the weather, find the number of bunches of shingles necessary for both sides of a gable roof the length and the rafter of which are respectively:

1. 30′, 12′.	4. 42′, 15′ 6″.	7. 35′ 8″, 12′ 6″.
2. 32′, 14′.	5. 42′, 16′ 8″.	8. 40′ 8″, 15′ 9″.
3. 35′, 16′.	6. 45′, 14′ 6″.	9. 60′ 6″, 18′ 6″.

If shingles cost $4 per thousand, and slate $5.50 per square, find the difference in the cost between these two materials for the three roofs whose length and rafter are given below; both materials to be laid 4½″ to the weather, and no allowance to be made for waste on the shingles.

10. 28′, 12′. 11. 36′ 6″, 16′ 4″. 12. 45′ 6″, 16′ 6″.

13. The roof in the figure is to be covered with 1″ boarding at $30 per M, and with tin at $6.25 per square. Find the total cost of the roof.



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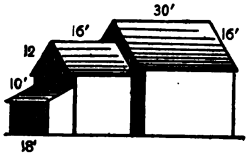
14. A contractor estimates the cost per square for roofing the building at the left as follows:



Roofing tin	$5.20
Painting two coats	1.40
Solder	1.00
Labor	2.25

What is his estimate of the cost of the roof?

(*d*) **Lathing.** Laths are 4 feet long, and are sold in bundles of 100. A bundle will usually cover 6 square yards.

In estimating the cost of lathing a room the openings, that is, the area of all doors and windows, should be deducted.

Illustrations:

1. How many bundles of laths will be required for the ceiling of a room 15′ long and 12′ 6″ wide?

For the area of the ceiling:

$$(15 \times 12.5) \text{ sq. ft.} = 187.5 \text{ sq. ft.} = 21 \text{ sq. yd.}$$

Since one bundle covers 6 sq. yd., $21 \div 6 = 3\frac{1}{2}$ bundles.

Half bundles are not sold, hence the ceiling requires 4 bundles. *Result.*

2. How many bundles of laths are required for the walls and ceiling of a room 15 ft. long, 12 ft. wide, and 9 ft. high, allowing for one door 7 ft. by 4 ft., and two windows, each 6 ft. 6 in. by 4 ft.?

The perimeter of the room $= 2 \times (15+12)$ ft. $= 54$ ft.

Total area of side walls (54×9) sq. ft. =		486 sq. ft.
Area of ceiling (15×12) sq. ft. =		180 sq. ft.
Total area of walls and ceiling =		666 sq. ft.
Area of door (7×4) sq. ft. =	28 sq. ft.	
Area of windows $2 \times (6\frac{1}{2} \times 4)$ sq. ft. =	52 sq. ft.	
Total area of openings =	80 sq. ft.	80 sq. ft.
Total area of lathing =		586 sq. ft.

586 sq. ft. $= (586 \div 9)$ sq. yd. $= 65^{+}$ sq. yd.

The estimate should cover full square yards, or 66 sq. yd.

Then: 66 sq. yd. $\div 6$ sq. yd. $= 11$ bundles. *Result.*

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BLACKBOARD PRACTICE

1. How many bundles of laths will be needed for the walls of a room 20′×14′×8′ if an allowance of 100 sq. ft. is made for openings?

2. If the laths for the room in example 1 cost \$.45 per bundle and the labor cost of putting them on is \$.25 per bundle, what is the cost of lathing the room?

3. A room is 18 ft. long and 14 ft. wide. There are three doors, each 7 ft. by 4 ft., and two windows, each 6 ft. by 4 ft. If the room is 9 ft. 6 in. high, how much will it cost to lath it at \$.70 per bundle including labor?

(*e*) **Plastering.** Plastering is usually estimated by the square yard. On new work plastering and lathing are usually figured together, but on old work the lathing is not usually included.

The methods used by contractors in estimating plastering vary widely, and it is impossible to give a general rule. Contracts should state the allowance to be made for openings, etc.

In the illustration below and in obtaining the results for the exercises that follow, a fraction of a square yard is considered a square yard; and full allowance is made for openings.

Illustration;

A room is 24 ft. long, 14 ft. wide, and 9 ft. high. There are 3 windows, each 6 ft. 8 in. by 4 ft., and 2 doorways, each 7 ft. 4 in. by 4 ft. Find the cost of plastering the room at \$.35 per sq. yd.

The perimeter of the room = $2\times(24+14)$ ft. = 76 ft.		
Total area of side walls = (76×9) sq. ft. =		684 sq. ft.
Area of ceiling = (14×24) sq. ft. =		336 sq. ft.
Total area of walls and ceiling =		1020 sq. ft.
Area of doors = $2\times(7\frac{1}{3}\times4)$ sq. ft. =	$58\frac{2}{3}$ sq. ft.	
Area of windows = $3\times(6\frac{2}{3}\times4)$ sq. ft. =	80 sq. ft.	
Total area of openings =	$138\frac{2}{3}$ sq. ft. or	139 sq. ft.
Total area of plastering =		881 sq. ft.

881 sq. ft. = (881 ÷ 9) sq. yd. = 98 sq. yd. (To the nearest square yard)

Cost of plastering = 98 × \$.35 = \$34.30. *Result.*

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BLACKBOARD PRACTICE

Find the number of square yards in a ceiling whose dimensions are:

1. 12′×12′.
2. 14′×15′.
3. 12′×16′.
4. 14′×16′.
5. 15′×17′.
6. 17′×18′.
7. 12′×10′ 6″.
8. 15′×12′ 7″.
9. 14′×14′ 8″.
10. 15′×18′ 10″.
11. 17′×21′ 6″.
12. 20′×22′ 4″.
13. 10′ 6″×12′ 5″.
14. 10′ 8″×11′ 6″.
15. 12′ 4″×14′ 8″.
16. 15′ 9″×16′ 6″.
17. 18′ 2″×19′ 4″.
18. 20′ 6″×24′ 8″.

At 35¢ per square yard find the cost of plastering a room whose dimensions are:

19. 10′×12′ 8″×8′.
20. 12′ 4″×15′ 6″×9′.
21. 12′ 6″×16′ 8″×8′.
22. 13′ 8″×14′ 9″×9′.
23. 14′ 10″×15′ 9″×9′.
24. 16′ 6″×18′ 8″×8′ 6″.
25. 18′ 6″×20′ 10″×9′ 6″.
26. 20′ 8″×22′ 8″×9′ 6″.
27. 22′ 4″×24′ 8″×10′ 6″.
28. 24′ 6″×28′ 9″×10′ 8″.

29. A room is 14 feet long, 10 feet wide, and 9 feet high. The total area of the openings is 76 square feet. What will it cost to plaster this room at 30¢ per square yard?

30. A room 24 feet long and 16 feet wide has two windows each 6 feet by 3 feet 6 inches, and two doors each 7 feet by 3 feet 6 inches. The room is 9 feet 8 inches high. At 35¢ a square yard, what will it cost to plaster the room?

31. A room 25 feet square and 10 feet high has one door 7 feet by 8 feet, one door 7 feet 4 inches by 3 feet 6 inches, one window 4 feet 6 inches by 6 feet, and two windows each 6 feet by 3 feet. At 32¢ a square yard, what will it cost to plaster the room?

(*f*) **Painting.** Painting is usually estimated by the square yard. No general rule for measurement and allowance for openings can be given, for local customs vary considerably. Your local contractor will give you accurate information as to his methods.

1 gallon of paint will cover 250 square feet two coats.

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In the following exercises no allowance is made for openings, because the labor of painting around doors and windows and painting window sashes is as great, if not greater, than the labor would be if there were no openings.

BLACKBOARD PRACTICE

Allowing 250 sq. ft. of surface for each gallon of paint, find the number of gallons necessary for painting a floor whose dimensions are:

1. $12' \times 15'$.
2. $14' \times 16'$.
3. $15' \times 18'$.
4. $16' \times 20'$.
5. $18' \times 22'$.
6. $19' \times 23'$.
7. $20' \times 32'$.
8. $30' \times 45'$.
9. $35' \times 60'$.

At 20¢ per square yard for two coats of paint, what will it cost to paint the walls of a room whose dimensions are:

10. $128' \times 14'$, 9′ high?
11. $16' \times 20'$, 9′ high?
12. $18' \times 24'$, 10′ high?
13. $24' \times 30'$, 12′ high?
14. $35' \times 60'$, 14′ high?
15. $45' \times 80'$, 18′ high?

At 10¢ per square yard find the cost of painting, with cold water paint, a shop whose length and breadth, and height, respectively, are:

16. 40′, 25′, and 10′.
17. 45′, 30′, and 10′.
18. 50′, 30′, and 12′.
19. 60′, 35′, and 12′.
20. 90′, 25′, and 14′.
21. 120′, 40′, and 16′.

One gallon of varnish covers 500 sq. ft. At $3.00 per gallon, find the cost of the varnish for varnishing a floor:

22. $12' \times 15'$.
23. $14' \times 16'$.
24. $14' \times 18'$.
25. $15' \times 18'$.
26. $15' \times 20'$.
27. $16' \times 20'$.
28. $18' \times 22'$.
29. $18' \times 25'$.
30. $22' \times 24'$.

With paint costing $1.60 a gallon, and estimating the cost of the labor to be three times the cost of the paint, find the cost of

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painting both sides of a fence two coats, whose length and height respectively are:

31. 75′ and 6′.	**33.** 100′ and 5′.	**35.** 250′ and 5′ 8″.
32. 90′ and 5′.	**34.** 120′ and 6′.	**36.** 750′ and 6′ 6″.

37. Estimate the cost of painting the ceiling of your schoolroom with cold water paint at $.10 per square yard.

38. Find the cost of painting the barn shown in the figure, if the roof is painted at a cost of 15¢ per square yard, and the sides and ends at 20¢ per square yard. The barn is 40′ long, 24′ wide, 12′ high at the corners, and 21′ high at the peak, and the rafters are 17′ long.

39. A barn is 80 feet long, 35 feet wide, 20 feet high at the eaves, and 15 feet high from the level of the eaves to the peak of the roof, and the rafter is 24 feet. Find the cost of painting the barn, including the roof, at 20 cents a square yard.

II. The Householder's Applications

(*a*) **Papering.** Wall paper is sold in rolls, and a standard single roll is a piece 8 yards long and 18 inches *wide*.

A roll is often spoken of as a piece.

Paper is usually shipped from the factory in double rolls, that is, two single rolls in one long strip. The plain "felt" papers are made 30 inches wide, and many imported papers vary from the standards of width that have been adopted by domestic manufacturers. Borders and ornamental decorations are sold by the yard. In contracts for wall papering there is a charge for the paper and a charge for hanging it on the wall. The cost of hanging varies with the quality of the paper.

Because of the loss in matching patterns it is impossible to estimate exactly the amount of paper a room will require. Several different methods of estimating are widely used, and among them is the following. This method makes a liberal allowance for matching and for the deduction for openings.

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Measure the perimeter of the room in yards, deducting total width of openings.

Allow two strips of paper to each yard in this result.

Measure the height of the room and determine how many strips can be cut from a single roll.

Divide the number of strips required by the number you can cut from a single roll.

The quotient is the number of single rolls required.

Illustration:

1. Find the number of rolls of paper required for the walls of a room 17′ by 13′, and 9′ high, making allowance for two doors and three windows, each 4′ wide.

Perimeter of room $= 2(17+13)$ ft. $= 2 \times 30$ ft. $= 60$ ft.
Subtracting width of openings, 60 ft. $- 5 \times 4$ ft. $= 40$ ft.
40 ft. changed to the nearest number of yards $= 14$ yd.
Number of strips required $= 2 \times 14 = 28$ strips.
Divide length of roll (24 ft.) by height of room (9 ft.) and we find that $2\frac{1}{2}$ strips per roll is a safe estimate. (Or 5 strips per double roll.)

Then, $\frac{\text{Number strips needed}}{\text{Number strips per roll}} = \frac{28}{2\frac{1}{2}} = 12$, the number of rolls. *Result.*

BLACKBOARD PRACTICE

1. How many rolls of paper 8 yards long and 18 inches wide will be required for the walls of a room 14 feet long and 12 feet wide, the height of the room being 8 feet, and the openings one door and two windows each 4 feet wide?

2. How many rolls of paper 30 inches wide will be needed for the walls of a room 18 feet square, if the height of the room is 9 feet, and the allowance for openings is 5 strips?

3. A hall is 32 feet long, and 14 feet wide, and it is to be papered with 30-inch felt paper. The height is 8 feet 6 inches, and the allowance for openings includes four doors each 4 feet wide, and one door 6 feet wide. How many rolls of paper will be needed for the walls and the ceiling?

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4. A bedroom 14 feet square is 9 feet high, and has two doors and two windows each 4 feet wide. How many rolls of paper 18 inches wide will be required for both the walls and the ceiling? If the paper costs 25¢ per roll find the total cost of the paper.

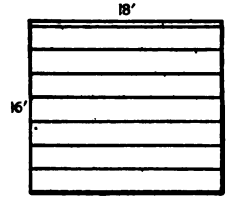
5. Find the cost of papering a kitchen 14 feet long, 10 feet wide, and 9 feet high; the paper costing 20¢ per roll, the cost of hanging it being $.15 per roll, and the allowance for openings being one fourth the total area of the side walls. Include in your estimate the cost for papering the side walls and the ceiling.

6. A dining room 18 feet long, 14 feet wide, and 9 feet high has 3 doors and 3 windows, each 4 feet wide. The paper used for the walls cost 50¢ per roll and the paper used for the ceiling cost 25¢ per roll. The price charged for hanging the paper was 35¢ per roll. What was the total cost of papering the room?

(*b*) **Carpeting.** Carpet is sold by the linear yard.

The better grades of carpet are made $\frac{3}{4}$ of a yard, or 27 inches, wide, and the cheaper grades are made 1 yard wide.

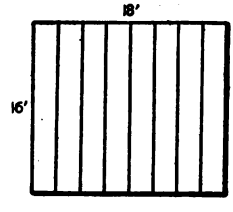
With plain carpet or with carpet in which the figures are small, there is no loss in matching patterns. If the figure is large, there must be an allowance on each strip after the first for the loss in matching the pattern.



If the strips run lengthwise:
16 ft. = (16×12) in.

$$\frac{16\times12}{27} = 7\tfrac{1}{9} \text{ strips.}$$

8 strips, *6 yards long* = 48 yd.



If the strips run crosswise:
18 ft. = (18×12) in.

$$\frac{18\times12}{27} = 8 \text{ strips.}$$

8 strips, $5\frac{1}{3}$ *yards long* = $42\frac{2}{3}$ yd.

With carpet at $1 per yard, the saving in the second case exceeds $5.00.

Floor coverings like linoleum and matting are sold by the linear yard and also by the square yard. Most linoleum is made two yards wide.

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BLACKBOARD PRACTICE

Find the number of yards of plain carpet 1 yard wide required for a room whose dimensions are:

1. 12′×15′.
2. 15′×18′.
3. 12′×18′.
4. 12′×16′.
5. 15′×17′.
6. 18′×20′.
7. 12′ 6″×14′ 6″.
8. 15′ 6″×18′ 9″.
9. 16′ 8″×20′ 6″.

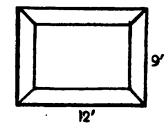
Find the number of yards of plain carpet $\frac{3}{4}$ of a yard wide required for a room whose dimensions are:

10. 12′×15′.
11. 15′×18′.
12. 12′×18′.
13. 12′×16′.
14. 15′×21′.
15. 15′×24′.
16. 15′×12′ 8″.
17. 16′×20′ 8″.
18. 18′×24′ 6″.

With plain carpet 27 inches wide find how much the carpet will cost for each of the following rooms, the dimensions of the room and the price per yard being respectively:

19. 14′×16′ ($1.20 per yard).
20. 16′×18′ ($1.00 per yard).
21. 15′×17′ ($1.10 per yard).
22. 15′×19′ ($1.30 per yard).
23. 18′×12′ ($1.60 per yard).
24. 16′×28′ ($1.00 per yard).
25. 24′×26′ ($1.90 per yard).
26. 24′×15′ ($1.65 per yard).
27. 25′×20′ ($1.75 per yard).
28. 27′×22′ ($2.00 per yard).

29. Find the cost of the material for the rug shown in the figure, the material being $\frac{3}{4}$ yard wide, and the allowance for waste at the corners being twice this width; and the cost of the material $1.20 per yard.



30. A room 15′ wide is carpeted with carpet 1 yard wide. The strips run lengthwise and the room is 18′ long. If $\frac{1}{4}$ of a yard is lost in matching each strip after the first strip, how many yards of carpet will be needed?

31. A floor is to be covered with carpet 27″ wide, and the loss in matching is $\frac{1}{8}$ of a yard per strip after the first strip. The carpet cost $2.25 per yard. The room is 16 feet long and 15 feet wide. Which is the more economical way to run the strips, and what is the cost of the carpet?

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III. The Contractor's Applications

Paving. Paving estimates are based upon the square foot or square yard.

While the contractor gives his estimates in square yards of the surface of the finished product, his own problem includes the material necessary for foundations and the work necessary to lay them.

BLACKBOARD PRACTICE

At 14¢ per square foot, find the cost of building a concrete sidewalk:

1. 100 feet long, 4 feet wide.
2. 150 feet long, 5 feet wide.
3. 240 feet long, 3½ feet wide.
4. 300 feet long, 6 feet wide.
5. 500 feet long, 4½ feet wide.
6. 800 feet long, 5 feet wide.
7. 50 yards long, 3 feet wide.
8. 75 yards long, 4½ feet wide.
9. 90 yards long, 5 feet wide.
10. 125 feet long, 1 yard wide.
11. 200 yards long, 5 feet 4 inches wide.
12. 350 feet long, 8 feet 5 inches wide.

At 21¢ per square foot, find the cost of a concrete floor for a cellar:

13. 35 feet long, 25 feet wide.
14. 65 feet long, 32 feet wide.
15. 40 feet long, 32 feet wide.
16. 84 feet long, 45 feet wide.

At $2.40 per square yard, find the cost of an asphalt paving for a street:

17. 100 feet long, 40 feet wide.
18. 150 feet long, 32 feet wide.
19. 250 feet long, 50 feet wide.
20. 100 yards long, 50 feet wide.
21. 175 yards long, 60 feet wide.
22. 450 feet long, 75 feet wide.
23. 1000 feet long, 50 feet wide.
24. 1500 feet long, 60 feet wide.
25. One half mile long, 60 feet wide.
26. Two miles long, 50 feet wide.
27. For a rectangular court 100′×60′.
28. For a square court 108 feet on each side.

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If brick cost \$32.00 per thousand laid, and each brick has a surface of 9″×3″, find the cost of paving a street:

29. 100 feet long, 45 feet wide.
30. 150 feet long, 48 feet wide.
31. 200 feet long, 60 feet wide.
32. 500 feet long, 60 feet wide.
33. 1200 feet long, 50 feet wide.
34. 1 mile long, 75 feet wide.

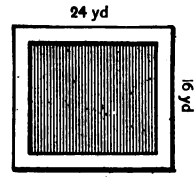
At \$0.90 per square yard find the cost of a macadam road:

35. 2 miles long, 16 feet wide.
36. $1\frac{1}{2}$ miles long, 20 feet wide.
37. 4 miles long, 18 feet wide.
38. 5 miles long, 16 feet wide.
39. 8 miles long, 18 feet wide.
40. 10 miles long, 16 feet wide.

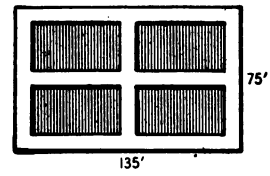
41. The courtyard of an apartment house is 42 ft. 3 in. long and 31 ft. 9 in. wide. Find the cost of paving this court with concrete at \$1.44 per square yard.

42. The floor of a manual training room is 36 ft. long and 24 ft wide, and the cost of paving it with concrete was \$168. How much did the paving cost per square yard?

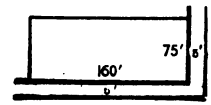
43. The figure at the right represents a grass-plot surrounded by a concrete walk. How many square yards are there in the surface of the walk if it is 3 ft. wide?



44. The figure at the right represents a courtyard, with concrete walks as indicated, each 5 ft. wide. How many square yards are there in the walks?



45. A houseowner builds a concrete walk on the two sides of his corner property. With the dimensions as in the figure, how much does the walk cost him at \$.24 per square foot?



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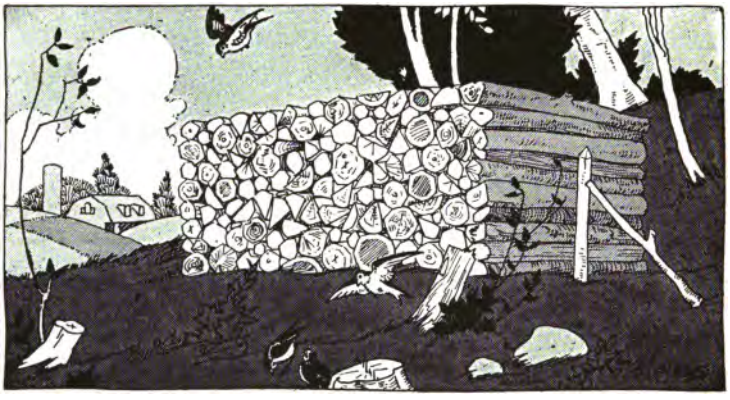
VOLUME MEASURE

I. The Farmer's Applications

(*a*) **Wood Measure.** The unit of measure for firewood is the Cord.

A cord of wood is 8 feet long, 4 feet high, and 4 feet wide.



The length of the sticks, 4 feet, is a standard length. When wood is cut in lengths shorter than 4 feet, a pile 8 feet long and 4 feet high is often considered a cord; and the price is determined by the length of the sticks.

BLACKBOARD PRACTICE

Find the number of cords of 4-foot wood in a pile:

1. 16′×4′.	**4.** 16′×5′.	**7.** 30′×4′.	**10.** 62′×5′.
2. 12′×4′.	**5.** 12′×6′.	**8.** 36′×5′.	**11.** 80′×4′.
3. 24′×4′.	**6.** 24′×5′.	**9.** 40′×6′.	**12.** 72′×8′.

Find the number of cords of 16-inch, or stove-wood, in a pile:

13. 12′×4′.	**16.** 18′×5′.	**19.** 36′×4′.	**22.** 80′×5′.
14. 16′×4′.	**17.** 20′×5′.	**20.** 40′×5′.	**23.** 100′×6′.
15. 18′×6′.	**18.** 24′×6′.	**21.** 48′×6′.	**24.** 120′×4′.

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25. A load of wood measures 5 feet long, 4 feet wide, and 3 feet high. How much must I pay for six such loads at $4.50 per cord?

26. A wagon-box is filled with 4-foot wood. The inside of the box is 8 feet long, 4 feet wide, and 3 feet deep. How many cords can be delivered by this wagon in seven loads?

(*b*) **Capacity of Bins.** While wheat, corn, and other grains are usually sold by weight, the farmer has frequent need for estimating the quantity of grain in his bins without actually weighing it. For this purpose he makes use of the approximate relations between such units as the bushel and the cubic foot, and he has frequent need for the following:

Approximate Equivalents

1 bushel shelled grain = $1\frac{1}{4}$ cu. ft.

1 bushel corn on the ear = $1\frac{5}{9}$ cu. ft.

1 ton well-settled hay = 450 cu. ft.

It will be helpful to remember that:

1. The capacity of a bin in bushels $= \frac{4}{5}$ of the number of cubic feet in its volume.
2. The number of cubic feet for a given number of bushels $= \frac{5}{4}$ of the number of bushels given.

BLACKBOARD PRACTICE

Find the number of bushels of shelled grain that can be placed in a bin whose dimensions are

1. $4' \times 3' \times 3'$.
2. $8' \times 5' \times 3'$.
3. $10' \times 4' \times 4'$.
4. $12' \times 6' \times 5'$.
5. $15' \times 8' \times 9'$.
6. $18' \times 12' \times 10'$.
7. $5'\ 6'' \times 8' \times 4'$.
8. $6'\ 6'' \times 4'\ 6'' \times 3'$.
9. $8'\ 8'' \times 3'\ 6'' \times 4'\ 4''$.
10. $10'\ 8'' \times 6'\ 8'' \times 5'\ 4''$.
11. $12'\ 8'' \times 6'\ 8'' \times 5'\ 4''$.
12. $16'\ 6'' \times 10'\ 4'' \times 8'\ 6''$.

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Find the number of cubic feet of space that must be provided for a quantity of grain amounting to:

13. 250 bu.	**16.** 520 bu.	**19.** 1000 bu.	**22.** 2500 bu.
14. 300 bu.	**17.** 650 bu.	**20.** 1250 bu.	**23.** 3000 bu.
15. 450 bu.	**18.** 900 bu.	**21.** 1500 bu.	**24.** 7200 bu.

II. The Contractor's Applications

(*a*) **Excavations.** The cost of cellars, ditches, wells, etc., is estimated by the cubic yard.

BLACKBOARD PRACTICE

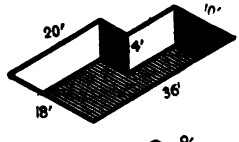
Find the cost, at \$.45 per cubic yard, of excavating a cellar whose dimensions are:

1. 24′×18′×8′.	**5.** 30′×27′×8′.	**9.** 45′×25′ 6″×8′ 6″.
2. 25′×20′×8′.	**6** 38′×27′×8′ 6″.	**10.** 48′×24′ 8″×9′ 6″.
3. 28′×20′×9′.	**7.** 40′×30′×9′ 6″.	**11.** 50′×25′ 6″×9′ 6″.
4. 30′×25′×9′.	**8.** 45′×32′×19′.	**12.** 64′ 6″×48′ 6″×9′.

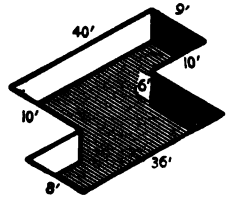
Find the cost, at \$.45 per cubic yard, of digging a ditch:

13. 45′ long, 5′ deep, 2′ wide. **15.** 65′ long, 6′ deep, 2′ wide.
14. 48′ long, 4′ deep, 2′ wide. **16.** 75′ long, 4′ deep, 3′ wide.
17. 2 miles long, 4′ deep, 1′ 6″ wide.
18. 3 miles long, 4′ 8″ deep, 1′ 6″ wide.

19. What is the cost of a mile of ditch $1\frac{1}{2}$ feet wide and 3 feet deep at \$.40 per cubic yard?



20. With the dimension shown in the figure at the right, find the cost of digging the cellar at \$.45 per cubic yard.



21. A contractor digs a cellar with dimensions as in the figure at the right, and he receives \$.35 per cubic yard for the work. How much does he receive?

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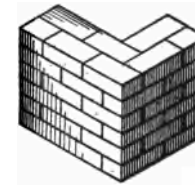
(*b*) **Brickwork.** The number of bricks required for a wall is estimated by the thousand.

As a rule, a cubic foot of wall laid in mortar requires 22 bricks of the ordinary size. This size is 8″ long, 4″ wide, and 2″ thick.

In estimating the cost of labor in bricklaying it is common practice to measure the outside of the walls, which really results in counting each corner twice. As the construction of a corner makes additional work, this practice is considered fair to both the workman and the builder. And no allowance is made for openings, because the openings add greatly to the labor required.



In estimating the amount of material in brickwork the corners are counted only once, and some allowance is made for the openings. There is some variation in the methods used in different localities, and your own local contractors will give you the method they use.

Illustrations:

1. Find the number of bricks required for a wall 36 feet long, 14 feet high, and 1 foot thick, allowing 22 bricks to the cubic foot. Find the cost of labor at $8.25 per thousand.

For the number of bricks: $36 \times 14 \times 1 \times 22 = 11.088$ M.
For the labor: $11.088 \times \$8.25 = \$91.48$.

2. Find the number of bricks and the cost of the labor at $7.90 per thousand for a foundation wall of a house 28′ by 20′, the walls to be 8′ high and 1′ thick.

Outside measure of wall = perimeter $= 2 \times (28 + 20)$ ft. $= 96$ ft.
For the number of bricks: $96 \times 8 \times 1 \times 22 = 16.896$ M.
For the labor: $16.896 \times \$7.90 = \$133.48$.

To estimate the number of brick needed for a wall we may use the following rules:

If the wall is to be 1 brick thick, multiply the area of the wall in square feet by 7; if the wall is to be 2 bricks thick, multiply the area of the wall by 14; if it is to be 3 bricks thick, multiply the area of the wall by 21, etc.

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BLACKBOARD PRACTICE

Find the number of bricks required for a wall with the following dimensions, using 22 bricks per cubic foot:

1. 30′ long, 12′ high, 1′ thick.
2. 35′ long, 10′ high, 1 thick.
3. 40′ long, 9′ high, 18″ thick.
4. 45′ long, 12′ high, 15″ thick.
5. 50′ long, 16′ high, 1′ thick.
6. 75′ long, 6′ high, 1′ thick.
7. 100′ long, 4′ high, 1′ thick.
8. 120′ long, 5′ high, 15″ thick.
9. 200′ long, 4′ high, 1′ thick.
10. 200′ long, $4\frac{1}{2}$′ high, $1\frac{1}{2}$′ thick.

If bricks are laid on edge, and if six bricks are allowed to every square foot of surface, find the number of bricks necessary for a driveway:

11. 60′ long, 7′ wide.
12. 75′ long, 8′ wide.
13. 90′ long, 6′ 6″ wide.
14. 100′ long, 7′ wide.
15. 125′ long, 8′ wide.
16. 130′ long, 9′ wide.
17. 145′ long, 10′ wide.
18. 150′ long, 10′ 6″ wide.

19. The rear wall of a storage house is 40′ wide, 34′ high, and 8″ thick. Find the number of bricks necessary to build the wall.

20. How many bricks are needed for a wall 3 ft. high, 240 ft. long, and 2 bricks thick?

21. A floor is supported through the center by four brick piers, each 12 feet high and 2 feet square. The bricks cost $7.75 per thousand, and the mortar and labor together cost $8.50 per thousand. Find the total cost of the four piers.

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(*c*) **Concrete.** Concrete walls and foundations are estimated by the cubic yard.

In concrete work the labor of making the "forms," the wooden frames into which the wet concrete is poured, is estimated and included by the contractor in his price. Corners are counted but once, and as a rule an allowance is made for the openings. The variations in the price of concrete work are due to the character of the mixture, some concrete being much richer in cement than others, and some being mixed with crushed stone instead of the cheaper cinders or sand and gravel.

Illustration:

The concrete foundation walls for an assembly hall 100 feet long and 50 feet wide are 8 feet high and 1 foot 6 inches thick. The corners are counted only once, and there are ten openings each 4 feet long and 2 feet deep. Find the total cost of the concrete walls at $7.50 per cubic yard.

Counting the corners only once, the total length of the finished wall is $2(100+50)$ ft. -4(thickness of wall in feet) $=300$ ft. -6 ft. $=294$ ft.

Volume of wall $(294\times8\times1.5)$ cu. ft. $=3528$ cu. ft.

Deducting openings: 10 openings each 4 ft. long, 2 ft. deep, and 1 ft. 6 in. thick:

$$10\times4\times2\times1.5 \text{ cu. ft.} = 120 \text{ cu. ft.}$$

Changing to cubic yards, 3408 cu. ft. $=126\frac{3}{9}$ cu. yd.

For the cost: $126\frac{3}{9}\times\$7.50=\$946.67$. *Result.*

BLACKBOARD PRACTICE

Counting the corners but once and making no allowance for openings, at $7.50 per cubic yard find the cost of 18-inch foundation walls for a building:

1. 30 feet long, 18 feet wide, 8 feet high.
2. 36 feet long, 20 feet wide, 9 feet high.
3. 40 feet long, 24 feet wide, 8 feet high.
4. 60 feet long, 32 feet wide, 9 feet high.
5. Find the cost of a concrete wall 150 feet long, 6 feet high, and 10 inches thick, at $8.25 per cubic yard.

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6. Find the cost of a concrete wall 200 ft. long, 3 ft. high, and 1 ft. wide, at $9.25 per cubic yard.

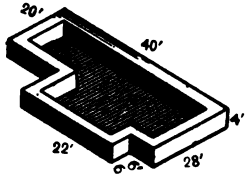
7. Find the cost of 24 concrete posts, each 6 ft. long, and 1 ft. square, at $.72 a cubic foot.

8. Find the cost of a concrete pillar 12 ft. high, 2 ft. wide, and 1 ft. thick, at $16.75 per cubic yard.

9. 100 concrete fence posts were each 7 ft. long, and 6 inches square. At $.45 a cubic foot, how much did the posts cost?

10. A wall is 240 ft. long, 5 ft. high, 2 ft. wide at the bottom, and 1 ft. wide at the top. Using the "average thickness" of the wall, how many cubic yards of concrete are there in it?

11. The figure at the right represents a foundation wall for a house. The wall is 15 inches thick, and costs $8 per cubic yard. No allowance being made for openings, find the total cost of the wall.



12. In concrete work a 1 : 2 : 4 mixture is a mixture made of 1 part cement, 2 parts sand, and 4 parts crushed stone. If a contractor uses this mixture for a wall containing 1400 cubic feet, how much cement, how much sand, and how much crushed stone does he use?

13. A contractor builds a wall for a house 48 feet long and 28 feet wide, the wall being 8 feet high and 18 inches thick. He makes no allowance for openings, and uses a 1 : 2 : 4 mixture. At $1.30 per bag, what does the cement cost him, a bag of cement being equal to 1 cubic foot?

(*d*) **Stonework.** Stonework is estimated by the Perch, or by the cubic foot, or by the cubic yard.

A Perch is a volume of 24^{+} cu. ft.

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This unit is based upon a volume $16\frac{1}{2}$ ft. by $1\frac{1}{2}$ ft. by 1 ft. = $24\frac{3}{4}$ cu. ft.

For convenience in figuring, a perch is often estimated at 25 cubic feet.

The labor for stonework is estimated exactly as it is for brickwork, the corners are counted twice, and no allowance is made for openings.

Material for stonework is estimated exactly as it is for brickwork, the corners are counted only once, and an allowance is made for the openings.

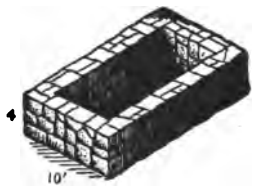
BLACKBOARD PRACTICE

Find, to the nearest perch, the stonework in a wall:

1. 40 ft. long, 5 ft. high, and 1 ft. thick.
2. 75 ft. long, 6 ft. high, and 1 ft. 6 in. thick.
3. 100 ft. long, 4 ft. 6 in. high, and 1 ft. 8 in. thick.
4. 225 ft. long, 5 ft. 6 in. high, and 1 ft. 4 in. thick.
5. 240 ft. long, 3 ft. 8 in. high, and 1 ft. 3 in. thick.
6. 400 ft. long, 4 ft. 4 in. high, and 1 ft. 6 in. thick.

Find, to the nearest perch, the stonework in a 24-inch foundation wall:

7. 8 ft. high, for a house 32 ft. by 24 ft.
8. 9 ft. high, for a house 45 ft. by 24 ft. 6 in.
9. 8 ft. 6 in. high, for a house 45 ft. by 32 ft.
10. 9 ft. 6 in. high, for a house 65 ft. 6 in. long by 32 ft. wide.
11. 8 ft. 9 in. high, for a house 45 ft. long and 28 ft. 6 in. wide.
12. 9 ft. 9 in. high, for a house 54 ft. 6 in. long and 24 ft. 8 in. wide.
13. Find the number of perches of stonework in a foundation wall represented by the figure at the right, the walls being 1 ft. thick.
14. If the mason received \$1.90 per perch for the labor, what was the cost of the labor for the foundation wall represented by the figure at the right?



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UNITED STATES GOVERNMENT WAR-SAVINGS CERTIFICATES

The War-Savings Plan was established by the United States Government to encourage habits of saving, and to provide money for increased government expenses. Under the plan **Thrift Stamps** are sold for twenty-five cents each, and with the first purchase a **Thrift Card** is given to the investor. When filled the Thrift Card holds sixteen Thrift Stamps, and represents a saving of four dollars. Thrift Stamps do not earn interest, but the **War-Savings Stamps** for which they are exchanged bear interest until the Government pays the loan four years later. Thus, the series of War-Savings Stamps which was sold in 1918 will be paid by the Government on January 1, 1923, and the series sold in 1919 will be paid on January 1, 1924.

A War-Savings Stamp is purchased with a filled Thrift Card, or with cash; and costs the investor four dollars plus a small number of cents as indicated in the table:

Cost of War-Savings Stamps in the Year 1919

January	**$4.12**	**May**	**$4.16**	**September**	**$4.20**
February	**4.13**	**June**	**4.17**	**October**	**4.21**
March	**4.14**	**July**	**4.18**	**November**	**4.22**
April	**4.15**	**August**	**4.19**	**December**	**4.23**

The stamps of this series will be worth $5.00 on January 1, 1924.

A War-Savings Certificate contains spaces for twenty stamps, and, when filled, represents the saving of eighty dollars, increased by the total of the cents paid for the stamps in the months in which they were purchased. An investor may buy a quantity of War-Savings Stamps at one time, without using the Thrift Cards; but no investor may hold more than two hundred War-Savings Stamps of a single issue.

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Investment Value of War-Savings Stamps and Certificates

Suppose you buy in the month of November, 1919, enough War-Savings Stamps to fill a War-Savings Certificate.

Your twenty stamps will cost $20 \times \$4.22 = \$84.40$.

On January 1, 1924, the Government will pay you $100 for your Certificate.

Therefore, the profit on your investment will be

$$\$100.00 - \$84.40 = \$15.60.$$

WRITTEN APPLICATIONS

1. Find the cost of 5 War-Savings Stamps bought in June, 1919.

2. Find the cost of 24 War-Savings Stamps bought in July, 1919.

3. Find the cost of 20 War-Savings Stamps bought in September, 1919.

4. Find the cost of 75 War-Savings Stamps bought in November, 1919.

5. Find the profit that will be made on 20 War-Savings Stamps bought in July, 1919, if the investor holds them until January 1, 1924.

6. How much profit will an investor make if he buys 20 War-Savings Stamps in August, 1919, 20 more in October, 1919, and 20 more in December, 1919, and holds them all until January 1, 1924?

7. A boy saved $1 each week for 15 weeks, and then $2 each week for 24 weeks. If he bought War-Savings Stamps with this money, purchasing them all in September, 1919, how much did he pay for them? How much profit will he make on them, if he holds them until January 1, 1924?

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THE SOLUTION OF PROBLEMS BY ANALYSIS

I. ORAL ANALYSIS

In solving a problem we may use two different methods, and by either method arrive at the same result.

Indirect Analysis. Illustration:

50 books cost a dealer \$100. How much will 75 books cost at the same price per book?

In working indirectly, we first find the cost of one book.

If 50 books cost \$100, 1 book costs \$100 divided by 50, or \$2.

If 1 book costs \$2, 75 books will cost 75 times \$2, or \$150. *Result.*

This method is often called Unitary Analysis, for in using it we seek for information concerning *a single article.*

Direct Analysis. Illustration:

50 books cost a dealer \$100. How much will 75 books cost at the same price per book?

In working directly we begin with the number asked for by the question.

How is 75 related to 50? 75 is $\frac{75}{50}$ or $\frac{3}{2}$ of 50.

50 books cost \$100; therefore, 75 books will cost $\frac{3}{2}$ of \$100, or \$150. *Result.*

This method is often called Comparison, for in using it we compare the given number and the required number and obtain thereby a relation between a given result and the required result.

Method and Accuracy. If you were required to find the cost of 91 books in the above illustration, you would have to use the indirect analysis. Use your judgment in selecting the method best suited to a problem. But remember, always, that ACCURACY is absolutely essential, and that a result is either exactly right or wholly wrong.

Estimating the Answer. Practical business men frequently make a rough estimate of an answer before making a careful solution. Unreasonable answers are avoided if an estimate of a result is made in advance.

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ORAL PRACTICE

(The following exercises should be solved indirectly)

1. If 5 pencils cost 25 cents, how much will 8 pencils cost?
2. If 7 oranges cost 28 cents, how much will 10 oranges cost?
3. If 4 coats cost $12, how much will 9 coats cost?
4. If 9 books cost $18, how much will 13 books cost?
5. If 8 barrels of flour cost $48, how much will 5 barrels cost?
6. If 7 tons of coal cost $35, how much will 4 tons cost?
7. How much will 8 bushels of oats cost if 11 bushels cost $6?
8. $180 were paid for 4 cows. How much will 7 cows cost at the same rate?
9. 9 pigs were sold for $54. At the same rate how much will 13 pigs sell for?
10. How much will 15 chickens cost if 9 chickens cost $4.50?
11. If 1 pound of coffee costs 30¢, how many pounds can be bought for $1.50?
12. What is the cost of 12 lemons when they are selling at 5 for 10 cents?
13. How much must I pay for 15 towels when they are selling at 4 for $1?
14. How many dozen eggs can be bought for $1.75 if $1 will buy 4 dozen?
15. If a bus runs 4 blocks in 5 minutes, how many blocks can it run in 30 minutes?

(The following exercises should be solved directly)

16. If 6 pounds of tea cost $3, how much will 12 pounds cost?
17. If 9 pencils cost 27 cents, how much will 3 pencils cost?
18. If 16 cards cost 80 cents, how much will 8 cards cost?
19. If 12 bottles of ink cost 60 cents, how many bottles can be bought for 40 cents?
20. If 18 pounds of butter is sold for $5.40, for what amount will 27 pounds sell?

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21. How much must I pay for 18 tablets, if 12 tablets are sold for $.96?

22. If oranges are selling at 3 for 10 cents, how much will 30 oranges cost?

23. If oranges are selling at 4 for 15 cents, how much will 4 dozen oranges cost?

24. If an automobile runs 45 miles in 3 hours, how far will it run in 6 hours?

25. How much must I pay for 6 oranges, if they are selling at 4 for 10 cents?

26. What is the cost of 7 pounds of butter, if 14 pounds cost $3.00?

27. If a wheel turns 18 times in 10 seconds, how many turns will it make in 1 minute?

28. If a watch loses 1 minute in 36 hours, how much does it lose in 6 days?

29. If 100 lb. of meat cost $18, how much will 75 lb. cost?

30. If a trolley car runs 4 miles in 30 minutes, how far does it run in 3 hours?

(Select the better method for solving the following exercises)

31. A boy bought 5 pencils at 3 cents each and had 10 cents left. How much money did he have before he bought the pencils?

32. A boy earned $1 and with it bought 2 books at 35¢ each. How much of it had he left?

33. A girl gave a merchant a 2-dollar bill in payment for 3 hair ribbons costing 40¢ each. How much should she receive in change?

34. A grocer bought 25 pounds of butter at 30¢ per pound, and sold it for 34¢ per pound. How much did he make on the transaction?

35. A boy spent $1.25 for feed for his chickens, and sold 5 dozen eggs at 40¢ per dozen. How much more did the eggs bring than the feed cost?

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36. Working Saturdays in a store a boy earned 50¢ on each of 28 days. He bought a coat from his employer for $9. How much was then due him in cash?

37. A man earned $1000 in one year, and spent $750 for his board, clothing, and general expenses. If he saved the balance, what fraction of his earnings did he save?

38. A man spent $\frac{3}{4}$ of his income on his board, clothing, and general expenses, and saved the balance which was $250. How much did he spend?

39. Two thirds of a man's money amounts to $4000, and is invested in Liberty Bonds. If the balance is in the bank, how much has he in the bank?

40. A watch gains $\frac{1}{2}$ of a minute in 3 days. At this rate how many minutes will it gain in 30 days?

41. If $1\frac{1}{4}$ pounds of meat cost a housekeeper $.25, what was the cost of the meat per pound?

42. A man bought a lot for $2000, and on the lot he built a house costing $6000. What part of the cost of the house is the cost of the lot?

43. In the preceding problem find what part of the total cost of the house and the lot is represented by the cost of the lot; by the cost of the house.

44. One third of a man's property is left to his family, and the family receives $40,000. What is the total amount of the property he left?

45. Two fifths of the value of a farm is invested in the buildings, which cost the owner $3000. What is the total value of the land and the buildings?

46. A builder paid $12 per thousand for 15,000 brick, and one half the cost of the brick for a concrete foundation. What was the total cost of the brick and the concrete foundation?

47. 30 men can dig a trench for a sewer in 5 days. In how many days can 10 men dig this trench?

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48. 10 men can dig a field of potatoes in 4 days. In how many days can 5 men dig the same field of potatoes?

49. It took 5 men 3 hours to load a car with lumber. On the following day 2 men load a similar car with lumber. How long does it take them?

50. 3 men work 8 hours a day to paint a house in 5 days. 4 men are working 10 hours a day to paint the same kind of house. In how many days should the second house be painted?

II. WRITTEN ANALYSIS

The Problems of Everyday Life in business and in the home are mainly of a simple character, and a great majority of them require nothing beyond an intelligent use of arithmetic through fractions and common measures.

Accuracy is the first essential in all arithmetical processes.

Systematic Arrangement is one of the greatest aids to accuracy.

The student should learn at the outset:

1. To be accurate.
2. To be systematic.
3. To test every result by checking.

Business men do not forget or forgive errors, — they replace inaccurate clerks with those who are accurate and careful. Speed is desirable if it is not obtained at a sacrifice of accuracy, but a person's accuracy and not his speed determine his real worth in a business house.

Checking is the practice of testing each step of the work after it is completed. In engineering, every process and calculation is checked by another person or persons after the engineer or draftsman has made the original calculations, and a careful record is kept of the individuals who have taken part in every problem. Careless and inaccurate assistants are speedily dismissed, since errors might cause the loss of thousands of dollars or hundreds of lives.

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Systematic Method. In the following exercises:

1. *Read the problem carefully.*
2. *Estimate the answer in advance when it can be done.*
3. *Make a careful, systematic solution.*
4. *Check every step in the process and in the calculations.*

WRITTEN APPLICATIONS

I. The Business Man's Use of Analysis

1. 60 overcoats are bought for \$8 each, and are sold at a gain of \$2.50 each. How much is received for all of the coats?

2. A bag of salt containing 100 pounds costs a grocer \$.75. How much does he gain by selling the salt in 10-pound bags at 12 cents each?

3. From a bolt of bunting containing 48 yards, a dealer sold 16 yards. What fractional part of the whole bolt did he sell?

4. If a merchant buys dress goods at \$1.20 per yard and sells it at \$1.45 a yard, how many yards must he sell to gain a profit of \$10?

5. 10 dozen eggs are bought by a country grocer for \$.55 per dozen. How many yards of cloth at \$.06¼ per yard should he give in payment for the eggs?

6. A coal dealer bought 50 tons of coal at the mine for \$4.25 per ton. The freight being \$.42 per ton, how much did the coal cost him at his yard?

7. A dry goods merchant sold ¾ of a yard of silk for \$2.25. At the same rate, how much should he charge for 2¼ yards of the silk?

8. A piano dealer offers to sell a piano for \$400 cash, or for \$450 in installments. What fractional part of the cash price does he add to the price if he is paid in installments?

9. A dealer in sewing machines received 24 payments of \$1.25 each for a machine. His profit on this machine was \$10. How much did the machine cost him?

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10. A shoe dealer sold 45 pairs of slippers at $\frac{3}{4}$ of the regular price. If he received \$45 for them, what was the regular price?

11. A merchant's profits for the year amount to \$2160, and this amount exceeds by $\frac{1}{8}$ his profits for the preceding year. How much were his profits during the preceding year?

12. From a ham weighing $16\frac{3}{4}$ pounds, a butcher sold $2\frac{1}{2}$ pounds to one woman and half as much to another. How many pounds of this ham were left?

13. An insurance firm charges 6¢ per \$100 for liability insurance on a certain risk. How much do they charge for such insurance amounting to \$12,500?

14. Three sales of $5\frac{1}{2}$ yards, 8 yards, and $12\frac{3}{4}$ yards, respectively, were made from a bolt containing 40 yards of cloth. How much is the rest of it worth at 24¢ per yard?

15. A dealer bought a box containing 40 dozen lemons for \$6 and then sold them at 25¢ a dozen. How much did he make on them?

16. A dealer in gasoline used a 5-gallon can in emptying a tank, and he filled the can 38 times. How much was the gasoline worth at 14¢ per gallon?

17. A firm paid \$10.76 for a barrel of flour weighing 196 pounds, and sold it all in 8 equal sacks at 8¢ per pound. Find the price of each sack and also the gain on all of it.

18. 25 cases of a cereal, containing 2 dozen packages to the case, cost \$1 a dozen. $\frac{1}{8}$ of the cereal spoiled, and the rest was sold at 15¢ per box. Did the grocer gain or lose, and how much?

19. A coal company bought 600 long tons of coal, 2240 pounds to the ton, at \$4 per ton; and sold it all by the short ton of 2000 pounds, receiving \$4.50 per ton for it. What was the profit on this transaction?

20. Two men together own $\frac{5}{12}$ of a motorboat, and one of them owns 3 times as large a share as the other. What fraction of the value of the boat does each man own?

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II. The Housekeeper's Use of Analysis

21. A lady paid \$1.00 for $\frac{2}{3}$ of a yard of silk. What was the cost of the silk per yard?

22. A woman exchanged 10 yards of cloth worth 45¢ a yard for handkerchiefs worth \$2.25 per dozen. How many handkerchiefs should she receive?

23. A butcher charged 48¢ for a slice of ham weighing $1\frac{1}{2}$ pounds. How much is this per pound?

24. A woman exchanged 8 dozen eggs worth 55¢ a dozen for granulated sugar worth 9¢ a pound. How many pounds of sugar should she receive?

25. A housekeeper found that 8 tons of coal at \$5 per ton supplied her range for one year. How much did the coal cost to supply the range one week?

26. A woman finds that her stair carpet must cover 15 stairs, and she allows $\frac{2}{3}$ yd. for each stair. How much does the carpet cost at \$1.75 per yard?

27. In canning fruit a woman uses 24 pounds of sugar, which is one half of the weight of the fruit used. How much does the fruit cost at 8¢ per pound?

28. If the sugar used in problem 27 cost $9\frac{1}{2}$¢ per pound and the fruit filled 40 cans, find the total cost of each can.

29. A housewife bought a sewing machine, paying \$1 per week for 28 weeks. The cash price was \$23.50. How much could she have saved by paying cash?

30. In a month of 30 days a family used 2 quarts of milk and 9¢ worth of cream daily. What is their expense for milk and cream, if the milk costs 14¢ a quart?

31. A wife has \$16 a week with which to pay her maid and buy groceries, meat, and milk. $\frac{1}{4}$ of this amount goes to her maid, \$7.25 for groceries, and the rest for meat and milk. How much is paid for these last two items?

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32. A dealer sold a housekeeper $4\frac{1}{2}$ dozen eggs at 30¢ a dozen, $\frac{1}{2}$ bushel of potatoes at \$.80 a bushel, and $\frac{1}{2}$ peck of apples at 30¢ a peck. How much change should he give her from a two-dollar bill?

33. A widow spends $\frac{1}{4}$ of her annual income for rent, \$25 a month for food, and \$50 a year for clothing. The balance, \$160, she saves each year. Find the amount of her annual income.

34. A woman finds that she can rent an apartment for \$24 per month, including the heat, or that she can rent a small house for \$18 per month which will require 9 tons of coal at \$7 per ton during the year. Which is the cheaper plan for her to follow if no charge is made for caring for the heater?

35. A housekeeper owning her house rents two rooms for \$2.25 a week each, two others at \$2 a week, and a fifth at \$1.75 a week. Her annual expenses for running the house are: Taxes, \$65.40; interest, \$54; repairs, \$20; and heat and light, \$100. How much profit does she make in one year by renting these rooms?

III. The Farmer's Use of Analysis

36. A quart of milk weighs about 2.2 pounds. How many quarts are there in 125 pounds of milk?

37. A farmer sold 11 head of cattle, which was 5 more than $\frac{1}{4}$ of his whole herd. How many cows were there in his herd?

38. A farmer finds that $\frac{3}{5}$ of an acre produces 90 bushels of potatoes. At that rate how many bushels do 6 acres produce?

39. $\frac{3}{5}$ of a pasture was sold to a farmer, and the man who sold it to him retained 10 acres of it for his own use. How many acres were there in the original pasture?

40. $\frac{2}{3}$ of a crop of grain was sold for \$50, and the 50 bushels remaining unsold brought \$30 later in the season. What prices were received per bushel at each sale?

41. A real estate operator offers a farmer \$2000 for $\frac{1}{5}$ of his land, and \$60 an acre for the remaining 80 acres. What is the total offer for all of the land? What is the offer per acre for $\frac{1}{5}$ of the land?

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42. A farmer requires some "1–2–4" concrete; that is, 1 part cement, 2 parts sand, and 4 parts gravel. If he uses 1 barrel of cement and 2 barrels of sand, how many barrels of gravel should he use?

43. If he requires 700 cubic feet of concrete, how many cubic feet of each ingredient will he use? (A "1–2–4" mixture has 1 part + 2 parts + 4 parts, or 7 parts in all.)

44. A farm laborer earns $1.25 a day besides his board, and his average daily expense for clothing, etc., is $.50. He already has $55 and wishes to save enough to make it $100. In how many days should he save the desired amount?

45. A farmer has 720 bushels of wheat, and two thirds of the number of bushels of wheat he has is equal to three fourths of the number of bushels of corn that he must buy. How much will this corn cost him at $.60 per bushel?

46. A farmer offers to exchange a quantity of wheat for a mowing machine costing $45.50, a rake costing $32.00, and a wagon costing $87.00. If his wheat is worth $1.75 a bushel, how many bushels must he give in exchange for these implements?

47. The foundation for a silo requires 280 cubic feet of concrete, and the 1–2–4 mixture is used. Find the amount of cement, sand, and gravel needed to build the foundation.

48. A farmer raises a crop of 110 barrels of fancy apples for which he is offered $2.50 a barrel He declines the offer and buys enough half-peck baskets at 3¢ each, in which to pack the entire crop, each barrel containing $2\frac{1}{2}$ bushels. He retails these baskets of apples at $.20 each. Allowing for the cost of the baskets, how much more did he receive by retailing his apples?

49. A farmer uses a 2-ton truck for delivering 2400 bushels of potatoes, and averages 2 tons to the load on each trip. If the truck uses, on the average, a gallon of gasoline to every 8 miles, and if the distance traveled on each round trip is 5 miles, find the cost of delivering the potatoes; the gasoline costing 22¢ a gallon, and the labor $.75 each trip.

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THE EQUATION

Statement of Equality. When we make a statement that a number of units of one kind is equal to a number of units of another kind we make a *statement of equality*.

For example:

"Ten dimes are equal to one hundred cents."

"Thirty-two ounces are equal to two pounds."

Finding the Value of a Unit. From a statement of equality like those illustrated above, we find the value of a single unit or thing by division.

For example:

We know that 10 dimes are equal to 100 cents.

Expressing the words "are equal to" by the familiar sign of equality, "=,"

We may write 10 dimes = 100 cents.

Dividing by 10, 1 dime = 10 cents

An Equation is a statement of equality between two quantities or between two numbers.

Using a Letter in an Equation. Suppose we make the statement that five wagons cost three hundred dollars. With the symbols that we have long used, we write briefly

5 wagons cost \$300.

Suppose we use the first letter "w" of the word "wagons" and since cost means "cost an amount equal to" use the sign "=" for the word "cost."

We have then, $5\,w = \$300$.

Dividing each by 5, $1\,w = \$60$.

These two "equations" are the briefest possible form of expression for the statement in analysis that "If 5 wagons cost \$300, 1 wagon will cost $\frac{1}{5}$ of \$300, or \$60."

Applying the same principle:

If $4\,a = 20$,	If $5\,b = 60$,	If $10\,x = 70$,	If $4\,x = 56$,
$a = 5$.	$b = 12$.	$x = 7$.	$x = 14$.

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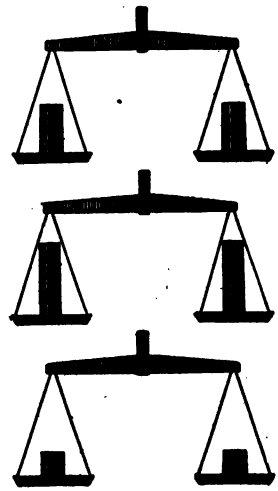
Comparison of an Equation and a Balance. A little investigation will convince you that the members of an equation may be compared to the weights on the pans of a balance.



Suppose in (*a*) that each pan of the balance has 4 one-pound weights. Then, the scales exactly balance

And, arithmetically, $4 = 4$. (*a*)

Suppose in (*b*) that we add two more one-pound weights to each pan. The scales are still in exact balance.

And, arithmetically, $4 = 4$.
Adding, $2 = 2$. (*b*)
Or, $6 = 6$.

Suppose in (*c*) that we have removed two one-pound weights from the original number in each pan. The scales are still in exact balance.

And, arithmetically, $4 = 4$. (*c*)
Subtracting, $2 = 2$.
Gives $2 = 2$.

Clearly, therefore,

1. *If equals are added to equals, the sums are equal.*
2. *If equals are subtracted from equals, the remainders are equal.*

If we multiply the original number of weights in each pan by the same number, we shall have the same number in each.

That is, originally, $4 \text{ lb.} = 4 \text{ lb.}$
And for twice as many weights in each, $2 \times 4 \text{ lb.} = 2 \times 4 \text{ lb.}$
Or, $8 \text{ lb.} = 8 \text{ lb.}$

Also, if we divide the original number of weights in each pan by the same number, we shall continue to have the same number in each.

That is, originally, $4 \text{ lb.} = 4 \text{ lb.}$
And for half as many weights in each, $4 \text{ lb.} \div 2 = 4 \text{ lb.} \div 2$.
Or, $2 \text{ lb.} = 2 \text{ lb.}$

Clearly, therefore,

3. *If equals are multiplied by equals, the products are equal.*
4. *If equals are divided by equals, the quotients are equal.*

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Changing a Number from One Member of an Equation to the Other Member.

We know that $10+4=14.$ (1)

Subtracting 4 from each member,

$$10+4-4=14-4.$$

Simplifying, since $4-4=0,$ $10=14-4.$ (2)

Observe that in (1) 4 is seen in the first member with a + sign, and in (2) 4 is seen in the right member with a − sign.

Therefore, 4 might have been written in the other member with its sign changed from + to −.

Again, take $10-4=6.$ (1)

Adding 4 to each member, $10-4+4=6+4.$

Simplifying, since $4-4=0,$ $10=6+4.$ (2)

Here, also, −4 might have been written in the second member with its sign changed from − to +.

Hence, a number may be changed from one member of an equation to the other if its sign is changed either from + to −, or from − to +.

Unknown Numbers. Each of the following equations has a letter whose value is unknown. Finding the value of an unknown letter is **solving an equation.**

Illustrations:

1. Solve $x+4=10.$

Change +4 to the other member and change its sign,

$$x=10-4.$$

Subtracting, $x=6.$ *Result.*

2. Solve $x-3=9.$

Change +3 to the other member and change its sign,

$$x=9+3.$$

Adding, $x=12.$ *Result.*

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3. Solve $2x+3=15$.

Change $+3$ to the other member and change its sign,

$$2x=15-3.$$

Subtracting, $2x=12.$

Dividing by 2, $x=6.$ *Result.*

4. Solve $5x-2=1.$

Change -2 to the other member and change its sign,

$$5x=1+2.$$

Adding, $5x=3.$

Dividing by 5, $x=\frac{3}{5}.$ *Result.*

BLACKBOARD PRACTICE

Solve the following equations:

1. $x+5=9.$	8. $x-3=8.$	15. $5x=35.$	22. $3x-1=14.$
2. $x+6=9.$	9. $x-8=9.$	16. $6x=3.$	23. $4x-3=13.$
3. $x+8=9.$	10. $2x=10.$	17. $4x=7.$	24. $6x+4=16.$
4. $x+2=6.$	11. $2x=16.$	18. $5x=11.$	25. $7x-3=19.$
5. $x+4=8.$	12. $3x=15.$	19. $2x+3=9.$	26. $5x+4=21.$
6. $x-1=9.$	13. $4x=24.$	20. $3x+2=8.$	27. $8x+1=25.$
7. $x-5=7.$	14. $6x=18.$	21. $5x+1=16.$	28. $9x-2=30.$

ORAL PRACTICE

Using Letters to Represent Quantities

1. Numbers of the same kind may be added. Thus: \$10+\$5 = \$15; 10¢+5¢=15¢; 10 lb.+51 lb.= ? lb.; 10 yd.+5 yd.= ? yd.

2. Can you add \$5 and 3 yd.? 10 yd.+3 lb.? 8 rd.+5¢? Why is it that you cannot add in such cases?

3. Sometimes we *indicate* a sum because we cannot add the forms that are given. Thus, we may be asked for the sum of 10 ft.+6 in. We cannot add, however, until both measures are expressed in the same unit.

4. Read these *indicated sums.* 5 yards+12 feet. 8 rods+ 5 yards+2 feet. 2 ton+3 hundredweight+75 pounds.

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5. Can you see that each of the expressions in problem 4 is merely a compound denominate number with the plus sign between the denominations?

6. If you are 12 years old now, how old will you be in 5 years? 12 years+5 years may be written in the form (12+5) years.

7. If you are 12 years old now, how old will you be in x years? 12 years+x years may be written in the form (12+x) years. Can you do any more than merely indicate the addition?

8. If you are 12 years old now, how old will you be in y years? How old will you be in z years? How old will you be in $m+n$ years? How old will you be in $n+p$ years?

9. If you are 12 years old now, how old were you 5 years ago? 12 years−5 years = ? years. 12 years less 5 years may be expressed in the form (12−5) years.

10. If you are 12 years old now, how old were you y years ago? 12 years−y years may be expressed (12−y) years. Give the expression for your age z years ago.

11. Jack has 25 cents and Edith has 20 cents. Both have the sum of 25 cents and 20 cents, or (25+20) cents. How many cents have both? How many more cents has Jack than Edith?

12. Suppose that Jack has 25 cents, and that Edith has x cents. We may express the number of cents that both have by writing (25+x) cents. But until we know how many cents are represented by x cents, can we add the number of cents that both have?

13. Mary solved 15 examples, and Jane solved twice as many. Therefore, Jane solved 2 times 15, or (2×15) examples, or 30 examples. (2×15) is an indicated multiplication, but we can do the work asked for and get what number?

14. Suppose we were told that Mary solved x examples and that Jane solved twice as many. Indicating the number that Jane solved, we write (2 times x) examples. This can be written in the simpler form "2 x." (We mean 2 times 1 hat when we say "2 hats." We mean 2 times 1 x when we say or write "2 x.")

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Using an Equation in Analysis. In solving a problem by using an equation you should observe the following steps.

1. *Read the problem carefully to find what is wanted.*
2. *Represent the unknown number by x.*
3. *The problem will give a statement about the unknown number. Write that statement, using x for that unknown number.*
4. *Solve the equation.*

WRITTEN APPLICATIONS

I. Given: A Product and One of Its Factors.

To Find: The Other Factor.

Illustration:

Five times a certain number is 35. What is the number?

Let	x = the unknown number.
Then	$5x$ = five times that unknown number.
But we know that	35 = five times that unknown number.
Therefore,	$5x = 35$.
Hence,	$x = 7$. *Result.*

1. Seven times a number is 112. What is the number?
2. Nine times a number is 144. What is the number?
3. What number multiplied by 13, gives a product of 156?
4. $175 was divided among 7 children, each receiving the same amount. How many dollars did each receive?
5. 1350 tons of coal were shipped in cars holding 45 tons each. How many cars were required to ship this coal?
6. Twelve laborers were paid $216 for their work on a street improvement. If each laborer received the same amount, how much did each laborer receive?
7. A mile track was laid out in 16 sections for a boys' relay race. If the sections were all equal how many feet were there in each section?

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II. Given: A Sum and One of Its Parts.

To Find: The Other Part.

Illustration:

What number increased by 35 is equal to 90?

Let x = the unknown part.
Then $x+35$ = the sum of both parts.
But we know that 90 = the sum of both parts.
Therefore, $x+35=90$.
Hence, $x=90-35$.
Or, $x=55$. *Result.*

8. What number increased by 25 equals 110?

9. What number increased by 37 equals 145?

10. What number decreased by 21 equals 93?

11. What number decreased by 34 equals 115?

12. A boy earns 45 cents, and then has 88 cents. How much had he at first?

13. A boy saved 98 cents and after buying some school supplies with it had only 62 cents of it left. How much did he spend for the supplies?

14. A thrifty young man finds that by saving \$150 more he will have \$1000. How much has he at the present time?

15. A farmer's daughter canned 145 quarts of strawberries, and kept 60 cans for home use, and sold the rest. How many cans did she sell?

16. A man is 34 years older than his boy, and the sum of their ages is 66 years. How old is the boy?

17. John and his father together weigh 235 pounds, and John weighs 90 pounds. How many pounds does his father weigh?

18. Two men bought a boat and one of them furnished \$24 more of the money than the other. If the boat cost \$136, how much did each man furnish?

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III. Given: The Sum of Two Numbers and Their Relation to Each Other.

To Find: The Numbers.

Illustration:

The sum of two numbers is 45, and one of them is twice the other. What are the numbers?

Let	x = the smaller number.
Then	$2x$ = the larger number.
And their sum is	$= 3x$ (or $x + 2x$).
But we know that their sum	$= 45$ (the given sum).
Therefore,	$3x = 45$.
And	$x = 15$, the smaller number. } *Results.*
Also, 2 times the smaller, or	$2x = 30$, the larger number. }

19. The sum of two numbers is 100, and one of them is three times the other one. What are the numbers?

20. Find two numbers, one of which is five times the other, if the sum of both numbers is 60.

21. A boy is three times as old as his sister, and the sum of their ages is 24 years How old is each?

22. A man is three times as old as his boy, and the sum of their ages is 48 years. Find the age of each.

23. A man weighs three times as much as his boy, and together they weigh 240 pounds. Find the weight of each.

24. Two boys solved a number of problems, and one solved twice as many as the other. Together they solved 60. How many did each solve?

25. A man and a boy working together at a task earned $15. If the man was paid four times as much as the boy was, how much was paid to each?

26. A farmer paid $250 for a horse and a wagon, and the cost of the horse was four times the cost of the wagon. What was the cost of each?

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27. A young man saved a certain sum and bought a house lot. Then he saved five times as much and built a house. The house and the lot cost him $3000. How much did the lot cost? How much did the house cost?

IV. Miscellaneous Problems for Analysis and Solution by Equation.

28. Three times a given number, increased by 10, equals 25. Find the number.

29. Five times a given number, increased by 5, equals 80. Find the number.

30. Four times a certain number, decreased by 10, equals 50. Find the number.

31. Seven times a certain number, decreased by 11, equals 59. Find the number.

32. Three times a number increased by 5 times the same number equals 56. Find the number.

33. Seven times a number decreased by four times the same number equals 36. Find the number.

34. Separate 50 into two such parts that the larger part shall be equal to four times the smaller part.

35. A boy saved $7, which was $1 more than twice the cost of a sled he purchased. What did the sled cost?

36. A man gave 10 cents to each of a number of boys and had 30 cents left. If he had 80 cents at first, how many boys received 10 cents each?

37. A man's salary is $100 and after paying a debt of $10 he deposited five times the amount of the debt in the bank and kept the rest of his salary for expenses. How much did he keep for his expenses?

38. A housekeeper purchased seven dining room chairs and gave a 50-dollar bill in payment. She received in change three 5-dollar bills. What was the price of each chair?

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39. A house and lot together cost $4800, and the house cost 3 times as much as the lot. How much did the lot cost?

40. Jack has 10 cents more than Tom, and Tom has 15 cents more than Harry. If all three have $1.00, how many cents has each boy?

41. Jerry's father weighs 20 pounds more than 3 times as much as Jerry weighs. If both weigh 250 pounds, how much does each of them weigh?

42. Mary made 3 times as many buttonholes as Emily, and Emily made 3 times as many as Kate. If all three made 52 buttonholes, how many did each of them make?

43. Earl and Dick had war gardens, and Earl earned from his garden $8 more than Dick earned from his. How much did each earn, if both earned $28?

44. A truck and its load of coal weigh 7350 pounds, and it was found that the coal weighed just twice as much as the truck. Find the weight of each, and the number of tons of coal in the load.

45. On a pleasure trip a man travels three times as far by rail as he travels by boat, and the entire trip is 160 miles. What is the number of miles traveled by rail and the number by boat?

46. An automobilist finds at the end of the season that his expenses for gasoline were six times the amount paid for repairs. If the amount paid for both gasoline and repairs was $175, how much was paid for each item?

47. A housekeeper finds that her annual expenses for food are twice as much as her expense for rent, and that her expense for clothing and for her maid equals that paid for rent. If she spends $2000 annually, how much does she spend for rent, and for food?

48. A man said to his boy, "Your age is twice that of your sister's, and my age is equal to twice the sum of your age and your sister's age. Also, the sum of the ages of all three of us is 54 years." What is the age of the sister, the age of the boy, and the age of the man?

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PERCENTAGE

Per Cent means hundredths or "*by the hundred.*"

If a merchant made a profit of $10 on every sale of $100 worth of goods, he made "10 on the hundred," or "10 per cent."

If you paid $6 for the use of $100 for a given length of time, you paid "6 on the hundred," or "6 per cent."

The expression "per cent" comes from the Latin "per centum," meaning "by the hundred."

The symbol "%" is used instead of the words "per cent."

Thus: 10 per cent is written 10%.
6 per cent is written 6%.

A per cent may be written in the form of a common fraction.

Thus: $10\% = \frac{10}{100} = \frac{1}{10}$.

$20\% = \frac{20}{100} = \frac{1}{5}$.

$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{1}{3}$, etc.

For the numerator of the fraction write the given number of hundredths. For the denominator of the fraction write 100.

Reduce the fraction to lowest terms.

Per cents may be written in the form of decimals.

Thus: 5% = .05, since "per cent" means "hundredths."
12% = .12.
37.5% = .375, etc.

Move the decimal point in the given per cent two places to the left. Omit the symbol %.

ORAL PRACTICE

Read:

1. 12%.	**5.** $12\frac{1}{2}$%.	**9.** $62\frac{1}{2}$%.	**13.** 99%.
2. 15%.	**6.** $33\frac{1}{3}$%.	**10.** 75%.	**14.** 120%.
3. 18%.	**7.** $37\frac{1}{2}$%.	**11.** $83\frac{1}{3}$%.	**15.** $133\frac{1}{3}$%.
4. 25%.	**8.** 50%.	**12.** $87\frac{1}{2}$%.	**16.** 200%.

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Give, in the lowest terms, the equivalent fractions for:

17. 20%.	**21.** 50%.	**25.** 15%.	**29.** 55%.
18. 25%.	**22.** 60%.	**26.** 24%.	**30.** 68%.
19. 30%.	**23.** 75%.	**27.** 32%.	**31.** 72%.
20. 35%.	**24.** 80%.	**28.** 48%.	**32.** 84%.

Give the equivalent decimals for:

33. 10%.	**36.** 55%.	**39.** $37\frac{1}{2}$%.	**42.** $\frac{1}{2}$%.
34. 15%.	**37.** 75%.	**40.** $62\frac{1}{2}$%.	**43.** $\frac{3}{5}$%.
35. 20%.	**38.** 85%.	**41.** $87\frac{1}{2}$%.	**44.** $\frac{7}{10}$%.

Common fractions may be expressed as per cents.

Thus: $\frac{7}{100}$ may be written .07 = 7%.
$\frac{19}{100}$ may be written .19 = 19%.
$\frac{5}{8}$ may be written .625 = $62\frac{1}{2}$%.

Change the given fraction to a decimal.
Move the decimal point two places to the right.
Annex the symbol %.

Decimal fractions may be expressed in the form of per cents.

Thus: .12 may be written 12%.
.625 may be written 62.5%, etc.

Move the decimal point two places to the right.
Annex the symbol %.

ORAL PRACTICE

Express as per cent:

1. .10.	**9.** .$03\frac{1}{3}$.	**17.** $\frac{1}{5}$.	**25.** $1\frac{1}{4}$.
2. .25.	**10.** .$66\frac{2}{3}$.	**18.** $\frac{3}{4}$.	**26.** $1\frac{3}{8}$.
3. .35.	**11.** 1.25.	**19.** $\frac{3}{5}$.	**27.** $2\frac{1}{4}$.
4. .375.	**12.** 2.75.	**20.** $\frac{5}{8}$.	**28.** $3\frac{5}{8}$.
5. .875.	**13.** 3.00.	**21.** $\frac{7}{8}$.	**29.** $4\frac{7}{8}$.
6. .625.	**14.** 3.25.	**22.** $\frac{3}{8}$.	**30.** $4\frac{2}{3}$.
7. .725.	**15.** 3.125.	**23.** $\frac{7}{10}$.	**31.** $5\frac{3}{4}$.
8. .935.	**16.** 4.375.	**24.** $\frac{5}{12}$.	**32.** $5\frac{5}{6}$.

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Much labor is saved by using fractional forms for the more common per cents. The following equivalents should be memorized:

TABLE OF EQUIVALENTS

$5\% = \frac{1}{20}$	$16\frac{2}{3}\% = \frac{1}{6}$	$50\% = \frac{1}{2}$
$6\frac{1}{4}\% = \frac{1}{16}$	$20\% = \frac{1}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$
$8\frac{1}{3}\% = \frac{1}{12}$	$25\% = \frac{1}{4}$	$75\% = \frac{3}{4}$
$10\% = \frac{1}{10}$	$33\frac{1}{3}\% = \frac{1}{3}$	$83\frac{1}{3}\% = \frac{5}{6}$
$12\frac{1}{2}\% = \frac{1}{8}$	$37\frac{1}{2}\% = \frac{3}{8}$	$87\frac{1}{2}\% = \frac{7}{8}$

ORAL PRACTICE

1. What is $\frac{1}{10}$ of 100? .1 of 100? 10% of 100?
2. What is $\frac{1}{4}$ of 60? .25 of 60? 25% of 60?
3. What is $\frac{3}{8}$ of 80? .375 of 80? $37\frac{1}{2}\%$ of 80?
4. What is $\frac{5}{8}$ of 200? .625 of 200? $62\frac{1}{2}\%$ of 200?
5. What fractional part of a yard is 1 foot? What per cent of a yard is 1 foot?
6. What fractional part of a gallon is 1 quart? What per cent of a gallon is 1 quart?
7. What fractional part of a foot is 1 inch? What per cent of a foot is 1 inch?
8. A man spends one fifth of his salary for rent, and one fourth of it for food. Express each item as a per cent of his salary.
9. From 12 bushels of seed potatoes a field produced 180 bushels of potatoes. What per cent of the crop was the seed?
10. A profit of $300 was made on a sale amounting to $1500. What was the per cent of profit?
11. Of the 120 spectators at a ball game, 24 were boys. What per cent of the whole number of spectators is the number of boys?
12. A triangle has three equal sides. What per cent of the perimeter of the triangle is in the length of any one side?

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THE THREE CASES OF PERCENTAGE

I. To Find a Per Cent of a Number.

Illustrations:

1. Find 5% of 240.

5% = .05.
240
.05
12.00

Hence, 5% of 240 = 12.

2. Find 12% of 750.

12% = .12.
750
.12
1500
750
90.00

Hence, 12% of 750 = 90.

3. Find $8\frac{3}{4}$% of 96.

$8\frac{3}{4}$% = .0875.
96
.0875
480
672
768
8.4000

Hence, $8\frac{3}{4}$% of 96 = 8.4.

Express the given per cent as a decimal.

Multiply the given number by this decimal.

The number of which a per cent is required is called **the Base.**

In the example at the right 405 is the base.

405 *Base.*
.09 *Rate.*
36.45 *Percentage.*

The number of hundredths in the required per cent is called **the Rate.**

In the example 9 hundredths, or .09, is the rate.

The product of the base by the rate is called **the Percentage.**

In the example 36.45 is the percentage.

Rate × Base = Percentage

ORAL PRACTICE

Find:

1.	5% of 100.	2% of 500.	8% of 40.	20% of 500.
2.	5% of 80.	2% of 800.	8% of 80.	20% of 800
3.	5% of 90.	2% of 20.	8% of 90.	30% of 1[illegible].
4.	5% of 95.	4% of 80.	9% of 60.	30% of [illegible]0.
5.	10% of 200.	4% of 120.	9% of 90.	30% of 400.

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BLACKBOARD PRACTICE

Find:

1. 5% of 80.	8% of 70.	10% of 45.	15% of 75.
2. 6% of 80.	8% of 95.	10% of 110.	16% of 120.
3. 6% of 90.	9% of 109.	12% of 125.	20% of 135.
4. 7% of 90.	10% of 130.	14% of 150.	25% of 165.

5. 15% of \$275.	9. 25% of \$125.50.	13. 40% of \$1250.
6. 15% of \$300.	10. 28% of \$325.75.	14. 60% of \$1500.
7. 18% of \$320.	11. 30% of \$610.50.	15. 75% of \$2150.
8. 20% of \$340.	12. 35% of \$670.75.	16. 80% of \$3750.

17. Find 10%, 20%, 40%, and 80% of \$1000.
18. Find $12\frac{1}{2}$%, 25%, $37\frac{1}{2}$%, and 50% of \$1500.
19. Find $33\frac{1}{3}$%, $66\frac{2}{3}$%, and $87\frac{1}{3}$% of \$2400.
20. Find $6\frac{1}{4}$%, $8\frac{1}{3}$%, $16\frac{2}{3}$%, and $83\frac{1}{3}$% of \$2400.
21. Find 110% of \$500; 125% of \$500; 150% of \$600.
22. Find 10% of 25% of \$200; 25% of 40% of \$2500.

Find:

23. $\frac{1}{2}$% of 20.	27. $\frac{1}{10}$% of 100.	31. $\frac{4}{5}$% of 200.
24. $\frac{1}{2}$% of 30.	28. $\frac{3}{10}$% of 300.	32. $\frac{3}{4}$% of 200.
25. $\frac{1}{4}$% of 20.	29. $\frac{3}{8}$% of 1200.	33. $\frac{2}{3}$% of 600.
26. $\frac{1}{3}$% of 30.	30. $\frac{5}{6}$% of 400.	34. $\frac{4}{9}$% of 900.

WRITTEN APPLICATIONS

1. Of a class numbering 120 boys, 5% failed to graduate. How many boys failed to graduate?

2. A house is valued at \$5000 and the annual rent of this house is 8% of its value. How much is the rent of this house for one year?

3. A carload of grain costing \$500 is damaged and is sold at a reduction of $12\frac{1}{2}$ per cent. How much is received for the grain?

4. A grocer makes a profit of 8 per cent on a stock of goods that cost him \$9500. What is the amount of his profit?

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5. A boy bought candy for \$9.00 and sold it at a profit of $12\frac{1}{2}$ per cent. Find the amount of his profit.

6. An estate amounted to \$24,000. One of the heirs received 15 per cent of this estate. What amount did he receive?

7. A house rented for \$20 per month, but the owner increased the rent 20 per cent. How much did the house then rent for per month?

8. A tank that holds 1200 gallons is 85 per cent full. How many gallons are in the tank?

9. 540 persons attended a lecture, $33\frac{1}{3}\%$ of the number being men, 50 per cent women, and the rest children. Find the number of men, the number of women, and the number of children that attended the lecture.

10. A grocer had \$7000 invested in his business. In each of two successive years he cleared $12\frac{1}{2}$ per cent, and in the third year he cleared 15 per cent. Find the total profit for the three years.

11. A clerk receives \$800 for his first year's work, but in each of the four years following he was given an increase of 10 per cent over the preceding year Find the total amount he earned in these five years.

12. 15 per cent of a man's property is invested in real estate, $33\frac{1}{3}$ per cent in bonds, 40 per cent in his business, and the rest is in the savings bank The total value of his property is \$54,000. Find the amounts in each of his different investments, and also the amount he has in the savings bank.

13. A man bought an automobile for \$3000. In the first year that he ran it he paid 4% of the purchase price of the car for gasoline and oil, 5% of the price for insurance, 1% of it for a license, and 6% of it for garage charges. Find the total expense of keeping and running the car for that year.

14. A family saved \$450 in a year, but spent the balance of their income as follows: for rent, 25%; for food, 35%; for clothing, 18%; for reading and amusements, 7%; and for miscellaneous expenses, 5%. Find the amount of their income.

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II. To Find the Per Cent One Number Is of Another Number.

From the first case:

$$\text{Rate} \times \text{Base} = \text{Percentage}$$

From which, by division:

$$\text{Rate} = \text{Percentage} \div \text{Base}$$

To Find the Rate Divide the Percentage by the Base.

Illustration:

1. What per cent of 60 is 15?

The base is known to be 60. The percentage is known to be 15.

Applying the rule: $\frac{\text{Percentage}}{\text{Base}} = \frac{15}{60} = \frac{1}{4} = .25$ or 25%. The Rate.

You should note that the process gives the *decimal expression* of the *fractional relation* of the two given numbers.

ORAL PRACTICE

1. What part of 20 is 2? How many hundredths of 20 is 2? What per cent of 20 is 2?

2. What part of 100 is 25? How many hundredths of 100 is 25? What per cent of 100 is 25?

3. What part of 150 is 30? How many hundredths of 150 is 30? What per cent of 150 is 30?

What per cent of

4. 5 is 4?	9. 12 is 4?	14. 120 is 40?	19. \$9 is \$3?
5. 6 is 3?	10. 36 is 6?	15. 144 is 18?	20. \$18 is \$9?
6. 8 is 6?	11. 45 is 9?	16. 160 is 10?	21. \$54 is \$9?
7. 9 is 3?	12. 54 is 18?	17. 210 is 70?	22. \$72 is \$27?
8. 15 is 3?	13. 63 is 21?	18. 300 is 100?	23. \$85 is \$17?

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BLACKBOARD PRACTICE

What per cent of

1.	30 is 6?	30 is 10?	40 is 10?	60 is 12?
2.	40 is 5?	40 is 8?	40 is 20?	75 is 15?
3.	50 is 10?	75 is 20?	80 is 32?	90 is 50?
4.	125 is 50?	125 is 75?	125 is 100?	125 is 125?
5.	128 is 8?	156 is 13?	250 is $12\frac{1}{2}$?	350 is 175?
6.	150 is 30?	180 is 36?	200 is 40?	240 is 48?
7.	225 is 75?	256 is 16?	324 is 81?	520 is 5.2?
8.	250 is 12.5?	375 is 7.5?	4200 is 84?	4200 is 8.4?
9.	300 is 120?	300 is 180?	300 is 225?	4500 is 1350?

To the nearest tenth, what per cent of

10.	27 is 13?	35 is 14?	42 is 12?	54 is 16?
11.	35 is 15?	46 is 19?	54 is 19?	54 is 25?
12.	64 is 18?	69 is 24?	74 is 19?	82 is 40?
13.	92 is 21?	95 is 29?	110 is 37?	120 is 44?
14.	110 is 31?	120 is 35?	145 is 53?	160 is 56?
15.	\$25 is \$11?	\$75 is \$32?	\$90 is \$17?	\$100 is \$19?
16.	\$35.50 is \$11.35?	\$47.10 is \$13.25?	\$54.65 is \$11.75?	
17.	\$125.50 is \$8.19?	\$256.75 is \$111.25?	\$1565.50 is \$125.00?	

WRITTEN APPLICATIONS

1. A baseball team won 75 games of the 125 games played. What per cent of the total number of games played was the number won?

2. A boy weighed 100 pounds last year, but at the present time his weight is 120 pounds. What per cent of his present weight is his former weight?

3. From a salary of \$1500 a clerk saved in one year \$300. What per cent of his salary did he save?

4. A dairyman kept 80 cows last year but has 100 at the present time. By what per cent did he increase the number kept last year to give the number he has now?

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5. A horse that cost a dealer \$160 was sold for \$200. What was the gain in dollars? What was the per cent of gain?

6. A farm of 200 acres is so divided that 40 acres are in pasture. What per cent of the whole farm is the pasture?

7. A baseball team won 42 games and lost 14. What per cent of the total number of games played is the number of games won?

8. A club with a membership of 30 admits 5 new members. What per cent of the new membership is the former membership?

9. A contractor is paid \$600 when he begins the construction of a dwelling that is to cost \$3600. What per cent of the total cost is this payment?

10. A mixture of 1400 cubic feet of concrete contains 200 pounds of cement. What per cent of the mixture is the cement?

11. 65% of the cost of a house and lot was spent on the house. If the house and lot cost \$8000, find the cost of the lot and the cost of the house.

12. A cow gave 16,257 pounds of milk in one year, and her milk was found to contain 642 pounds of butter fat. What was the per cent of butter fat in her milk?

13. A corporation had \$150,000 invested in their plant and in one year they sold \$250,000 worth of their product. What per cent of their investment was their sales?

14. A real estate broker received \$1750 as his commission for selling a business block. If the block sold for \$35,000, what was the rate per cent of his commission?

15. A man who failed in business paid \$2437.50 on a debt amounting to \$3750. What per cent of the whole amount did he pay? How many "cents on the dollar," did he pay?

16. A business man made a total profit of \$15,000 in one year, but his expenses were \$7500 for help, \$1800 for rent, and \$1200 for miscellaneous items. Find the per cent of his total profits that was left after paying his expenses.

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17. 70% of a journey of 1200 miles was made by water, and the rest of it was made by rail. How many miles were traveled by rail?

18. A painter can paint a barn in 12 days. What per cent of it can he paint in 5 days? What per cent in 3 days? What per cent in 7 days? What per cent in 11 days?

19. A broker failed and the total amount of his debts was $120,000. If the total value of his property is $80,000, what per cent of his debts can be paid with his property?

20. From a crop of 1200 bushels of potatoes a farmer sold 400 bushels to hotels, 600 bushels to grocers, and the rest to individuals. What per cent of the whole crop did he sell individually? To grocers? To hotels?

21. A man bought a lot for $1500 and built a house on it costing $6000. What per cent of the cost of the house is the cost of the lot? What per cent of the total cost of the house and lot is the cost of the house?

22. A salesman is offered a salary of $2000, and in addition he is promised $100 commission for every $10,000 worth of goods he sells during the year. He sells goods worth $120,000. What per cent of his salary is the amount he receives in commissions?

23. A man paid $6000 for a house. His annual expenses for maintaining and owning it included $180 for interest, $75 for taxes, and $20 for water. What per cent of the cost of the house is the amount of these three items?

24. In a league of four baseball teams the standing on a certain day was obtained from the following: The Crescents had won 24 games and had lost 10; the Stars had won 25 and had lost 11; the Imperials had won 20 and had lost 13; and the Orioles had won 26 and lost 8. Find the total number of games each team played and the per cent of games won by each team.

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III. To Find the Number, of which a Given Number is a Given Per Cent.

From the first case:

Rate × Base = Percentage

By Division:

Base = Percentage ÷ Rate

To Find the Base, Divide the Percentage by the Rate.

Illustrations:

1. 8% of a number is 60. What is the number?

Since 8% of the number is 60,

1% of the number is $60 \div 8 = 7.5$.

Hence, 100%, or the required number, $100 \times 7.5 = 750$. *Result.*

2. Find the number of which 540 is $12\frac{1}{2}\%$.

Since $12\frac{1}{2}\%$ of the number is 540,

1% of the number is $540 \div 12\frac{1}{2} = 43.2$.

Hence, 100%, or the required number, $100 \times 43.2 = 4320$. *Result.*

ORAL PRACTICE

Find the number of which

1. 10 is 10%.
2. 15 is 10%.
3. 20 is 10%.
4. 10 is 20%.
5. 15 is 20%.
6. 20 is 20%.
7. 10 is 25%.
8. 15 is 25%.
9. 20 is 25%.
10. 25 is 25%.
11. 25 is $12\frac{1}{2}\%$.
12. 30 is $12\frac{1}{2}\%$.
13. 35 is 20%.
14. 40 is 25%.
15. 40 is 20%.
16. 40 is 40%.
17. 50 is $33\frac{1}{3}\%$.
18. 60 is $62\frac{1}{2}\%$.
19. 60 is $16\frac{2}{3}\%$.
20. 75 is $37\frac{1}{2}\%$.
21. 60 is $33\frac{1}{3}\%$.
22. 66 is $37\frac{1}{2}\%$.
23. 70 is 70%.
24. 80 is 80%.
25. 100 is $16\frac{2}{3}\%$.
26. 120 is $33\frac{1}{3}\%$.
27. 150 is 50%.
28. 160 is 80%.
29. 270 is 90%.
30. 240 is 120%.

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BLACKBOARD PRACTICE

Find the number of which

1.	50 is 25%.	75 is 50%.	125 is 50%.	150 is 60%.
2.	60 is 20%.	80 is 40%.	100 is 60%.	180 is 90%.
3.	60 is 30%.	80 is 50%.	140 is 50%.	210 is 60%.
4.	75 is 50%.	100 is $33\frac{1}{3}$%.	180 is 60%.	240 is $62\frac{1}{2}$%.
5.	75 is 60%.	90 is 75%.	144 is 80%.	180 is $62\frac{1}{2}$%.
6.	50 is $12\frac{1}{2}$%.	75 is $33\frac{1}{3}$%.	125 is $62\frac{1}{2}$%.	200 is 125%.
7.	75 is $37\frac{1}{2}$%.	100 is $67\frac{1}{2}$%.	180 is 90%.	210 is 105%.
8.	90 is 40%.	120 is 60%.	240 is 80%.	250 is 120%.
9.	120 is 100%.	125 is 60%.	190 is 95%.	250 is 200%.
10.	120 is 150%.	135 is $67\frac{1}{2}$%.	175 is 100%.	275 is 200%.
11.	200 is 100%.	225 is $112\frac{1}{2}$%.	250 is 125%.	350 is 200%.

WRITTEN APPLICATIONS

1. If 12 boys join a class, and this number is 20% of the original class, how many were in the original class?

2. A man gains 15 pounds, which is 12% of his former weight. How much was his former weight?

3. A house is sold at a loss of \$500, which is $12\frac{1}{2}$% of the cost of it. How much did the house cost?

4. After forty-five miles of a telephone line is completed the builder reports that 85% still remains to be built. How long will the line be when it is completed?

5. After twelve tenants had moved into a new apartment house, $33\frac{1}{3}$% of the entire capacity of the house was still vacant. How many more tenants can the house accommodate?

6. 420 bushels of corn are sold by a dealer, which was 30% of the entire stock he had on hand. How many bushels of corn did he have at first? How many bushels has he left?

7. A horse was sold for \$200, which was 120% of the amount paid for him. How much was paid for him? How much was the profit on the sale? What was the per cent of profit on the sale?

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8. A merchant paid bills amounting to \$960, and this sum was only $62\frac{1}{2}\%$ of the total amount he owed. What was the total amount of his indebtedness? What per cent of it remains unpaid?

9. A baseball team lost 48 games in an entire season, and this number was $37\frac{1}{2}\%$ of the total number of games played. Find the total number of games played, and the total number won.

10. A man bought a lot and built a house on it that cost him \$7500. The cost of the house was $83\frac{1}{3}\%$ of the total cost of both the house and the lot. How much was the cost of the lot?

11. A merchant's total sales for a year amounted to \$49,650, and his expenses for running the business were \$5650. His sales, less his expenses, were 110% of the amount paid for his goods. How much was his profit for the year?

GENERAL REVIEW OF PERCENTAGE

ORAL PRACTICE

Find:

1. 10% of 20.
2. 20% of 10.
3. 10% of 50.
4. 50% of 10.
5. 10% of 90.
6. 90% of 10.
7. 25% of 36.
8. 40% of 70.
9. 50% of 80.
10. 60% of 70.
11. 70% of 90.
12. 80% of 100.
13. $16\frac{2}{3}\%$ of 60.
14. $33\frac{1}{3}\%$ of 75.
15. $37\frac{1}{2}\%$ of 80.
16. $62\frac{1}{2}\%$ of 80.
17. $83\frac{1}{3}\%$ of 96.
18. $97\frac{1}{2}\%$ of 140.

What per cent of

19. 10 is 5?
20. 10 is 8?
21. 15 is 10?
22. 20 is 10?
23. 30 is 10?
24. 30 is 15?
25. 50 is 30?
26. 54 is 36?
27. 60 is 40?
28. 70 is 35?
29. 75 is 15?
30. 75 is 25?
31. 90 is 60?
32. 100 is 80?
33. 100 is $6\frac{1}{4}$?
34. 100 is $12\frac{1}{2}$?
35. 100 is $16\frac{2}{3}$?
36. 100 is $33\frac{1}{3}$?

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Find the number of which

37. 20 is 10%.
38. 30 is 10%.
39. 40 is 10%.
40. 40 is 20%.
41. 50 is 20%.
42. 60 is 25%.
43. 10 is $12\frac{1}{2}$%.
44. 15 is $12\frac{1}{2}$%.
45. 20 is $16\frac{2}{3}$%.
46. 30 is $16\frac{2}{3}$%.
47. 48 is $12\frac{1}{2}$%.
48. 48 is $16\frac{2}{3}$%.
49. 60 is $37\frac{1}{2}$%.
50. 70 is $33\frac{1}{3}$%.
51. 75 is $62\frac{1}{2}$%.
52. 80 is $62\frac{1}{2}$%.
53. 100 is $66\frac{2}{3}$%.
54. 125 is $83\frac{1}{3}$%.

BLACKBOARD PRACTICE

Find, to the nearest .01:

1. $6\frac{1}{4}$% of 25.
2. $6\frac{1}{4}$% of 35.
3. $6\frac{1}{4}$% of 45.
4. $6\frac{1}{4}$% of 55.
5. $6\frac{1}{4}$% of 65.
6. $6\frac{1}{4}$% of 75.
7. $8\frac{1}{3}$% of 50.
8. $8\frac{1}{3}$% of 75.
9. $8\frac{1}{3}$% of 90.
10. $8\frac{1}{3}$% of 105.
11. $8\frac{1}{3}$% of 115.
12. $8\frac{1}{3}$% of 125.
13. $12\frac{1}{2}$% of 75.
14. $12\frac{1}{2}$% of 90.
15. $12\frac{1}{2}$% of 105.
16. $12\frac{1}{2}$% of 115.
17. $12\frac{1}{2}$% of 130.
18. $12\frac{1}{2}$% of 145.

19. $16\frac{2}{3}$% of 50.
20. $16\frac{2}{3}$% of 75.
21. $16\frac{2}{3}$% of 85.
22. $16\frac{2}{3}$% of 90.
23. $16\frac{2}{3}$% of 95.
24. $16\frac{2}{3}$% of 105.
25. $33\frac{1}{3}$% of 50.
26. $33\frac{1}{3}$% of 70.
27. $33\frac{1}{3}$% of 80.
28. $33\frac{1}{3}$% of 95.
29. $33\frac{1}{3}$% of 110.
30. $33\frac{1}{3}$% of 115.
31. $62\frac{1}{2}$% of 75.
32. $62\frac{1}{2}$% of 90.
33. $62\frac{1}{2}$% of 105.
34. $62\frac{1}{2}$% of 115.
35. $62\frac{1}{2}$% of 130.
36. $62\frac{1}{2}$% of 145.

37. $83\frac{1}{3}$% of 60.
38. $83\frac{1}{3}$% of 65.
39. $83\frac{1}{3}$% of 70.
40. $83\frac{1}{3}$% of 75.
41. $83\frac{1}{3}$% of 80.
42. $83\frac{1}{3}$% of 90.
43. $83\frac{1}{3}$% of 95.
44. $83\frac{1}{3}$% of 105.
45. $87\frac{1}{2}$% of 105.
46. $87\frac{1}{2}$% of 110.
47. $87\frac{1}{2}$% of 120.
48. $87\frac{1}{2}$% of 125.
49. $87\frac{1}{2}$% of 130.
50. $87\frac{1}{2}$% of 135.
51. $87\frac{1}{2}$% of 140.

52. 105% of \$50.
53. 110% of \$75.
54. 115% of \$90.
55. 120% of \$100.
56. 120% of \$40.50.
57. 125% of \$56.15.
58. 125% of \$62.50.
59. $106\frac{1}{4}$% of \$25.00.

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60. 125% of \$110.
61. 125% of \$125.
62. 105% of \$15.50.
63. 110% of \$20.25.
64. 115% of \$35.10.
65. 112½% of \$35.50.
66. 112½% of \$50.75.
67. 137½% of \$125.50.
68. 137½% of \$162.50.
69. 162½% of \$162.50.

70. 5% of 10% of \$50.
71. 5% of 10% of \$75.
72. 5% of 10% of \$100.
73. 10% of 20% of \$100.
74. 10% of 25% of \$120.
75. 10% of 30% of \$125.50.
76. 15% of 30% of \$250.00.
77. 20% of 30% of \$375.00.
78. 30% of 50% of \$500.00.
79. 40% of 50% of \$1000.00.

What per cent of

80. 250 is 25?
81. 450 is 50?
82. 550 is 110?
83. 625 is 125?
84. 750 is 150?
85. 800 is 160?
86. 990 is 200?
87. 1200 is 240?
88. 1350 is 270?
89. 1500 is 450?
90. \$50 is \$12.50?
91. \$90 is \$12.50?
92. \$100 is \$12.50?
93. \$125 is \$12.50?
94. \$150 is \$16.25?
95. \$175 is \$18.75?
96. \$225 is \$27.50?
97. \$250 is \$32.50?
98. \$300 is \$37.50?
99. \$400 is \$62.50?
100. \$12.40 is \$3.10?
101. \$18.75 is \$3.75?
102. \$22.50 is \$4.50?
103. \$37.50 is \$6.75?
104. \$50.00 is \$6.25?
105. \$62.50 is \$6.25?
106. \$75.00 is \$12.50?
107. \$87.50 is \$37.50?
108. \$112.50 is \$37.50?
109. \$137.50 is \$62.50?

Find the number of which

110. 42 is 15%.
111. 65 is 20%.
112. 90 is 18%.
113. 120 is 25%.
114. 12.5 is 10%.
115. 15.6 is 25%.
116. 125.7 is 30%.
117. 225.25 is 40%.
118. .15 is 25%.
119. .004 is 20%.
120. .117 is 37½%.
121. .0025 is 62½%.

Find the amount of which

122. \$1.25 is 8%.
123. \$2.75 is 10%.
124. \$3.50 is 12½%.
125. \$4.50 is 20%.
126. \$12.00 is 12½%.
127. \$50.00 is 62½%.
128. \$75.00 is 80%.
129. \$225.00 is 16⅔%.
130. \$375.00 is 33⅓%.
131. \$625.00 is 62½%.

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WRITTEN APPLICATIONS

The Use of Percentage in Everyday Life

I. The Business Man's Problems in Percentage.

1. A grocer makes an annual profit of \$3000, which is 15% of the total amount of business that he transacts. What amount of business does he transact in one year?

2. A merchant does an annual business of \$27,500, and loses in poor accounts \$550. What per cent of the amount of business he does is the amount that he loses?

3. A real estate firm sold $\frac{1}{3}$ of a building tract of 120 acres to one man, and $\frac{1}{4}$ of it to another man. What per cent of the tract was left unsold?

4. From a field containing 15 acres and costing \$4200 a single acre is sold at a profit of 15%. What amount is received for the acre sold?

5. On a bill of goods amounting to \$780, a freight bill of \$16, and a cartage charge of \$23, were paid. What per cent of the bill were the freight and cartage charges?

6. A grocer paid \$3.84 for a crate of strawberries containing 32 quarts. If he sold them for 15 cents a quart, what per cent of the cost price was his profit? What per cent of the selling price was his profit?

7. A merchant sold a piano for \$75 less than the marked price. If this reduction was 20% of the marked price, what was the marked price? How much did he receive for the piano? If his profit was 20% on the cost, how much did the piano cost him?

8. A clerk received an increase of 20% in his salary, and he observed that if the increase had been 30% instead of 20%, it would have given him \$200 additional. What was the amount of his original salary?

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9. What is the amount saved on 140 tons of coal, when a reduction of 5% is made by the dealer from the regular price of $6.75 per ton? How much is the saving per ton?

10. A speculator in land increased his holdings 30%. He then sold 20% of all his holdings and had left 1300 acres. How many acres did he hold at first, and how many acres did he sell?

11. A dealer sold a case of 60 bars of soap for $3.00, and his profit on the sale was 20%. What was the price he paid per bar? What was the price he received per bar?

12. A merchant failed in business owning property amounting to $45,600, and owing debts amounting to $67,500. The cost of settling his business was $600. What per cent did his creditors lose?

13. A secretary's salary in 1916 was $1800. In 1917 his salary was increased 25%, but in 1918 it was decreased 25% because of poor business. What amount did he receive in 1918? What per cent of his 1916 salary was his 1918 salary?

14. A merchant's expenses for the year were $1875. His coal bill was 25% of his total expense, and was also 15% of his total profits. How much was his profit on the year's business?

15. During his second year in business a merchant's sales increased 20% over the first year; in his third year the increase was 30% over the first year's sales, and in the fourth year 25% over the first year's sales. The sales in the first year amounted to $12,000. How much were his total sales during the four years?

16. $37\frac{1}{2}$% of 2400 yards of cloth were sold at $1.40 per yard; 25% of it at $1.60 per yard, and the rest of it at $1.50 per yard. How much was received for the entire lot? How much was the profit on the lot if the cost of the cloth was $1.25 per yard?

17. $4.20 per barrel is paid for flour at the mill and the freight charge is $.12$\frac{1}{2}$ per barrel. The flour is sold for $5.00 per barrel. What is the per cent of profit made on the flour? What per cent of the cost of the flour is the freight charge?

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II. The Farmer's Problems in Percentage.

18. From a crop of potatoes 85%, or 1190 bushels, were sold at \$0.65 per bushel, and the rest at \$0.55 per bushel. How much was the total received for them?

19. 630 bushels of wheat were sold from a crop, and 70% of the whole crop remained unsold. How many bushels were there in the whole crop?

20. A farmer sold a colt for \$240, and the buyer resold the colt at a profit of \$50. What per cent of profit did the buyer make on the purchase?

21. A hog weighed 240 pounds live weight, and dressed 80% of that amount. At \$14.40 per hundredweight alive, how much is it worth per pound dressed?

22. A farmer sprayed one half of a field of potatoes and the yield from this part was 308 bushels. From the unsprayed half of the same field he obtained 140 bushels. What was the per cent gained by spraying?

23. Two steers weighed 1100 pounds each, live weight. The dressed weight of one of them was 62% of the live weight, and the dressed weight of the other was 51% of its live weight. At $11\frac{1}{2}$¢ per pound dressed weight, how much did both bring?

24. The lumber and other materials for a small barn cost a farmer \$368.75, and the labor for erecting it cost him \$165. What per cent of the total cost of the finished barn was the amount paid for the labor?

25. For a ton of fertilizer a farmer used 1200 pounds of bone, 500 pounds of sulphate of ammonia, and 300 pounds of muriate of potash. Find the percentage of each of the three ingredients.

26. For one of his crops the farmer who made the fertilizer in example 25 used 200 pounds of this fertilizer to the acre. What amount of each ingredient did he use for each acre?

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27. A ton of a fertilizer is to contain 2% of nitrogen and 8% of phosphoric acid. The nitrogen will cost the farmer 16¢ per pound, and the phosphoric acid 6¢ per pound. How much will the nitrogen and the acid together cost for the ton of fertilizer?

28. Potatoes have been found to remove from the soil .2% of their weight in nitrogen. How many pounds of nitrogen at this rate will be removed from the soil by a crop of 500 bushels of potatoes? (60 pounds per bushel.)

29. Corn is known to remove from the soil 1.9% of its weight in nitrogen, and wheat removes 2.4% of its weight in nitrogen. How many pounds of nitrogen were removed from a field yielding 2 tons of corn and 1.5 tons of wheat?

30. A cow gives 7200 pounds of milk in a year, which tests 3.3% of butter fat; another cow gives 5400 pounds of milk, which tests 5.1% of butter fat. Of these two cows which is the more profitable, considering only the butter fat?

31. Each of two cows in a herd produced 4500 pounds of milk in one year. One cow's milk tested 3.3% butter fat, and the other's milk tested 5.4%. Find the difference in the amount of butter fat produced by these two cows.

32. In a herd of 32 cows, 18 gave 40 pounds of milk each daily, 9 gave 30 pounds each daily, and 5 gave 22 pounds each daily. The milk from the whole herd averaged 3.6% butter fat. Find the total quantity of butter fat produced daily.

33. In a herd of 20 cows, 8 gave 40 pounds of milk each daily, 7 gave 30 pounds each daily, and 5 gave 24 pounds each daily. The first group "tested" 5.2%, the second group 4.5%, and the third group 3.6%. Find the total number of pounds of butter fat produced daily by this herd.

34. Because of the salt, water, etc., a pound of butter fat has been found to make, on the average, $1\frac{1}{8}$ pounds of butter. At 55¢ per pound what is the value of the butter produced from 5000 pounds of milk that tests 5.4% butter fat?

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PROFIT AND LOSS

The Cost of an article, in a business sense, is the amount paid for it.

The Selling Price of an article is the amount received for it.

A book is purchased for \$1.00, and is sold for \$1.25.

The cost of the book is \$1.00.

The selling price of the book is \$1.25.

The Profit on a sale is the amount by which the selling price exceeds the cost price.

The profit in the illustration is \$1.25 − \$1.00 = \$.25.

The Loss on a sale is the amount by which the cost price exceeds the selling price.

If the book in the illustration had been sold for \$0.80, the loss on the transaction would have been \$1.00 − \$.80 = \$.20.

Problems in Profit and Loss are merely problems in percentage in which

The Base is the cost.

The Rate is the per cent of gain or of loss.

The Percentage is the profit or the loss.

Business Practice in Profit and Loss. In ordinary business practice it is almost a universal custom to figure the profits on the gross cost of the article sold. The gross cost includes the actual cost of the article itself, together with such expense as freight, cartage, insurance, selling expense, etc.

Illustration:

15 rugs at \$16 each, cost at the factory	\$240.00
Freight on same	8.00
Cartage on same	1.00
Selling expense (estimated from known experience), 4%	9.60
Gross cost	\$258.60

Gross cost of 1 rug = \$258.60 ÷ 15 = \$17.24.

Suppose the dealer desires a profit of 25% on this cost.

Selling Price = \$17.24 + (25% of \$17.24) = \$17.24 + \$4.31 = \$21.55.

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Many large firms and department stores vary the practice to avoid complicated bookkeeping, as will be shown later on.

For convenience and brevity in memorizing the principles that govern problems in profit and loss, they may be stated as follows:

For Profit:

Rate of Gain × Cost = Gain. (1)
(100% + Rate of Gain) × Cost = Selling Price. (2)

For Loss:

Rate of Loss × Cost = Loss. (3)
(100% − Rate of Loss) × Cost = Selling Price. (4)

Applications of these principles.

Illustration of (1):

At a rate of 25% find the gain on a book costing \$2.00.
25% of \$2, or .25 × \$2.00 = \$0.50. Gain.

Illustration of (2):

Find the selling price of a book that cost \$2.00 if the gain is 25%.
100% + 25% = 125%. 125% = 1.25. 1.25 × \$2.00 = \$2.50. Selling Price.

Illustration of (3):

At a rate of 25% find the loss on a book costing \$2.00.
25% of \$2, or .25 × \$2.00 = \$0.50. Loss.

Illustration of (4):

Find the selling price of a book that cost \$2.00 if the loss is 25%.
100% − 25% = 75% = .75. .75 × \$2.00 = \$1.50. Selling Price.

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BLACKBOARD PRACTICE

With the rate indicated, find the amount of profit on goods costing

1. \$40 at 10%.
2. \$60 at 10%.
3. \$65 at 12%.
4. \$75 at 15%.
5. \$90 at 20%.
6. \$95 at 20%.
7. \$100 at $12\frac{1}{2}$%.
8. \$125 at $16\frac{2}{3}$%.
9. \$150 at 15%.
10. \$200 at 20%.
11. \$275 at 15%.
12. \$350 at 25%.
13. \$125.50 at 5%.
14. \$240.25 at 8%.
15. \$360.75 at 7%.
16. \$4500.00 at $12\frac{1}{2}$%.
17. \$20,000 at 16%.
18. \$35,000 at 25%.

With the rate of profit as indicated, find the selling price of goods costing

19. \$100 at 15%.
20. \$125 at 20%.
21. \$130 at 16%.
22. \$160 at 25%.
23. \$190 at 20%.
24. \$200 at 15%.
25. \$200 at 10%.
26. \$250 at 15%.
27. \$300 at $12\frac{1}{2}$%.
28. \$400 at $16\frac{2}{3}$%.
29. \$450 at 20%.
30. \$750 at 25%.
31. \$245.50 at 12%.
32. \$375.20 at 15%.
33. \$454.95 at 15%.
34. \$580.61 at 20%.
35. \$7500.00 at 20%.
36. \$12,750 at $33\frac{1}{3}$%.

Find the amount of the loss when the selling price of goods costing

37. \$500 is reduced 10%.
38. \$600 is reduced 15%.
39. \$750 is reduced 15%.
40. \$900 is reduced 20%.
41. \$925 is reduced 25%.
42. \$975 is reduced 30%.
43. \$1275.50 is reduced 10%.
44. \$2450.75 is reduced $12\frac{1}{2}$%.
45. \$3575.25 is reduced $16\frac{2}{3}$%.
46. \$4110.75 is reduced $16\frac{2}{3}$%.
47. \$25,750.00 is reduced $33\frac{1}{3}$%.
48. \$27,580.00 is reduced $37\frac{1}{2}$%.

Find the selling price when goods costing

49. \$1200 are reduced 15%.
50. \$1500 are reduced $12\frac{1}{2}$%.
51. \$1750 are reduced 15%.
52. \$2500 are reduced 15%.
53. \$3750 are reduced 15%.
54. \$4250 are reduced 20%.
55. \$1250.50 are reduced 10%.
56. \$1975.50 are reduced $12\frac{1}{2}$%.
57. \$2150.80 are reduced $12\frac{1}{2}$%.
58. \$3675.20 are reduced 15%.
59. \$5490.50 are reduced 15%.
60. \$6252.75 are reduced $16\frac{2}{3}$%.

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APPLICATIONS OF PROFIT AND LOSS IN BUSINESS

I. To Find the Cost of Goods When the Actual Gain or Loss, and the Rate of Gain or Loss are Known.

We have learned that *Rate of Gain* × *Cost* = *Gain.*

From which

$$\frac{\text{Gain}}{\text{Rate of Gain}} = \text{Cost.}$$

Application :

What is the cost of goods which give 25% profit when sold at a gain of $100?

In this question:
Gain = $100.
Rate of Gain = 25%, or .25.

Then:
$\frac{\text{Gain}}{\text{Rate of Gain}} = \frac{\$100}{.25} = \$400$, the Cost.

We have also learned that *Rate of Loss* × *Cost* = *Loss.*

From which

$$\frac{\text{Loss}}{\text{Rate of Loss}} = \text{Cost.}$$

Application :

What is the cost of goods which show 15% loss when sold at a loss of $30?

In this question:
Loss = $30.
Rate of Loss = 15%, or .15.

Then:
$\frac{\text{Loss}}{\text{Rate of Loss}} = \frac{\$30}{.15} = \$200$, the Cost.

BLACKBOARD PRACTICE

What is the cost of goods which show:

1. $75 profit at 10%?
2. $125 profit at 12%?
3. $250 profit at 15%?
4. $500 profit at 12½%?
5. $625 profit at 16⅔%?
6. $30 loss at 5%?
7. $45 loss at 7½%?
8. $67.50 loss at 6¾%?
9. $125 loss at 12½%?
10. $250 loss at 16⅔%?

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II. To Find the Rate of Gain When the Cost Price and the Selling Price are Known.

We have learned that *Selling Price − Cost Price = Gain.*
And that *Rate of Gain × Cost = Gain.*

From the last expression,

$$\text{Rate of Gain} = \frac{\text{Gain}}{\text{Cost}}.$$

Application:

What is the rate of gain when goods costing \$60 sell for \$75?

In this question:
Selling Price = \$75
Cost Price = \$60
Gain = \$15

Then:
$\frac{\text{Gain}}{\text{Cost}} = \frac{\$15}{\$60} = .25 = 25\%$, Rate of Gain.

III. To Find the Rate of Loss When the Cost Price and the Selling Price are Known.

We have learned that *Cost Price − Selling Price = Loss.*
And that *Rate of Loss × Cost = Loss.*

From the last expression,

$$\text{Rate of Loss} = \frac{\text{Loss}}{\text{Cost}}.$$

Application:

What is the rate of loss when goods costing \$120 sell for \$100?

In this question:
Cost Price = \$120
Selling Price = \$100
Loss = \$20

Then:
$\frac{\text{Loss}}{\text{Cost}} = \frac{\$20}{\$100} = .20 = 20\%$, Rate of Loss.

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BLACKBOARD PRACTICE

Find the rate of gain when goods costing

1. \$300 sell for \$400.
2. \$450 sell for \$540.
3. \$500 sell for \$550.
4. \$500 sell for \$575.
5. \$750 sell for \$1000.
6. \$876 sell for \$1168.

Find the rate of loss when goods costing

7. \$300 sell for \$270.
8. \$375 sell for \$300.
9. \$450 sell for \$400.
10. \$600 sell for \$500.
11. \$1200 sell for \$900.
12. \$1800 sell for \$1500.

Marking Goods with Private Cost Marks.

Merchants often mark their goods with characters known only to the employees of the firm. Such systems are based upon some word or words which can be easily memorized. For example:

The letters of the word r u d i m e n t a l
may be used for the figures 1 2 3 4 5 6 7 8 9 0.

Such a word is called a **Key**. Using this key:

rul	represents	\$1.20,	raum	represents	\$19.25,
dnm	"	\$3.75,	umll	"	\$25.00,
ell	"	\$6.00,	mlll	"	\$50.00, etc.

Some firms use a "repeater" to help conceal their cost marks. If a repeater "×" is used, the mark umll is written umlx.

If both the cost price and the selling price are written on an article or a tag, it is customary to write the cost price over the selling price. Thus: $\frac{\text{rdm}}{\$1.75}$. In most cases merchants write the selling price in plain figures.

Using the key show how to mark goods costing:

1. \$1.00 to gain 20%.
2. \$1.20 to gain $33\frac{1}{3}$%.
3. \$1.12 to gain 25%.
4. \$1.25 to gain 20%.
5. \$1.50 to gain 30%.
6. \$2.50 to gain 30%.
7. \$300 to gain $16\frac{2}{3}$%.
8. \$12.50 to gain 20%.
9. \$15.00 to gain 25%.
10. \$18.75 to gain 20%.
11. \$22.50 to gain 15%.
12. \$24.00 to gain $33\frac{1}{3}$%.
13. \$37.50 to gain 40%.
14. \$50.00 to gain $37\frac{1}{2}$%.

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WRITTEN APPLICATIONS

The Business Man's Problems in Profit and Loss

1. 15% profit is gained on a shipment of grain which cost a dealer $2400. How much is his gain?

2. A shipment of grain which cost $2400 was sold for $2670. What is the per cent of gain?

3. At the rate of 15% profit a dealer makes a profit of $360 on a shipment of grain. How much did the grain cost him?

4. A dealer sold 1700 bushels of wheat at a profit of 15% and received $1173 for it. How much did the wheat cost him per bushel?

5. A piece of real estate which cost $4200 increased $16\frac{2}{3}\%$ in value. Find the amount of the increase in value.

6. Lumber that cost $40 per thousand is damaged 5% and is then sold so as to make a gain of 15% on its real value. What is the selling price per thousand?

7. A dealer bought a horse for $200 and sold it at a price which would give him 20% profit. He received only 80% of that price. How much did he lose? What per cent of the cost did he lose?

8. A merchant sold 400 collars at $.085 each, and gained 25% by the transaction. How much did he pay for the collars?

9. A profit of 80 cents was made on a book that sold at 120% of its cost. What was the selling price of the book?

10. A horse was purchased at 10% less than his actual value, and then sold at a profit of 15% more than his value. What was the gain per cent?

11. A grocer bought sugar at 9 cents per pound, and then sold it all in bags of 5 pounds each. What price must he ask per bag in order to gain 20%?

12. A set of books that cost $35 was sold for $43.75. At what price must a set costing $48 be sold in order to make the same per cent of profit?

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13. A dealer marked a book so as to gain 25% on its cost price, but he finally sold it for \$3.50, which was a loss of $12\frac{1}{2}$% on the cost. What was the marked selling price of the book?

14. A dealer bought coal at \$8 per short ton. He sold it all at \$9 per short ton, and paid 35 cents per ton for hauling. What was the rate per cent of his gain?

15. A farmer bought 120 acres of land at \$60 an acre, and spent \$1800 in improvements. He then sold the land and the improvements and gained 15% on his total investment. How much did he receive per acre for the property?

16. 50 volumes of books were sold at \$2.40 each. On one half of them the dealer gained 20% of the cost price, but on the other half he lost 20%. How much did he gain or lose in the transaction?

17. A carload of ice cost \$3.90 per ton, and the freight charge was 3 cents per hundredweight. The ice was sold at \$0.45 per hundredweight. What was the per cent of profit? What was the profit on one thousand pounds?

18. A farm cost \$7500, and 40% of the cost price was spent in improvements. The farm was then sold for \$12,600. Find the amount of the gain and the gain per cent on the transaction.

19. A builder sold two houses for \$4500 each, gaining 20% on one of them and losing 20% on the other. Find the cost price of each house, and the actual gain or loss on the transaction.

The Retail Merchant's Method of Marking Goods.

In recent years large retailers have adopted the practice of marking goods so that the price received shall return

(1) The cash cost of the goods;

(2) The expenses of selling; and

(3) A reasonable percentage of profit.

In this classification the "expenses of selling" include such items as rent, heat, light, clerk hire, interest, insurance, advertising, etc. (called overhead expense).

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The Selling Price is the Base in using the Retail Merchant's Method of calculating profits.

Illustration:

A sewing machine cost a retailer $18, and the freight charge was $1. If the retailer knows that his selling expense is 12%, at what price shall he mark the machine to gain a profit of 15%?

Let the Selling Price = 100%.
Reduce this price by the selling cost and the profit desired.
Then, 100% − 12% − 15% = 73%, the Wholesale Cost.
Also, $18 + $1 = $19, the Cash Cost.
We want that number of which $19 is 73%.
Or, $19 ÷ .73 = $26.03, the Selling Price. Result.

Tables for finding a Selling Price are readily obtained, and merchants save much labor by using them. In the following table a few cases are tabulated for convenience:

% of Selling-Expense	% Profit Desired					
	10	12	15	20	25	30
10	80	78	75	70	65	60
12	78	76	73	68	63	58
15	75	72	70	65	60	55
16	74	72	69	64	59	54
18	72	70	67	62	57	52
20	70	68	65	60	55	50
25	65	63	60	55	50	45

Method of Using the Table.

Illustration:

A net profit of 20% is desired on goods costing in cash $254, the selling expense of the business being 16%. Find the selling price.

Under "% Profit Desired" in column "20" and opposite "16" we find "64."
Then $254 is 64% of the selling price.
Hence, $254 ÷ .64 = $396.87, the Selling Price. Result.

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BLACKBOARD PRACTICE

Find the selling price when the cash cost, the per cent of selling expense, and the per cent of profit desired are, respectively:

1. \$100, 10%, and 12%.
2. \$100, 15%, and 12%.
3. \$125, 12%, and 25%.
4. \$240, 16%, and 20%.
5. \$500, 25%, and 25%.
6. \$1150.50, 18%, and 20%.
7. \$1875.90, 12%, and 25%.
8. \$2750.00, 10%, and 12%.
9. \$10,575.50, 12%, and 15%.
10. \$24,500.00, 15%, and 20%.

11. A merchant bought furniture costing \$1500, and paid a freight charge of \$35. If he estimates his selling expense at 16%, at what price must he mark the furniture to gain a profit of 12%?

12. The freight on goods costing \$2400 was $\frac{1}{8}$%, and the selling expense was 25%. At what price did the merchant mark these goods, if his profit on them was 20%?

13. A house furnisher paid \$3750 for a lot of Wilton rugs, and sold them so that his profit was 15%. If the expense of running his business was 20%, at what price did he sell his rugs to give the profit indicated?

COMMERCIAL DISCOUNT

Discount is the amount deducted from a bill of goods or from a debt.

Trade Discount is the discount made from the published price of an article.

This form of discount has its origin in the constantly varying cost of the materials and the labor that enter into the manufacturing. A manufacturer publishes a catalogue, at a heavy expense, so from time to time he sends to his customers a new discount sheet to which the customer refers in making orders. Thus, the changing cost in manufacturing is adjusted in fairness to both manufacturer and retailer without the cost of a new catalogue.

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Cash Discount is the discount allowed for the immediate payment of a bill.

Merchandise is frequently offered to a buyer at a reduced price if payment for the same is made upon receipt of the goods.

Time Discount is the discount allowed for the payment of a bill within a specified time.

All three kinds of discount, trade, cash, and time discount, are frequently given the general name, **Commercial Discounts.**

Discounts are Reckoned as some rate per cent of the amount to be paid, or as some fraction of the amount.

For example: A 10% discount on a bill of \$500 means that the amount due is \$500 − (10% of \$500) = \$500 − \$50 = \$450.

Or, "$\frac{1}{10}$ off" on a bill of \$500 means that the amount due is

$$\$500 - (\tfrac{1}{10} \text{ of } \$500) = \$500 - \$50 = \$450.$$

Fractional expressions for discounts are inconvenient in cases like 8%, 15%, etc.

The Net Amount of a Bill is the amount of the bill less the discounts.

In the example above the net amount is \$450.

Business Practice in Commercial Discount.

Illustrations:

A bill of goods, based on catalogue prices, amounts to \$456.50, and the trade discount is 20%. Find the net amount of the bill.

Gross amount of bill =	\$456.50	
Discount = 20% of \$456.50 = \$456.50 × .20 =	91.30	
Net amount of bill =	\$365.20	*Result.*

A bill of goods is subject to a trade discount of 30%, and an additional cash discount of 5%. If the amount of the bill at the list price is \$734.80, find the net cost of the goods.

Gross amount of bill =	\$734.80
Discount = 30% of \$734.80 = \$734.80 × .30 =	220.44
Amount of bill less trade discount =	\$514.36
Cash discount = 5% of \$514.36 = \$514.36 × .05 =	25.72
Net amount of bill =	\$488.64

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BLACKBOARD PRACTICE

Find the discount on a bill amounting to

1. \$100 at 15%.
2. \$125 at 10%.
3. \$150 at 20%.
4. \$150 at 25%.
5. \$260 at 30%.
6. \$375 at 40%.
7. \$275.50 at 10%.
8. \$354.20 at $12\frac{1}{2}$%.
9. \$375.25 at $16\frac{2}{3}$%.
10. \$465.90 at $33\frac{1}{3}$%.
11. \$525.75 at 30%.
12. \$775.40 at $33\frac{1}{3}$%.

Find the net amount of a bill of goods when the list price and discount are:

13. \$27.50 and 10%.
14. \$45.90 and 15%.
15. \$96.75 and $12\frac{1}{2}$%.
16. \$115.10 and 20%.
17. \$175.87 and 25%.
18. \$290.05 and 30%.
19. \$54.50 and $12\frac{1}{2}$%.
20. \$75.90 and 15%.
21. \$110.50 and 20%.
22. \$250.50 and 25%.
23. \$375.90 and $37\frac{1}{3}$%.
24. \$625.45 and 40%.

WRITTEN APPLICATIONS

The Business Man's Problems in Discounts

1. Find the discount given on a suit marked \$24, if the rate of discount is 20%.

2. Find the net price of a suit marked \$24, if the rate of discount is 20%.

3. Find the rate of discount when a set of books marked \$24 is sold for \$19.20.

4. What is the rate of discount when a book marked \$2.00 is sold for \$1.60?

5. A piano listed at \$450 is sold at a discount of 10% for cash. How much does the dealer receive for it?

6. Find the net amount paid for 25 tons of coal at \$6.40 per ton if 5% discount is allowed for cash.

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7. A farmer bought a mowing machine at a discount of $16\frac{2}{3}\%$ for cash. The discount amounted to \$27. How much did he pay for the machine?

8. An auto truck is offered at \$2750 if paid for in six months, or at \$2475 if paid for on delivery. What is the rate of discount offered for cash?

9. A farmer bought a spraying outfit at a discount of 20% from a list price of \$2.60. What was the cost of the outfit?

10. What is the net cost of a bill of farming implements amounting to \$450.90 at list prices, if the discount from list prices is 1%?

11. Find the selling price of a sewing machine that was bought at 20% discount from a list price of \$25 and sold at a profit of 40% on the net cost price.

12. A farmer bought a potato planter listed at \$83, and by paying cash received a discount of 5%. The freight charge was \$1.15. How much did the planter cost him?

13. A merchant orders a bill of goods at list prices amounting to \$1125. The discount on one third of the bill is 15%, and on the other two thirds 20%. What is the net amount of the bill?

14. A farmer paid \$300 for a gas engine. The dealer who sold it to him made a profit of 25%, and the dealer bought it at a discount of 10% from the list price. What was the list price of the engine?

15. A dealer bought an automobile at $16\frac{2}{3}\%$ discount from the list price. The discount amounted to \$240. The dealer's selling price included the freight charges of \$20, and also a profit of 20% on the net cost to him. Find the selling price.

16. A fur coat that cost a dealer \$150 was marked to sell at a profit of $33\frac{1}{3}\%$, but it was sold late in the season at a discount of 25% from the marked price. What was the dealer's profit on the coat?

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17. 100 bars of soap are billed to a grocer at \$6.25, and a discount of 20% is allowed him. What was the cost per cake? If the soap is sold at 10 cents per cake, what is the per cent of profit?

18. A dealer paid \$20 for a dozen hats. Three of them were sold at a profit of 20%, but the others were damaged by fire and sold at a loss of 25%. How much was the actual loss on the transaction?

19. A merchant is offered a time discount of $12\frac{1}{2}\%$ on a bill of \$4800 if paid within 30 days, or a cash discount of \$640 if paid immediately. The merchant accepts the latter offer. How much does he save by the choice?

20. A dealer in books bought 5000 volumes at \$1.00 each, less 40%. He sold 2000 volumes at \$1.00 each, 1600 at \$.75 each, 1000 at 50¢ each, and the remainder at 25¢ each. How much profit did he make on the whole lot?

21. 100 sewing machines were bought at \$22.50 each. The buyer was offered his choice of a discount of 10% for cash or 5% if paid within 30 days. How much did he save by choosing the better terms?

22. A merchant discounted his bills for a year, and in that time he bought goods amounting to \$67,500. If the average rate of discount was 5%, and if the expense of running his business was 15% of the net cost of his goods, find the amount he saved, and the cost of running his business.

23. Goods listed at \$2000 were bought at a discount of 15%, and then sold at a profit of 20% on the net cost. If the same goods had been bought at a discount of 20%, and then sold at a profit of 30% on the net cost, how much more would the seller have made than in the first case?

24. A merchant received three different bills of goods, the first for \$1180 with 10% discount for cash, the second for \$750 with 20% trade discount, and the third for \$960 with $12\frac{1}{2}\%$ trade discount and 5% for cash. Find the total net amount of all three bills.

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Discount Series.

In many business transactions it is impossible for a manufacturer to publish a discount sheet by which he agrees to bind himself for any length of time, for the changes in manufacturing costs often come unexpectedly. For example, a manufacturer of rugs may have published a price list in which he offers his product at a discount of 10%, when he is suddenly confronted with the need for making further discounts or lose valuable business to a competing manufacturer. To meet the competition he gives notice to his customers of an additional discount, and sends his bills out with the original 10% reduction, and an additional 5%. On such a bill he gives, therefore, 10% and 5%, and the commercial expression is either " with 10% and 5% off," or " less 10 and 5."

Two or more discounts allowed on a bill are called Discount Series.

Illustration:

A bill of goods amounting to $2450 is sold subject to discounts of 20%, 10%, and 5%. Find the net amount of the bill.

20% of $2450 = $490.	$2450 − $490 = $1960.	
10% of $1960 = $196.	$1960 − $196 = $1764.	
5% of $1764 = $88.20.	$1764 − $88.20 = $1675.80.	*Result.*

The first discount is from the list price.
The second discount is from the first remainder.
The third discount is from the second remainder, etc.

BLACKBOARD PRACTICE

From the following amounts obtain the net amounts at the discounts indicated.

1. $75 at 10% and 5%.
2. $90 at 10% and 5%.
3. $100 at 20% and 10%.
4. $125 at 20% and 15%.
5. $150 at 25% and 20%.
6. $350 at 15%, 10%, and 5%.
7. $400 at 10%, 8%, and 5%.
8. $575 at 20%, 8%, and 5%.
9. $790 at 10%, 10%, and 10%.
10. $950 at 25%, 20%, and 10%.

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WRITTEN APPLICATIONS

1. Find the net cost of a piano listed at \$600, with discounts of 20% and 10%.

2. Find the net cost of a bill of dry goods amounting to \$240 at list prices with discounts of 20%, 15%, and 10%.

3. If a dealer buys a sewing machine at discounts of 20% and 10% from the list price, and then sells it at the list price, what is his per cent of profit?

4. The list price of a cultivator was \$18 and a dealer bought it at discounts of 10% and 5% from the list price, and sold it for \$20. How much was his profit?

5. A firm listed a set of books at \$90, less 20% and 10%. Another firm listed the same set of books at \$90, less 15% and 15%. Which is the better offer?

6. Goods amounting to \$1500 at list prices are bought by a wholesaler at discounts of 10% and 5%. He sells them at 8% and 2% from the same list prices. How much does he gain?

7. Discounts of 15%, 10%, and 5% are allowed on a bill of goods amounting to \$1200. Would the discount be greater or less if 5%, 10%, and 15% were allowed instead?

8. A piano is listed at \$750 with discounts of 20% and 20%. If the dealer sells it for \$600, what is the per cent of his profit?

9. A firm offers a machine at 25% off from a list price of \$275. A rival firm's discounts from the same list price for the same machine are 15%, 10%, and 5%. Which is the better offer?

10. Ten sewing machines listed at \$45 each are sold at 20 and 10 off. The buyer then sold them all at a profit of 25%. What was his net gain on all of them?

11. An agent for farm implements bought a 250-gallon sprayer listed at \$250, and was given discounts of 20% and 5%. He sold the sprayer to a farmer at a profit of 20% less 5% discount for cash. How much did the agent make on the sale? What did the farmer pay for the sprayer?

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Application of Discounts to Bills.

The following bill illustrates modern business practice in bills and discounts:

PHILADELPHIA, Jan 10 1919

PILLSBURY FURNITURE CO.

DEALERS IN

FINE FURNITURE, RUGS AND DRAPERIES

Sold to Raymond Bros.
Indianapolis Indiana

50 Morris Chairs @ $9.00		$450	00
10 Sheraton Tables @ 12.25		122	50
		$572	50
Less 10%		57	25
Net		$515	25

Sometimes the different items in a bill of merchandise are subject to discounts at different rates. In such cases the discount on each item must be deducted separately, and the total net amount of the bill is the sum of the net amounts charged for the different items.

WRITTEN APPLICATIONS

In each of the following exercises make out a bill, taking care to rule a sheet so that it is an exact copy of a modern bill-head. In all such work you must continually strive for

1. **Accuracy.**
2. **Clearness.**
3. **Neatness.**

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1. J. M. French bought of the Great Northern Furniture Company, 12 dining room tables at \$16.50 apiece; 36 chairs at \$2.25 apiece; 15 couches at \$10.75 apiece; 18 morris chairs at \$10.25 apiece. Terms, 15% and 10%. Find the net amount of the bill.

2. William L. Earle & Co. bought of the Sunset Fruit Company, 100 cases lemons at \$4.25 per case; 60 cases oranges at \$5.60 per case; 200 bunches bananas at \$1.35 per bunch; 30 dozen pineapples at \$1.20 per dozen. Terms, 20% and $12\frac{1}{2}\%$. Find net amount of bill.

3. Gibson & Wheeler bought of the Whitman Carpet Co., 200 yd. Brussels carpet at \$1.80 per yard; 300 yd. ingrain carpet at \$1.40 per yard; 145 $8'3'' \times 10'6''$ rugs at \$20 apiece; and 80 $6' \times 9'$ rugs at \$14 apiece. Terms, 25% and 5%. Find the net amount of the bill.

4. Mitchell & Hallman bought of Kent & Fraser, dealers in Men's Furnishings, 15 dozen ties at \$4.25 per dozen; 3 dozen ties at \$12.00 per dozen; 4 dozen ties at \$15.00 per dozen; 6 dozen gloves at \$15.00 per dozen; 18 dozen gloves at \$12.25 per dozen; 35 dozen leather belts at \$8.00 per dozen; 5 gross handkerchiefs at \$10.50 per gross. Terms, 10% and 5%. Find the net amount of the bill.

5. The Glenwood Hardware Company bought of the French Manufacturing Company, 5 dozen shovels at \$7.50 per dozen; 10 dozen iron rakes at \$2.60 per dozen; 25 dozen scythes at \$6.25 per dozen; 10 dozen scythe snaths at \$8.00 per dozen; 4 dozen jack planes at \$9.00 per dozen; 5 dozen smoothing planes at \$7.50 per dozen; 24 dozen chisels at \$6.10 per dozen; 12 dozen hammers at \$4.50 per dozen; 8 dozen screwdrivers at \$3.25 per dozen; 5 dozen cross-cut saws at \$8.50 per dozen; 3 dozen back saws at \$9.50 per dozen; and 4 dozen braces at \$9.00 per dozen. This bill was sold at net list prices, with 2% off for cash. Find the amount of the bill if the buyer took advantage of the cash discount.

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INTEREST

Interest is money paid for the use of money.

If you borrow $1000, and repay the loan at the end of 1 year, and if you pay $60 for the use of the money, the $60 is called Interest.

The Principal is the money on which interest is paid.

In the illustration $1000 is the principal.

The Time is the period for which the money is borrowed or loaned.

In the illustration 1 year is the time.

The Rate of Interest, or **the Rate**, is the number of hundredths of the principal paid as interest for one year.

In the illustration $60 is paid for the use of $1000.
What decimal part of $1000 is $60?

$$\frac{\$60}{\$1000} = \frac{6}{100} = .06.$$

But, expressed as a per cent, .06 = 6%.
That is, the rate of interest in the illustration is 6%.

Simple Interest is interest charged only on the principal. In this book the word " interest " used alone will always mean *simple* interest.

Problems in interest are really problems in percentage with the element of *time* added. In a problem in interest

The Principal = The Base
The Rate = The Rate Per Cent
The Interest = The Percentage

Several methods for computing interest are in common use, and it is difficult to give a single process that will meet all needs. The following methods are all practical, and you should master the one based upon a principal of one dollar, and the method of " Exact Interest."

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METHODS OF FINDING INTEREST

I. When the Time is Given in Years, Months, and Days

Illustrations:

1. Find the interest on \$900 for 3 years at 5%.

The interest for 1 year at 5% = .05 × \$900 = \$45.00.
The interest for 3 years at 5% = 3 × \$45.00 = \$135.00. *Result.*

2. Find the interest on \$900 for 3 years 5 months at 6%.

Expressed in years the time = $3\frac{5}{12}$ years.
The interest on \$900 for 1 year at 6% = .06 × \$900 = \$54.00.
The interest on \$900 for $3\frac{5}{12}$ years = $3\frac{5}{12}$ × \$54.00 = \$184.50. *Result.*

3. Find the interest on \$1200 for 4 years 3 months 12 days at 5%.

The time expressed in years = $4\frac{17}{60}$ years.
The interest on \$1200 for 1 year = .05 × \$1200 = \$60.00.
The interest on \$1200 for $4\frac{17}{60}$ years = $4\frac{17}{60}$ × \$60 = \$257.00. *Result.*

For a general expression of the process just illustrated we have
Multiply the principal by the rate.
Multiply the product by the time expressed in years.

BLACKBOARD PRACTICE

With the principal, the time, and the rate, as follows, find the interest on:

1. \$1000 for 4 yr. at 5%.
2. \$1200 for 3 yr. at 6%.
3. \$1750 for 5 yr. at $4\frac{1}{2}$%.
4. \$125.50 for 3 yr. at 5%.
5. \$225.75 for 5 yr. at 4%.
6. \$457.25 for 4 yr. at 5.4%.
7. \$700 for 2 yr. 6 mo. at 6%.
8. \$1400 for 3 yr. 3 mo. at 3%.
9. \$2100 for 4 yr. 10 mo. at 4%.
10. \$135.80 for 3 yr. 4 mo. at 5%.
11. \$457.75 for 22 yr. 7 mo. at 4%.
12. \$675.50 for 4 yr. 5 mo. at 5%.
13. \$300 for 2 yr. 6 mo. 15 da. at 4%.

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14. $450 for 3 yr. 8 mo. 18 da. at 5%.
15. $600 for 5 yr. 3 mo. 6 da. at 4½%.
16. $900 for 2 yr. 2 mo. 3 da. at 4½%.
17. $1250 for 3 yr. 11 mo. 24 da. at 5%.
18. $1800 for 2 yr. 9 mo. 20 da. at 5%.
19. $2500 for 3 yr. 7 mo. 25 da. at 6%.

II. Interest Based upon a Principal of One Dollar

The value of this method lies in the ease with which we may compute the interest on $1 for any given time at 6%.

1 month = $\frac{1}{12}$ year,
6 days = $\frac{1}{5}$ month,
1 day = $\frac{1}{6}$ of $\frac{1}{5}$ month, = $\frac{1}{30}$ month.

Then:

The interest on $1 for 1 year at 6% = $.06.
The interest on $1 for 1 month at 6% = $.005.
The interest on $1 for 6 days at 6% = $.001.
The interest on $1 for 1 day at 6% = $.000$\frac{1}{6}$.

These units multiplied by the given time expressed in years, months, and days, give the interest on $1 for that time at 6%.

Illustrations:

1. Find the interest on $250 for 3 yr. 5 mo. 12 da. at 6%.

Interest on $1 for 3 years at 6% = 3 × $.06 = $.18
Interest on $1 for 5 months at 6% = 5 × $.005 = .025
Interest on $1 for 12 days at 6% = 12 × $.000$\frac{1}{6}$ = .002
Interest on $1 for 3 years 5 months 12 days at 6% = $.207
Interest on $250 for 3 yr. 5 mo. 12 da. = 250 × $.207 = $51.75. *Result.*

2. Find the interest on $625.40 for 4 yr. 7 mo. 19 da. at 6%.

Interest on $1 for 4 years at 6% = $.24.
Interest on $1 for 7 months at 6% = $.035.
Interest on $1 for 19 days at 6% = $.003$\frac{1}{6}$.
Interest on $1 for 4 yr. 7 mo. 19 da. at 6% = $.278$\frac{1}{6}$.
Interest on $625.40 for 4 years 7 months 19 days at 6% = .278$\frac{1}{6}$ × $625.40 = $173.96543 or, $173.97. *Result.*

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3. Find the interest on \$450 from June 10, 1916, to April 28, 1919, at 6%.

The time between the two dates is found by subtraction as indicated at the right.

	yr.	mo.	da.
	1919	4	28
	1916	6	10
	2	10	18

That is, the time is 2 years 10 months 18 days.

Interest on \$1 for 2 years at 6% =	\$.12	\$450
Interest on \$1 for 10 months at 6% =	.05	.173
Interest on \$1 for 18 days at 6% =	.003	1350
Interest for the whole time at 6% =	\$.173	3150
		450
		\$77.850 *Result.*

BLACKBOARD PRACTICE

Find the interest on:

1. \$150 for 2 yr. 5 mo. 12 da. at 6%.
2. \$225 for 2 yr. 9 mo. 18 da. at 6%.
3. \$290.50 for 3 yr. 8 mo. 24 da. at 6%.
4. \$210.50 for 2 yr. 10 mo. 6 da. at 6%.
5. \$342.12 for 3 yr. 7 mo. 15 da. at 6%.
6. \$497.18 for 2 yr. 8 mo. 17 da. at 6%.
7. \$654.32 for 3 yr. 5 mo. 25 da. at 6%.
8. \$872.19 for 3 yr. 7 mo. 17 da. at 6%.
9. \$1024.17 for 3 yr. 11 mo. 5 da. at 6%.
10. \$3512.19 for 1 yr. 1 mo. 19 da. at 6%.
11. \$4647.20 for 2 yr. 10 mo. 27 da. at 6%.
12. \$869.15 for 1 yr. 8 mo. 26 da. at 6%.
13. \$250 from January 1, 1918, to March 1, 1919, at 6%.
14. \$375 from April 1, 1917, to July 1, 1919, at 6%.
15. \$500 from March 10, 1916, to May 16, 1918, at 6%.
16. \$275.50 from June 1, 1918, to July 19, 1919, at 6%.
17. \$457.80 from October 1, 1917, to January 10, 1920, at 6%.
18. \$625.50 from March 1, 1916, to October 16, 1919, at 6%.
19. \$1275 from April 11, 1915, to January 25, 1919, at 6%.

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20. $2500 from June 11, 1918, to July 29, 1919, at 6%.

21. $3000 from September 11, 1917, to June 30, 1918, at 6%.

22. $5400 from September 11, 1916, to January 19, 1920, at 6%.

To Change a Result Obtained at 6% to a Result at Any Other Given Rate.

The method just illustrated depends wholly upon a calculation at 6%, hence it is necessary to learn a method whereby a result may be changed to a rate other than 6%. The change is readily made by a simple operation in division with either addition or subtraction.

Illustrations:

Find the interest on $1200 for 1 yr. 5 mo. 10 da. at 7%.

By the method just learned:		
Interest on $1200 for 1 yr. 5 mo. 10 da. at 6% =	$104.00	
Now 7% = 6% + $\frac{1}{6}$ of 6% = 6% + 1%.		
Then, since $104 = interest at 6%,		
the interest at 1% = $104 ÷ 6 =	17.33	
Adding, interest at 7% =	$121.33	*Result.*

Interest at the rates most commonly used may be found readily from 6% results as follows:

For interest at 3%	Divide interest at 6% by 2.
For interest at 4%	Subtract $\frac{1}{3}$ of interest at 6%.
For interest at 4½%	Subtract $\frac{1}{4}$ of interest at 6%.
For interest at 5%	Subtract $\frac{1}{6}$ of interest at 6%.
For interest at 5½%	Subtract $\frac{1}{12}$ of interest at 6%.
For interest at 7%	Add $\frac{1}{6}$ of interest at 6%.

In Pennsylvania many interest contracts are made at a rate of 5.4%.

$$5.4\% = \frac{5.4}{6} \text{ of } 6\% = .9 \text{ of } 6\%.$$

Therefore, for interest at 5.4%, multiply interest at 6% by .9.

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BLACKBOARD PRACTICE

Change the following 6% results to results at the indicated rate:

To 3%:	To 4%:	To 5%:	To 7%:
1. $274.60.	5. $450.60.	9. $327.30.	13. $320.16.
2. $342.16.	6. $542.10.	10. $343.17.	14. $384.42.
3. $454.48.	7. $642.12.	11. $460.10.	15. $428.11.
4. $564.29.	8. $746.19.	12. $574.70.	16. $542.19.
To $4\frac{1}{2}$%:	**To $5\frac{1}{2}$%:**	**To 5.4%:**	**To $6\frac{1}{2}$%:**
17. $64.96.	21. $120.60.	25. $110.19.	29. $627.12.
18. $72.48.	22. $248.40.	26. $370.12.	30. $518.28.
19. $68.76.	23. $436.08.	27. $340.16.	31. $642.10.
20. $85.19.	24. $511.19.	28. $510.17.	32. $754.09.

The Amount is the sum of the principal and the interest.

If $1000 is loaned at 6% for 3 yr. the interest is $180.

$1000+$180=$1180, the amount due at the end of the third year.

III. When the Time is Included between Two Given Dates

The two methods that follow are of great practical value when the interest period is short, particularly when the time is less than one year. Both methods are extremely accurate, and the first is known as

(*a*) The Method of Exact Interest.

Illustration:

Find the interest on $450 from Nov. 12, 1918, to March 19, 1919, at 6%.

Counting the exact number of days between the two dates, we have Nov. (18)+Dec. (31)+Jan. (31)+Feb. (28)+March (19)=127 days.

Then, $\frac{127}{365}\times 6\%$ of $450 = the required interest.

By cancellation: $\frac{127\times 6\times 450}{365\times 100}=9.12^{+}$

Therefore, the required interest = $9.12. *Result.*

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This method is used by the United States Government and by many of the leading trust companies. It is also the method used by many of the states in their financial problems.

BLACKBOARD PRACTICE

Find the exact interest on

1. \$200 for 48 da. at 5%.
2. \$300 for 38 da. at 5%.
3. \$325 for 57 da. at 6%.
4. \$275 for 72 da. at 6%.
5. \$425 for 69 da. at 5%.
6. \$425 for 82 da. at 6%.
7. \$570 for 70 da. at 5½%.
8. \$625 for 87 da. at 4½%.
9. \$750 for 112 da. at 5.4%.
10. \$875 for 164 da. at 6%.
11. \$235 from April 7 to June 12 at 6%.
12. \$345 from May 10 to October 16 at 5%.
13. \$457 from June 3 to August 12 at 6%.
14. \$125.60 from March 11 to September 20 at 5%.
15. \$175.90 from April 17 to December 19 at 6%.
16. \$240.24 from May 9 to December 31 at 6%.
17. \$275.37 from October 10 to March 17 at 5½%.
18. \$310.79 from August 13 to January 17 at 6%.
19. \$427.52 from September 19 to February 5 at 5.4%.
20. \$510.43 from December 30 to May 9 at 6%.
21. \$570.64 from July 12 to January 19 at 6%.
22. \$627.45 from April 15 to March 14 at 5%.
23. \$895.56 from May 23 to February 4 at 6%.
24. \$900.00 from July 19 to October 27 at 5%.
25. \$975.50 from November 19 to March 28 at 6%.
26. \$1000.00 from December 18 to September 18 at 6%.
27. \$1125 from January 19 to August 17 at 5%.
28. \$1240.12 from February 17 to January 11 at 5½%.

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(*b*) The "Banker's Time" Method.

Bankers often calculate time by months and days, using the number of months and the exact number of days for the fractional part of the month in the given time.

Illustration:

Find the interest on $450 from Sept. 12, 1917, to June 19, 1918, at 6%.

From September 12 to June 12 is 9 months.
From June 12 to June 19 is 7 days. Hence, the time is 9 mo. 7 da.
Interest on $1 for 9 mo. at 6% = $.045
Interest on $1 for 7 da. at 6% = $.001\frac{1}{6}$
Interest on $1 for 9 mo. 7 da. at 6% = $.046\frac{1}{6}$
Then, $.046\frac{1}{6} \times \$450 = \$20.775$, or $20.78. *Result.*

BLACKBOARD PRACTICE

Using "Banker's Time" method and a rate of 6%, find the interest on:

1. $150 from July 10 to October 8.
2. $200 from May 7 to August 15.
3. $375 from February 9 to June 17.
4. $450 from April 17 to October 11.
5. $625 from June 10 to December 20.
6. $750 from October 11 to February 20.
7. $925 from January 11 to October 15.
8. $1100 from May 15 to December 18.
9. $1500 from August 11 to January 19.
10. $1650 from November 18 to May 11.
11. $1875 from January 31 to October 9.
12. $1576.50 from December 18 to February 10.
13. $3750.25 from May 11 to June 5.
14. $5500 from December 30 to February 28.
15. $8500 from October 18 to December 31.

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IV. Interest Calculated from the Interest Table

For the use of bankers and business men whose work requires frequent calculations of interest, there are published complete tables from which the interest on any amount for any given time may be rapidly calculated. The following are incomplete portions of a page from one of the standard tables in common use. The table illustrated is based upon 360 days to the year, and a rate of 6%.

Years	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	60	120	180	240	300	360	420	480	540
2	120	240	360	480	600	720	840	960	1080
3	180	360	540	720	900	1080	1260	1440	1620
...	...	...	...	...	...	...	...	...	...
Months	**$1000**	**$2000**	**$3000**	**$4000**	**$5000**	**$6000**	**$7000**	**$8000**	**$9000**
1	5	10	15	20	25	30	35	40	45
2	10	20	30	40	50	60	70	80	90
3	15	30	45	60	75	90	105	120	135
...	...	...	...	...	...	...	...	...	...
Days	**$1000**	**$2000**	**$3000**	**$4000**	**$5000**	**$6000**	**$7000**	**$8000**	**$9000**
1	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50
2	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	43.00
3	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	.50
4	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00
5	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50
6	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
7	1.167	2.333	3.50	4.667	5.833	7.00	8.167	9.333	10.50
8	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00
...	...	...	...	...	...	...	...	...	...

In this table the totals are based upon a principal of $1000. For hundreds and tens of dollars, therefore, we may take decimal parts of the totals given, remembering that

$100 is .1 of $1000.
$10 is .01 of $1000, etc.

After finding the required interest at 6% by use of the table, the result may be changed to any other rate as already shown.

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Illustration:

Find the interest on \$3250 for 3 yr. 3 mo. 8 da. at 6%.

The following form is convenient:

	3 yr.	3 mo.	8 da.
Interest on \$3000 =	\$540	\$45	\$4
Interest on \$200 =	36	3	.267
Interest on \$50 =	9	.75	.067

Interest on \$3250 = \$585 + \$48.75 + \$4.334 = \$638.08. *Result.*

BLACKBOARD PRACTICE

Using the table, find the interest on

1. \$150 for 2 yr. at 6%.
2. \$275 for 3 yr. at 6%.
3. \$400 for 3 yr. at 6%.
4. \$550 for 4 yr. at 5%.
5. \$750 for 4 yr. at 7%.
6. \$450 for 2 yr. 7 mo. at 6%.
7. \$625 for 3 yr. 5 mo. at 5%.
8. \$750 for 3 yr. 2 mo. at 7%.
9. \$900 for 3 yr. 5 mo. at 5½%.
10. \$1250 for 2 yr. 5 mo. at 6%.
11. \$1225 for 2 yr. 5 mo. 10 da. at 5%.
12. \$2345 for 3 yr. 6 mo. 5 da. at 6%.
13. \$3550 for 3 yr. 5 mo. 3 da. at 6%.
14. \$4575 for 3 yr. 5 mo. 5 da. at 5%.
15. \$5870 for 3 yr. 6 mo. 15 da. at 6%.
16. Find the interest on \$450 from January 10 to September 10 at 6%.
17. Find the interest on \$1200 from March 11 to August 19 at 5%.
18. Find the interest on \$6000 from November 10, 1915, to September 12, 1916, at 5%.
19. A note for \$250 is given on March 12, 1918, for six months, and is renewed on September 12 for six months longer. Find the interest for the whole time at 6%.
20. Two notes for \$500 each are given on May 1, 1918. One is due March 3, and the other June 1, 1919. Find the total amount of both notes, the interest rate being 5%.

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SAVING AND INVESTING MONEY

I. Savings Bank

A Savings Bank is an institution that receives and safeguards the savings of individuals, paying them a stated rate of interest for the use of their money.

Savings Banks are under the rigid control of the State Governments.

Savings in the United States. The growth of the savings banks in the United States is an interesting study. In 1850 there were 108 savings banks in this country, and 251,354 depositors had to their credit a total of $43,431,130. In 1918 there were 1819 savings banks and 11,379,553 depositors had to their credit a total of $5,471,579,948. The average amount on deposit per person in 1850 was $172.78, but in 1918 this average had grown to $480.82.

Interest on Deposits. Most savings banks compute interest twice each year, usually on January 1 and July 1. The depositor may withdraw his interest, but if he does not withdraw it, the bank credits it to his account.

While the custom varies, most savings banks designate certain periods in which deposits begin to draw interest. In such cases deposits made after fixed dates do not begin to draw interest until the beginning of the next stated period. For purpose of illustrating, the exercises that follow are based upon the following dates:

Deposits made	Draw Interest
From Jan. 1 to Jan. 15,	From Jan. 1.
From Jan. 16 to Apr. 1,	From Apr. 1.
From July 1 to July 15,	From July 1.
From July 16 to Oct. 1,	From Oct. 1.

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How Savings Banks Calculate Interest.

Illustration:

On July 1, Donald deposited $10 in a savings bank; on September 20, $30; and on November 5, $20. Find his total credit on April 1, if the bank pays 4%, and credits interest on January 1 and July 1, if interest is allowed as indicated below.

July 1, Deposit	$10.00
Interest on $10, to Jan. 1	.20
Sept. 20, Deposit	30.00
Interest on $30, to Jan. 1	.30
Nov. 5, Deposit	20.00
(No interest allowed. See note.)	.00
Amount on deposit, Jan. 1	60.50
Interest on $60 to April 1	.60
Amount of deposit, April 1	61.10

The deposit of $20 in November having been made after Oct. 1, does not begin to bear interest until Jan. 1, hence no interest was allowed. Savings banks usually disregard the cents when calculating interest; thus, if a total deposit is $125.19, they calculate interest on $125.

WRITTEN APPLICATIONS

1. A deposit of $200 was made in a savings bank on January 1, 1918. If the bank credits interest on July 1 and January 1, find the amount of this deposit Jan. 1, 1919, the rate being 4%.

2. A deposit of $350 was made in a savings bank on Jan. 1, 1916. If the bank credits interest on April 1, July 1, October 1, and January 1, find the amount of this deposit on Jan. 1, 1917, the rate being 4%.

3. If, in the bank mentioned in example 2, a deposit of $100 is made on each of the days on which interest is credited, find the amount to the credit of the depositor on Jan. 1, 1917, the first deposit having been made on Apr. 1, 1916.

4. $1000 is deposited in a bank paying 3% on Jan. 1, 1916. $400 is withdrawn on July 2. Find the amount to the credit of the depositor on Jan. 1, 1917, if the bank calculates and credits its interest only semi-annually.

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II. The Liberty Loan Bonds

A Bond is a formal written promise to pay a specified sum at a given time.

The Liberty Loan Bonds are United States Government Bonds, and include four issues.

The First Liberty Loan bears interest at $3\frac{1}{2}\%$ and is due June 15, 1947.

The Second Liberty Loan bears interest at 4% and is due November 15, 1942.

The Third Liberty Loan bears interest at $4\frac{1}{4}\%$ and is due September 15, 1928.

The Fourth Liberty Loan bears interest at $4\frac{1}{4}\%$ and is due October 15, 1938.

The Victory Loan bears interest at $4\frac{3}{4}\%$ and is due May 20, 1923.

As the third and fourth loans were issued at a rate of interest higher than that of the second, the Government offered to exchange 4% bonds for $4\frac{1}{4}\%$ bonds, provided the change was made before November 9, 1918.

Investment Value of the Liberty Loan Bonds.

Illustration:

Suppose you bought a Fourth Liberty Loan Bond of the \$100 denomination.

Your annual income from the bond at the rate of $4\frac{1}{4}\%$ will be

$$4\frac{1}{4}\% \text{ of } \$100 = .0425 \times \$100 = \$4.25.$$

On October 15, 1938, your bond will be paid, by the United States Government, and you will be paid \$100 in gold.

The interest on the Liberty Loan Bonds is payable semi-annually, or every six months. Interest on these bonds is paid by coupons, which are attached to the bond, and for which any bank will pay cash. The bonds are issued in denominations of \$50, \$100, \$500, \$1000, \$5000, \$10,000, \$50,000, and \$100,000.

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WRITTEN APPLICATIONS

1. Find the annual income on $1000 invested in First Liberty Loan Bonds.

2. Find the annual income on $3500 invested in Third Liberty Loan Bonds.

3. A man invested $200 in each of the first four Liberty Loans. What is the annual income from these investments?

4. A man has $3000 invested in the First Liberty Loan, $4000 in the Second, $5000 in the Third, and $8000 in the Fourth. What is his total annual income from these four investments?

5. The total subscription to the First Liberty Loan Bonds was $3,035,226,850. Find the annual interest payment made by the Government on this loan.

6. The total number of subscribers to the First Liberty Loan was 4,500,000. What was the average amount subscribed by each subscriber?

7. The total subscriptions to the Third and Fourth Liberty Loans were $4,176,516,850, and $6,989,047,000, respectively. Find the annual interest payment made by the Government on these two loans.

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EIGHTH GRADE

BANKING

A Bank is an institution for lending, issuing, or caring for money, and for performing other financial service.

From the standpoint of organization and control, banks may be classed as *national*, *state*, and *private*.

A **National Bank** is a bank authorized and inspected by the United States Government. It has the right to issue national bank notes which are used as money.

A **State Bank** is a bank authorized and inspected by a State Government.

A **Private Bank** is conducted by an individual or company without the inspection or control of either the Federal or State Governments.

From the nature of the business which they do, banks are classed as *Commercial banks*, *Savings banks*, or *Trust companies*.

A **Commercial Bank**, sometimes called a Bank of Deposit, receives money for safe-keeping, makes loans, cashes checks and drafts, collects accounts, issues letters of credit, and performs other kinds of financial service.

A **Savings Bank** receives and invests savings and pays interest on deposits at stated times.

A **Trust Company** is empowered by state law to accept and execute trusts, to receive deposits of money and other personal property, and to lend money on real and personal security.

Federal Reserve Banks are central or regional banks under the control of the United States Government. They are often referred to as "the bankers' banks," and their character and work will be discussed after general banking has been described.

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THE PRACTICE OF GENERAL BANKING

Opening an Account is the first step in securing the services of a bank. The depositor, or customer, places money in the bank to be credited to his account. If he is not personally known to some official in the bank, he must first be "identified," that is, he must be introduced by a reliable person known to the bank. His signature is recorded by the bank in order to verify it when signed to checks or to other documents.

A Deposit Slip, describing in detail the items of his deposit, is made out by the depositor and presented to the teller.

A Pass Book is given him, in which are entered the sums deposited from time to time.

A Check Book is also given him to enable him to order the bank to pay various sums from his account.

OPENING AN ACCOUNT WITH A BANK

DEPOSITED IN
FORT DEARBORN NATIONAL BANK
For Account of
Frederick Anderson
Chicago Feb 10 1919

Checks on Chicago P. O. and Express Orders		Items Outside Chicago	
$38	29	$39	40
48	37	19	34
Currency		75	00
Gold		20	00
Silver		15	75
Total		$256	15

At the left is shown a deposit slip filled out to open an account for Mr. Frederick Anderson in the Fort Dearborn National Bank, Chicago, Illinois. The deposit slip provides a space in which the depositor enters local checks, as well as postal and express money orders; while checks payable by banks outside of Chicago are listed in a separate group. The deposit of $256.15 is credited to Mr. Anderson on the books of the bank, and a similar entry is made in his pass book.

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WRITTEN APPLICATIONS

Make out in your own name deposit slips for each of the following:

1. Currency, $48.00; Gold, $15.00; Silver, $18.85. Checks: First National of Minneapolis, $24.90; Commonwealth Trust Co. of Philadelphia, $151.65; First National of Chicago, $200.00.

2. Currency, $165.00; Gold, $35; Silver, $18.30. Checks: Fourth National of Minneapolis, $49.75; Franklin Trust Co. of New York, $54.10; Farmer's National of Albany, $162.40; City Bank of San Francisco, $119.87.

3. Currency, $245.00; Silver, $19.70. Checks: First National of Denver, $136.00; Mechanics National of Worcester, $450.00; Corn Exchange of New York, $178.50; Fidelity Trust Co. of Kansas City, $67.00.

4. Currency, $275; Gold, $20; Silver, $19.90. Checks: First National of Detroit, $287.10; Commercial Trust Co. of Philadelphia, $98.15; Franklin Realty Trust Co. of Boston, $175.90; Malden Savings Bank, $196.12.

5. Checks: Second National of Cleveland, $276.50; Rittenhouse Trust Co. of Philadelphia, $115; Chemical National Bank of New York, $1500. Currency, $276. Silver, $269.40.

6. Currency, $376; Gold, $45; Silver, $198.70. Checks: Hanover National of New York, $1100; Mechanics National of Worcester, $378.50; Central Union Trust Co. of New York, $1156.20; Fort Dearborn National of Chicago, $3120.57.

7. Currency, $119; Gold, $45; Silver, $37.20. Checks: First National of Chicago, $145.00; First National of Galesburg, $100.00; Third National of Philadelphia, $154.50; Fletcher Trust Co. of Indianapolis, $95.75; First National of Boston, $85.10; Second National of Cleveland, $45.00; Chemical National of New York, $275.00; First National of Augusta, Ga., $78.75.

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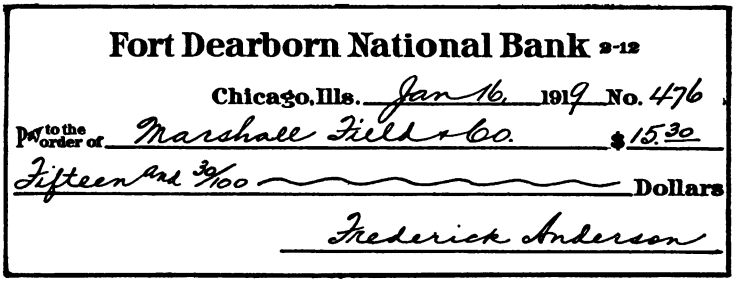
A Bank Check. A bank check is a written order made by a depositor directing the bank to pay a certain sum from his deposit.



Fort Dearborn National Bank 2-12

Chicago, Ills. Jan 16, 1919 No. 476

Pay to the order of Marshall Field & Co. $15.30

Fifteen and 30/100 Dollars

Frederick Anderson

The use of checks simplifies the transacting of business, for it permits the payment of bills or other obligations without the transfer of actual cash. Another advantage in the use of checks is that the accurate record of the banks who handle them serves as a verification in case of need. Moreover, the loss of a check is not usually a serious matter, for the maker may stop the payment of it, and, at a later date, issue a duplicate.

Forms of Indorsement. Before a bank pays a check it requires upon the back of the check the signature of the person to whom payment is made. This indorsement is a receipt from the payee to the bank making the payment.

When a check is made payable to an individual " or order " it is *negotiable;* that is, it may be collected by any one known to the bank to which it is presented. When a person or firm mails a check for deposit, it is customary to indorse it with the phrase " For Deposit Only," or, " For Deposit to the credit of " followed by the name of the payee. No bank would pay a check indorsed in this way if, through loss or otherwise, it fell into the wrong hands

WRITTEN APPLICATIONS

1. Write a check for one hundred dollars, dated to-day, payable to Henry W. Breed, drawn on the Union Bank of Indianapolis, and signed by yourself.

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2. Write a check for forty-seven dollars and fifty-eight cents, dated to-day, payable to Charles H. Bronson, and drawn on the Commonwealth Trust Company of Philadelphia, and signed by Thomas B. Wilcox.

3. Write a check for two hundred fifty dollars, dated January 10, 1919, payable to Thomas D. Raymond, and drawn on the Mechanics National Bank of Birmingham, Ala., and signed by Charles H. Swift.

4. Write a check for one hundred seventy-eight dollars and eleven cents, dated October 12, 1918, payable to George Carr, and drawn on the First National Bank of Louisville, Ky., by William White.

5. Write a check for fourteen dollars, dated to-day, payable to yourself, and drawn on the West End Trust Company of Philadelphia, Pa., by Thomas Stocking. Indorse this check yourself, and show the subsequent indorsement of the Cook Hardware Company, to whom you gave it in payment of a bill.

6. Write a check for seventy-five dollars, dated July 11, 1919, payable to yourself, and drawn on the Corn Exchange Bank of New York City by J. M. Green. Show how you would indorse this check in mailing it to your bank for deposit.

Keeping a Record of a Bank Account. The depositor should keep a careful record of his deposits and withdrawals. Such a record is kept on the check stubs, and without it he is liable to draw checks for more money than he has to his credit.

WRITTEN APPLICATIONS

1. A merchant's bank balance on June 30 is $1275.90. During July he deposits $1871.95, and checks out $2705.10. What is his balance on July 31?

2. A merchant's bank balance on August 1 was $549.20, and during the month he deposited $2735.60 and checked out $1988.54. What was his balance on August 31?

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3. On the first day of the month a business firm has a balance on hand of $2140.15, and in that month their deposits are $480.19, $317.42, $694.48, $510.00, $178.10, $1039.54, $687.07, and $2119.56. In the same month the firm drew checks amounting to $4566.18. Find the balance on deposit to the firm's credit at the end of the month.

4. During the week of April 2, 1918, a manufacturer made deposits of $2715.10, $3510.19, $1750, $1862.57, $697.25, and $598.69, on the six business days respectively. He withdrew $125 on each of the first five days of the week, and $2490.50 on Saturday. Find the difference between the total of the deposits and the total of the withdrawals for that week.

5. On Monday morning at the beginning of business a merchant had a balance of $2917.12 in the bank. On that day he deposited $890 and checked out $1137.12. On Tuesday he deposited $762.40 and checked out $196.37; on Wednesday he deposited $1317 and checked out $2762.10; and on Thursday he deposited $1678.47 and checked out $584.14. Find the amount of his balance at the end of each of the four business days.

6. At the beginning of a week the balance on deposit in an account was $241.50, and deposits of $294.50, $161.20, $351.80, $96.42, $171.11, and $840 were made on six consecutive days. The daily withdrawals for the same six days were respectively $164.10, $35.90, $246.91, $101.87, $211.19, and $67.85. What was the balance remaining at the end of the week?

7. At the opening of business on a certain Monday morning a merchant's balance in the bank was $1974.38. On that day he deposited $690.11 and checked out $482.10. On Tuesday he deposited $542.13 and checked out $376.14; on Wednesday he deposited $493.81; on Thursday he checked out $750; on Friday he deposited $1109.18 and checked out $1500; and on Saturday he deposited $2118 and checked out $2545.50. What was the amount of his balance at the close of business on Saturday?

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BANK DISCOUNT

Borrowing Money from a Bank. In carrying on their business, most business men have need at times of more money than they have on hand. In such cases the banks loan them money on promissory notes, which vary in form.

Bank Discount is the simple interest on the money loaned, collected in advance for the time for which the loan is made.

The Term of Discount is the number of days from the day on which the note is discounted to the day on which the note is due.

The Face of a Note is the sum written in the note.

The Proceeds of a Note are the balance remaining after deducting the discount from the face of the note.

Illustration:

If \$2500 is borrowed from a bank for 60 days at 6%, the bank calculates the interest on \$2500 for 60 days, or \$25.

This interest, or *discount*, is deducted from the face of the note. And, \$2500 − \$25, or \$2475, is paid over to the borrower.

In this case:

The Face of the Note = \$2500.
The Term of Discount = 60 days.
The Rate of Discount = 6%.
The Bank Discount = \$25.
The Proceeds = \$2475.

At the end of the 60 days, when the note is due, the borrower pays the bank \$2500.

WRITTEN APPLICATIONS

Find the discount and the proceeds when the face, the term of discount, and the rate, respectively, are:

1. \$300, 30 days, 6%.
2. \$350, 60 days, 6%.
3. \$450, 60 days, 6%.
4. \$575, 90 days, 5%.
5. \$750, 60 days, 6%.
6. \$1000, 30 days, 6%.
7. \$1500, 60 days, 5%.
8. \$2250, 90 days, 6%.
9. \$2500, 90 days, 5%.
10. \$3000, 60 days, 6%.

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WRITTEN APPLICATIONS

1. Make out a 60-day note for $100, dated to-day, and payable by yourself at your local bank.

2. Make out a 60-day note for $350, dated to-day, and payable by John Fox at the First National Bank of Montgomery, Alabama.

3. Make out a 30-day note for $500, signed by Harrison Field, payable at the First National Bank of Chicago, and dated August 4, 1919.

4. Make out a note for $500, dated to-day, and signed by yourself. If this money is borrowed from your local bank, find the proceeds if the note runs 60 days.

5. Make out a note in which French & Monroe, merchants, borrow $2500 from the Fort Dearborn National Bank of Chicago for 60 days, and find the proceeds of this note if the interest rate is 5%.

6. Adams, Jackson & Co. borrow $3000 from the Corn Exchange Bank of New York on June 1, 1919. If the note runs 60 days, and if the rate of discount is 5%, find the proceeds of the note. Make a note to show the transaction.

7. Flint and Harrison borrow $4500 from the First National Bank of Chicago, and their note is dated March 10, 1919. The note is to run 60 days, and the bank discounts it on the date it is made, at 5½%. Find the proceeds of the note. Make a note to show the transaction.

If a note falls due on a Sunday or on a legal holiday its payment is due, in most states, on the next succeeding business day, and the term of discount includes that day. A bank usually demands that a note which it has discounted shall be payable at that bank.

Discounting Interest-bearing Notes. A business man frequently holds the note of another business man or firm, and needs to use the money before the note legally becomes due. While he cannot compel the payment of the note before the end of the time named in it, he can obtain the cash by discounting it at his bank.

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Illustration:

A merchant holds a note for $500 dated April 10. The note is to run 4 months and bears interest at 5%, but on May 29 the merchant finds immediate need for the money and discounts the note at his bank. The bank actually purchases the note, and determines the amount to be paid to the merchant by the following process.

The note is due 4 months after April 10, or on August 10.

The bank takes it on May 29, hence, they must hold it from May 29 to August 10, or 73 days.

The interest on the note from April 10 to August 10 is $8.33.

Therefore, the note is worth on August 10, $508.33.

The bank must wait 73 days for this amount, or must advance this amount less the interest upon it for that time at 6%.

The discount, therefore, is the interest on $508.33 for 73 days at 6%.

This discount is $6.10. (By Method of Exact Interest, p. 100.)

Therefore, the holder of the note receives for it $508.33 − $6.10, or $502.23, the proceeds.

The actual value of the note at maturity is its face value plus the interest at the rate named in the note for the full time of the note.

The actual amount received by the holder when he discounts it, or the *proceeds*, is the value of the note at maturity less the interest on that amount from the day he discounts it to the day it is due, at the bank's rate of discount.

WRITTEN APPLICATIONS

In each of the following find (1) the date of maturity, (2) the term of discount, (3) the bank discount, and (4) the proceeds:

1. A 60-day note for $1000 without interest and dated March 1 is discounted on the same day at 6%.

2. A 60-day note for $1100 without interest and dated April 20 is discounted on the same day at 5%.

3. A 90-day note for $100 bearing interest at 6% and dated March 10 is discounted April 9 at 5%.

4. A 60-day note for $1000 bearing interest at 5% is dated January 1 and is discounted on that day at the same rate.

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5. A 30-day note for $1200 bearing interest at 5% is dated March 10, and is discounted on March 16 at the same rate.

6. A 90-day note for $1500 bearing interest at 5% is discounted 30 days after date at 6%.

7. A 90-day note for $2000 bearing interest at 6% is discounted 30 days after date at 5%.

8. A note dated June 1 is to run 60 days. The face is $1200, and the interest rate 5%. The note is discounted June 15 at 5%.

9. Find the proceeds of the following note.

$250. SPRINGFIELD, ILLINOIS, Jan. 10, 1919.

Sixty days after date, for value received, I promise to pay George M. Stacey, or order, two hundred fifty dollars, with interest at 6%, at the First National Bank.

Discounted, Jan. 25, at 5%. F. W. WILSON.

10. Find the proceeds of the following note.

$450. RICHMOND, VIRGINIA, August 1, 1919.

Ninety days after date, for value received, I promise to pay Brown, Nash & Company, or order, four hundred fifty dollars, with interest at 5%, at the Fourth Street National Bank.

Discounted, September 20, at 6%. R. M. LANSING.

Finding the Face of a Note when the time, the rate, and the proceeds are given.

Illustration:

James Mills wishes to borrow money for 60 days to pay a debt of $1980. At 6% interest, what will be the face of his note?

The interest on $1.00 for 60 days at 6% = $.01.

Hence, the proceeds of $1.00 for 60 days at 6% will be

$1.00 − $.01 = $.99.

Dividing the given proceeds, $1980, by the proceeds of $1 for the given time and at the given rate, we have

$1980 ÷ $.99 = $2000.

That is, he must borrow $2000 in order that the proceeds immediately available shall be $1980.

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WRITTEN APPLICATIONS

1. The proceeds of a note for 90 days at 6% were $295.50. What was the face of the note?

2. The proceeds of a note for 60 days at 5% were $1487.50. What was the face of the note?

3. The proceeds of a note for 60 days at 6% were $495. What was the face of the note?

4. The proceeds of a note for 90 days at 6% were $641.87. What was the face of the note?

5. A merchant wishes to borrow $2000 for 3 months at 6%. For what amount shall he give his note?

6. A merchant wishes to raise $2000 by having his 60-day note discounted at his bank at 5%. For what amount shall he give his note?

7. For what amount must a 4-months note bearing interest at 6% be written if the maker wishes to raise exactly $1200 by discounting it on the day it is written?

8. For what amount must a real estate dealer make a 90-day note so that a trust company will discount it at 4½% and turn over $20,000 to him?

9. In paying for a new house a man needed $3500. If he borrowed this sum from his bank, and on a note made to run 90 days and bearing 6% interest, for what sum did he write his note so as to obtain just the sum he needed?

10. Write a note dated at Grand Rapids, Michigan, May 1, 1919, made by Robert Corson for $150, and payable at the First National Bank of Grand Rapids, with 6% interest; the note to be payable to the order of Everett Kellogg, and to run 90 days.

11. Write a note dated at Topeka, Kansas, November 15, 1919, payable after 60 days to the Monroe-Franklin Company, at the First National Bank of Topeka, with 6% interest, and signed by Fels & Babbitt for $1200. Discount this note on November 27 at 4%, and find the proceeds.

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COMPOUND INTEREST

Compound Interest is interest on both the principal and the unpaid interest, the principal and interest being combined at regular intervals.

Illustration:

$100 at 6% interest for 1 year brings interest = $6.

$100 + $6 = $106, the amount drawing interest at beginning of 2d year.

$106 at 6% interest for 1 year brings interest = $6.36.

$106 + $6.36 = $112.36, the amount drawing interest at beginning of 3d year.

$112.36 at 6% interest for 1 year brings interest = $6.74.

$112.36 + $6.74 = $119.10, the amount drawing interest at beginning of 4th year.

In the illustration the interest is calculated and added annually.

Interest may be added, or *compounded*, annually, semi-annually, quarterly, or at any other intervals agreed on in a contract.

Most of the savings banks allow compound interest when sums deposited bear interest for a full period.

Application of Compound Interest

I. To calculate the compound interest on a given principal, for a given time, at a given rate.

Illustration:

Find the compound interest on $300 for 3 yr. 6 mo. at 6%.

Principal	$300.00
Interest for 1 yr. at 6%	18.00
Principal at beginning of 2d yr.	318.00
Interest for 2d yr. at 6%	19.08
Principal at beginning of 3d yr.	337.08
Interest for 3d yr. at 6%	20.22
Principal at beginning of 4th yr.	357.30
Interest for 6 mo.	10.72
Amount of $300 for 3 yr. 6 mo. at 6%	368.02

Then: $368.02 − $300.00 = $68.02, the compound interest. *Result.*

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WRITTEN APPLICATIONS

Find, when compounded annually, the interest on

1. \$400 for 3 yr. at 6%.
2. \$500 for 3 yr. at 5%.
3. \$600 for 4 yr. at 4%.
4. \$1000 for 5 yr. at 6%
5. \$1500 for 4 yr. at 6%.
6. \$300 for 2 yr. 6 mo. at 6%.
7. \$400 for 3 yr. 4 mo at 5%.
8. \$450 for 3 yr. 6 mo. 10 da. at 6%.
9. \$525.50 for 2 yr. 5 mo. 20 da. at 6%.
10. \$754.50 for 3 yr. 2 mo. 15 da. at 6%.

11. Find the interest, compounded semi-annually, on \$1200 for 2 yr. 6 mo. at 3%.

12. Find the amount of \$450 for 2 yr. 3 mo., compounded annually at $1\frac{1}{2}$%.

II. To find the compound interest on a given principal, for a given time, and at a given rate, by use of the Interest Table.

The Compound Interest Table is constantly used by savings banks, bond brokers, and insurance companies. The table gives the amount of \$1 for periods from 1 to 20 years, and at different rates. The use of the table is readily understood from the

Illustration:

Find the compound interest on \$500 for 6 years at 4%.

In the column headed "4%" and opposite "6" in the column of "years" we find ..... 1.265319.

This number is the amount of \$1 for the given time at the given rate.

Therefore, the amount of \$500 = 500 × \$1.265319 = \$632.6595.

And: \$632.66 − \$500.00 = \$132.66, the compound interest required.

Result.

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COMPOUND INTEREST TABLE

Amount of $1, at various rates, compound interest, 1 to 20 years

YEARS	1%	1¼%	1½%	2%	2½%	3%
1	1.010000	1.012500	1.015000	1.020000	1.025000	1.030000
2	1.020100	1.025156	1.030225	1.040400	1.050625	1.060900
3	1.030301	1.037971	1.045678	1.061208	1.076891	1.092727
4	1.040604	1.050945	1.061364	1.082432	1.103813	1.125509
5	1.051010	1.064082	1.077284	1.104081	1.131408	1.159274
6	1.061520	1.077383	1.093443	1.126162	1.159693	1.194052
7	1.072135	1.090850	1.109845	1.148686	1.188686	1.229874
8	1.082857	1.104486	1.126493	1.171659	1.218403	1.266770
9	1.093685	1.118292	1.143390	1.195093	1.248863	1.304773
10	1.104622	1.132271	1.160541	1.218994	1.280085	1.343916
11	1.115668	1.146424	1.177949	1.243374	1.312087	1.384234
12	1.126825	1.160755	1.195618	1.268242	1.344889	1.425761
13	1.138093	1.175264	1.213552	1.293607	1.378511	1.468534
14	1.149474	1.189955	1.231756	1.319479	1.412974	1.512590
15	1.160969	1.204829	1.250232	1.345868	1.448298	1.557967
16	1.172579	1.219889	1.268986	1.372786	1.484506	1.604706
17	1.184304	1.235138	1.288020	1.400241	1.521618	1.652848
18	1.196148	1.250477	1.307341	1.428246	1.559659	1.702433
19	1.208109	1.266108	1.326951	1.456811	1.598650	1.753506
20	1.220190	1.281935	1.346855	1.485947	1.638616	1.806111

YEARS	3½%	4%	4½%	5%	6%	7%
1	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.272279	1.315932	1.360862	1.407100	1.503630	1.605782
8	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.511069	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.563956	1.665074	1.772196	1.885649	2.132928	2.409845
14	1.618695	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.675349	1.800944	1.935282	2.078928	2.396558	2.759032
16	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.794676	1.947901	2.113377	2.292018	2.692773	3.158815
18	1.857489	2.025817	2.208479	2.406619	2.854339	3.379932
19	1.922501	2.106849	2.307860	2.526950	3.025600	3.616528
20	1.989789	2.191123	2.411714	2.653298	3.207136	3.869684

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ORAL PRACTICE

Find in the table and read the amount of \$1 for

1. 3 years at 2%.
2. 4 years at 3%.
3. 4 years at 4%.
4. 5 years at 5%.
5. 6 years at 6%.
6. 7 years at 4%.
7. 8 years at 4%.
8. 10 years at 5%.
9. 10 years at $3\frac{1}{2}$%.
10. 10 years at 6%.
11. 10 years at $1\frac{1}{2}$%.
12. 11 years at $2\frac{1}{2}$%.
13. 12 years at 4%.
14. 15 years at 5%.
15. 16 years at 3%.
16. 17 years at $4\frac{1}{2}$%.
17. 19 years at 7%.
18. 20 years at 5%.

In compound interest the practice is to compound annually unless otherwise specified. If compounded semi-annually, or twice each year, the rate at each compounding is one half of the given rate; if quarterly, the rate is one fourth the given rate.

Illustration:

Find the amount of \$1000 for 5 years at compound interest, compounded semi-annually at 5%.

Since the interest is compounded semi-annually, there are 10 periods.
The rate for each is one half the given rate, or $2\frac{1}{2}$%.
Therefore, find the amount of \$2000 for 10 years at $2\frac{1}{2}$%.
Using the table:
Amount of \$1 for 10 years at $2\frac{1}{2}$% = \$1.280085 (10th line, 5th column).
Hence, the amount of \$1000 for the same time at the same rate is
$1000 \times \$1.280085 = \$1280.085$. \$1280.09. *Result.*

WRITTEN APPLICATIONS

Using the table, find the compound interest on

1. \$1200 for 3 yr. 6 mo. at 6%.
2. \$1200 for 4 yr. 4 mo. at 6%.
3. \$1500 for 5 yr. 6 mo. at 5%.
4. \$1500 for 4 yr. 9 mo. at $4\frac{1}{2}$%.
5. \$1500 for 6 yr. 6 mo. at 4%.

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6. $1875 for 4 yr. 9 mo. at 3½%.
7. $2000 for 5 yr. 4 mo. at 4½%.
8. $2100 for 6 yr. 3 mo. at 3½%.
9. $2400 for 4 yr. 10 mo. at 5%.
10. $256.50 for 4 yr. 6 mo. 15 da. at 4%.
11. $350.25 for 5 yr. 8 mo. 10 da. at 4%.
12. $450.80 for 6 yr. 6 mo. at 5%, compounded semi-annually.
13. $565.11 for 4 yr. 8 mo. at 4%, compounded semi-annually.

SAVINGS BANKS

Savings Banks receive deposits in small amounts and pay compound interest to the depositors.

DATE	DEPOSITS	WITHDRAWALS	INTEREST	BALANCE
1917 Oct. 10	600 00			600 00
1918 Jan 1				600 00
Mch 7		150 00		450 00
May 4	100 00			550 00
July 1			9 00	559 00
Aug 10		300 00		259 00
1919 Jan 1			5 18	264 18

Interest is credited by savings banks at regular intervals, usually on January 1 and July 1 of each year. It is common practice to credit interest only at the end of each six months' period, and to calculate the interest for a period upon the average balance on deposit during that period. However, there are no fixed rules governing the practice of all savings banks.

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A common form of bank book issued by savings banks is illustrated. The illustration shows deposits, withdrawals, balances resulting at each transaction or entry, interest credits, and the final condition of an account.

In the illustration the interest rate is 4%, computed semi-annually, that is, 2% interest is credited on July 1 and January 1.

As an illustration of a custom that is followed in many places, the interest for each six months is calculated on the smallest total balance to the credit of the depositor at any time during the interest period. Thus, the interest credited July 1 is the interest for six months at 2% on $450, the smallest balance on hand at any one time in that six months.

WRITTEN APPLICATIONS

From each of the following arrange a page of a savings bank book, and find the balance due the depositor on the date indicated.

1. Deposits: March 1, 1918, $200; July 1, 1918, $350; October 15, 1918, $150. Withdrawals: June 15, 1918, $50; August 30, 1918, $75; December 1, 1918, $50. Interest computed semi-annually January 1 and July 1, at 4%. Find the balance due the depositor on January 1, 1919.

2. Deposits: January 1, 1918, $200; March 1, 1918, $150; April 10, 1918, $200; July 10, 1918, $100; August 15, 1918, $190; December 10, 1918, $130. Withdrawals: March 10, 1918, $25; April 15, 1918, $75; September 9, 1918, $50. Find the depositor's balance on January 1, 1919, the interest being compounded January 1 and July 1, at $3\frac{1}{2}\%$.

3. Deposits: October 5, 1917, $200; January 1, 1918, $74; March 10, 1918, $100; May 5, 1918, $125; October 1, 1918, $275; April 7, 1919, $350; June 10, 1919, $100. Withdrawals: February 10, 1918, $100; June 6, 1918, $100; December 10, 1918, $125; March 1, 1919, $150. Find the depositor's balance on July 1, 1919, interest being compounded on January 1 and July 1, at 4%.

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FEDERAL RESERVE BANKS

The Federal Reserve Act was passed by the Congress of the United States in 1916.

I. Federal Reserve Banks.

The entire United States was divided by the act into twelve Federal Reserve Districts, each of which includes a city in which is located a Federal Reserve Bank. Every national bank in a district was required to subscribe for an amount of stock in the Federal Reserve Bank in that district equal to six per cent of its own paid-up capital and surplus.

The Paid-up Capital of a bank is the money which has been paid at par for the stock which has been issued. The expression, "at par," means the face value of the stock. Thus, if the capital of a bank is divided into shares of $100 each, the par value of the shares is said to be $100. *The Surplus* of a bank is accumulated through two sources: (1) from the profits gained from its general banking business, and (2) from the additional amounts by which the sale of capital stock exceeds the par value of the stock sold. Thus, if a bank earned a certain sum in a year, and if it also sold shares of its stock at an increase of $25,000 over its par value, both sums could be added to the surplus upon the vote of the directors.

The act establishing Federal Reserve Banks stipulates that the capital of each of these banks must not be less than $4,000,000.

By the act all of the National Banks in a Federal Reserve District may become members of the Federal Reserve Bank of that district, and any State Bank or any Trust Company which complies with certain specified requirements may become members.

II. Federal Reserve Notes.

Upon depositing any United States bonds with the Treasurer of the United States, as stipulated by the laws relating to National Banks, the Federal Reserve Bank receives from the Comptroller of the Currency circulating notes, or **currency**, equal in value to the value of the bonds deposited. As this currency is issued by the twelve Reserve Banks it will gradually replace certain existing forms, which is one of the objects of the plan.

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III. What the Federal Reserve Banks Accomplish.

The Federal Reserve Banks have the power to rediscount the commercial paper of their member banks.

You will recall that a business man may discount a note, or receive cash in advance payment without waiting until the note is legally due. In the same way a Federal Reserve Bank may discount this same note a second time, and thus provide a larger supply of cash to the member bank presenting the note. Thus the Federal Reserve Bank acts as "the banker's bank."

The constant redistribution of cash from certain points where there is an excess of cash to other points where the supply is short is one of the principal powers of the new bank. As a result, bankers are spared those periods of "tight money," so frequent in the past, particularly at the close of the year when the demands for moving large crops and for dividend payments are unusually heavy.

The Twelve Federal Reserve Cities are

Boston	Richmond	Minneapolis
New York	Atlanta	Kansas City
Philadelphia	Chicago	Dallas
Cleveland	St. Louis	San Francisco

The Reserve Districts correspond in number to the order of the cities given above. Thus, Boston is in District No. 1, New York in District No. 2, etc.

WRITTEN APPLICATIONS

1. How much minimum capital was required by the Federal Reserve Act to establish all of the twelve Federal Reserve Banks?

2. How many shares of stock of each Federal Reserve Bank, at par value of $100, must have been subscribed before each bank could begin business?

3. The paid-up capital and the surplus of a Chicago bank amount to $22,000,000. When this bank became a member of the Federal Reserve Bank in its district, how much Federal Reserve Bank stock did this Chicago bank take?

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4. The paid-up capital of a leading bank in New York City is $25,000,000, and its surplus is $51,400,000. How much Federal Reserve Bank stock was this bank required to take when it became a member of the Federal Reserve Bank?

5. The maximum amount of stock which an individual or a company may own in the Federal Reserve Bank is $25,000. If five state banks and three insurance companies are permitted to purchase the maximum amount, what amount of stock may they together own?

6. In 1918 the Federal Reserve Banks in New York, Chicago, Philadelphia, and Boston earned, respectively, $3,718,955.00, $1,509,871.00, $753,874.00, and $912,294.00. Find the total earnings of the four banks, and the average rate of their dividends if they paid a total of $4,021,677 to their stockholders.

EXCHANGE

Exchange is the process by which bills or debts may be paid or collected in distant places without the actual transfer of cash.

Domestic Exchange is the exchange between places in the same country.

Foreign Exchange is exchange between two places in different countries.

Domestic Exchange

The most common forms of domestic exchange are:

1. The Postal Money Order.
2. The Express Money Order.
3. The Telegraph Money Order.
4. The Personal Check.
5. The Bank Draft, or Cashier's Check.

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I. **The Postal Money Order** is a written order signed by the postmaster in one place directing the postmaster in another place to pay to the person named in the order a specified sum of money.

A small fee is charged for a postal money order, the charge depending upon the amount of money transferred. The following reproduction of the table of rates issued by the Government gives the various fees.

RATES FOR POSTAL MONEY ORDERS

From $ 0.01 to $ 2.50	$.03
From $ 2.51 to $ 5.00	.05
From $ 5.01 to $ 10.00	.08
From $10.01 to $ 20.00	.10
From $20.01 to $ 30.00	.12
From $30.01 to $ 40.00	.15
From $40.01 to $ 50.00	.18
From $50.01 to $ 60.00	.20
From $60.01 to $ 75.00	.25
From $75.01 to $100.00	.30

For amounts in excess of $100, additional orders must be purchased, that amount being the maximum amount for a single order.

ORAL PRACTICE

Give the cost of a postal money order for:

1. $3.50. 4. $9.75. 7. $28.50. 10. $52.50. 13. $110.00.
2. $4.57. 5. $11.75. 8. $35.00. 11. $74.00. 14. $135.00.
3. $7.50. 6. $21.80. 9. $38.90. 12. $81.91. 15. $187.15.

Give the total amount paid by the purchaser of a money order for:

16. $11.70. 18. $47.25. 20. $56.15. 22. $110.00. 24. $142.65.
17. $31.50. 19. $65.75. 21. $75.90. 23. $131.50. 25. $169.90.

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II. The Express Money Order is similar to the postal order, but the conditions limiting its payment are somewhat different. A postal money order is payable to the party named in it, and only at the place named; while an express money order is payable to the order of the person named in it, and at any office of the same company where such orders are issued.

The rates for express money orders up to fifty dollars are the same as those for postal money orders. Beyond that amount additional orders must be purchased, the rates for which are the same as in the case of a similar amount sent by postal money order. For example:

$90 sent by express money order would be sent in two orders, one for $50, the other for $40. The rate for both orders would be $.30, just as much as if the whole amount had been sent by a postal money order.

ORAL PRACTICE

Give the cost of an express money order for:

1. $35. 3. $60. 5. $75. 7. $87.50. 9. $100. 11. $110.50.
2. $45. 4. $69. 6. $92. 8. $95.25. 10. $125. 12. $145.00.

WRITTEN APPLICATIONS

1. Compare the cost of sending $150 by Postal Money Order and the cost of sending it by Express Money Order.

2. Find the total cost of sending five express money orders, the amounts being, respectively, $12.50, $42, $18.75, $31.05, and $29.

3. A letter may be registered for 10 cents. Compare the cost of sending $50 by registered mail with the cost of sending the same sum by Postal Money Order.

4. How much would a firm save by sending $450 by registered mail instead of sending it by Express Money Order? How much would be saved by sending $450 by Postal Money Order instead of by Express Money Order?

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III. The Telegraph Money Order is issued by agents of the telegraph companies, on a plan similar to that of the postal and express orders, and affords the quickest possible form of exchange.

The rates charged for telegraph money orders include (1) the cost of a fifteen word message from the office of deposit to the office of payment, and (2) a premium based upon the following table.

RATES FOR TELEGRAPH MONEY ORDERS

For $25.00 or less	$0.25
From $25.01 up to $50.00	.35
From $50.01 up to $75.00	.60
From $75.01 up to $100.01	.85

For amounts in excess of $100 add to the rate for $100 twenty-five cents per hundred up to $3000. For amounts in excess of $3000 add twenty cents per $100 or fraction of $100.

Illustration:

The rate for a fifteen-word day message between Philadelphia and Boston is "48 and 3.5"; that is, the charge is 48 cents for the first ten words and 3.5 cents for each word in excess of ten. What will be the cost of sending $65 from Philadelphia to Boston by telegraph money order?

Rate for 15-word message between the two points	$ .66
Rate for $65 (between $50.01 and $75.00)	.60
War Tax	.10
Total cost for exchange	$1.36

ORAL PRACTICE

With the 15-word message rate indicated find the cost of telegraph orders for each amount:

30 and 2.5:	36 and 2.5:	42 and 3.5:	60 and 3.5:	90 and 6:
1. $25.	6. $40.	11. $45.	16. $55.	21. $80.
2. $60.	7. $90.	12. $80.	17. $85.	22. $90.
3. $90.	8. $150.	13. $100.	18. $100.	23. $125.
4. $110.	9. $225.	14. $275.	19. $250.	24. $350.
5. $175.	10. $300.	15. $500.	20. $600.	25. $900.

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IV. The Personal Check, as we have learned, is a common form of exchange in business transactions.

The personal check is drawn on a bank in which the person who signs it has a deposit.

The payee presents it to his local bank for cash or credit.

This bank collects it from the bank on which it is drawn.

The collection of personal checks is made without charge when the transaction is wholly within a city or banking zone where both banks involved transact business. When a personal check is presented for payment at some distance from the bank upon which it is drawn, a small charge is made for collection.

A Certified Check is a check upon which the bank holding funds to pay it makes the statement that the sum called for will be held to meet it. A certified check becomes, therefore, an obligation of the bank carrying the maker's account.

WRITTEN APPLICATIONS

1. Write your personal check on one of your local banks, payable to the Western Electric Company, for one hundred sixteen dollars and seventy cents.

2. Write your personal check on one of your local banks, payable to John M. French & Co., for two hundred ninety-six dollars, and indicate the certification of the check by the cashier of your bank. (This process is usually done by stamping the word "Certified" and writing underneath it the signature of the bank's cashier and the date of the certification.)

3. Write a check on one of your local banks payable to the Grand Rapids Furniture Company to illustrate the payment of twenty-five hundred dollars by one of your leading business houses. Indicate the proper certification of the check, and on the back show how the company receiving it indorses it for deposit in their local bank.

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V. The Bank Draft is a check drawn by one bank upon another bank.

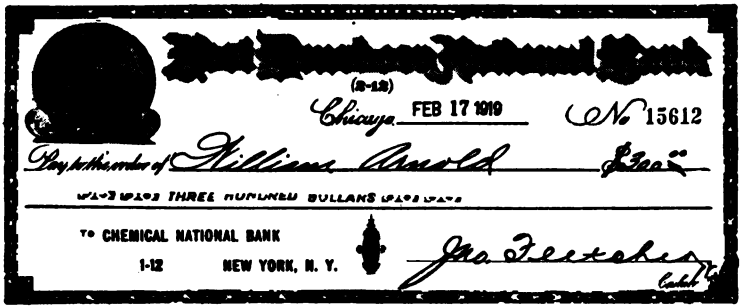
In order to simplify the transactions of business between different sections of the country, it is customary for banks in one section to keep deposits with correspondent banks in other sections. Consequently, one bank may draw checks upon another bank, just as individuals draw their checks on their local banks.

Illustration:

The Fort Dearborn National Bank of Chicago keeps a deposit with the Chemical National Bank of New York City.

Mr. John Brown of Chicago wishes to pay $300 to Mr. William Arnold of Philadelphia.

Mr. Brown, therefore, obtains a *bank draft* made out as follows:



Mr. Brown indorses the check on the back thus: "Pay to the order of William Arnold," and mails it to Mr. Arnold of Philadelphia. Any bank in Philadelphia will pay the $300 it calls for to Mr. Arnold, or will credit the amount to his account.

Finally, the bank receiving the draft in Philadelphia will send it to New York for collection, where, through the Clearing House, the $300 will be paid over to the Philadelphia bank by the Chemical National Bank; and the Chemical National Bank will charge the $300 to the account of the Fort Dearborn National Bank which issued the draft.

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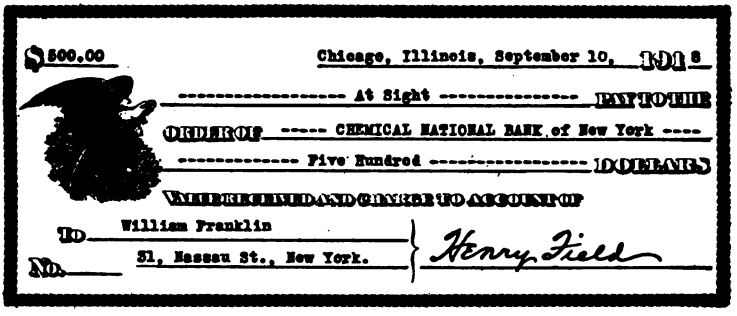
VI. The Sight Draft is a form of bank draft by which exchange is made between two individuals, one of whom " draws " upon the other to satisfy a debt.

The working process is as follows:

Mr. Henry Field of Chicago, Illinois, wishes to collect $500 from Mr. Wm. Franklin of New York City. Mr. Field makes out the following draft:



The local bank where Mr. Field transacts his business cashes this draft and pays the amount to him, less a small fee charged for exchange.

The Chemical National Bank collects the amount from its correspondent bank in New York and pays it over to the Chicago bank that paid the sum to Mr. Field.

If Mr. Franklin refuses to pay the draft the New York bank returns it to the Chemical National Bank, and this latter bank notifies the Chicago bank that payment is refused. The Chicago bank immediately notifies Mr. Field that he must " take up the draft," that is, refund the money that they paid him.

If Mr. Franklin pays the draft the transaction closes when the New York bank remits to the Chemical National Bank the amount that they have advanced to the Chicago bank.

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If Mr. Franklin's indebtedness to Mr. Field is not due for ninety days, the draft would have been written with the words "Ninety days after sight," instead of "at sight." Mr. Franklin then has the privilege of indorsing the draft with the word "Accepted" across the face, signing his name underneath; this process converting the draft to an "acceptance" which is virtually a promissory note due ninety days after the date of acceptance.

APPLICATIONS OF EXCHANGE BY BANK DRAFTS

We have found that the exchange of money by Postal or Express Orders is made at a small cost dependent upon the amount of money transferred. It is also true that exchange by bank draft may require a small expense, but, on the other hand, a saving is possible under certain conditions.

The demand for money in one city may be limited while the supply may be abundant. In another city the demand may be great but the supply limited. Consequently, the rate of exchange between the two cities will be affected, and the city which is in need of money will be made to pay a certain additional percentage in order to get it.

If New York has a large demand for drafts on Chicago, but Chicago has only a limited demand for drafts on New York, the supply of cash on deposit in Chicago by New York banks will be exhausted. As a check upon such a situation the rate for New York Exchange on Chicago will rise, and at the same time the rate for Chicago Exchange on New York will fall.

The Rate of Exchange is usually quoted as a number of cents per $1000 transferred.

In problems of exchange we must consider

(*a*) The amount to be transferred, and

(*b*) The value of money in one place compared with the value in the other place, or the rate of exchange.

Because of the expense of maintaining facilities for the transfer of money, most banks charge a small fee for a draft. Many banks, however, do not ask this fee from their regular depositors.

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Exchange Quotations.

If the cost of exchange is added to the amount transferred, exchange is said to be at a **Premium.**

If the cost of exchange is deducted from the amount transferred, exchange is said to be at a **Discount.**

If there is neither a premium nor a discount, exchange is said to be at **Par.**

To Find the Cost of a Draft

Illustration:

1. At $\frac{1}{2}\%$ premium find the cost of a draft for $5000.

$\frac{1}{2}\%$ premium means $\frac{1}{2}$ of 1% of $5000.

$\frac{1}{2}$ of 1% of $5000 = .5 of .01 of $5000 = .005 × $5000 = $25.

$5000 + $25 = $5025, the cost of a draft for $5000.

2. Find the cost of a draft for $2500 when exchange is selling at 50 cents discount.

50 cents discount means 50 cents discount on each $1000.
The discount on $2500, or 2.5 thousands, is 2.5 × $.50 = $1.25.
$2500 − $1.25 = $2498.75, the cost of a draft is for $2500.

WRITTEN APPLICATIONS

Find the cost of a draft for

1. $2000 at 20 cents premium.
2. $2000 at 20 cents discount.
3. $3000 at 30 cents premium.
4. $5000 at 50 cents discount.
5. $4000 at 75 cents premium.
6. $1000 at $\frac{1}{2}\%$ premium.
7. $1500 at $\frac{3}{4}\%$ discount.
8. $2000 at $\frac{1}{4}\%$ discount.
9. $3500 at .001 discount.
10. $6000 at .0015 discount.
11. What is the cost of a draft on San Francisco for $4500, if the rate of exchange is $\frac{1}{4}\%$ discount?

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12. A merchant in Philadelphia owes $3000 to a wholesale firm in Chicago. How much must he pay for a draft for the sum, if exchange is $\frac{3}{4}\%$ premium?

13. Find the total amount that must be paid to a bank for a draft of $4000, if the exchange rate is 75 cents premium, and a fee of 50 cents is charged by the bank for the transfer.

14. Find the total amount that must be paid to a bank for a draft of $4000, if the exchange rate is 75 cents discount, and a fee of $1.00 is charged by the bank for the transfer.

15. A merchant in Chicago draws on another in Philadelphia, the face of the draft being $4500. If the exchange rate is $\frac{1}{2}\%$ discount, what is the amount received by the Chicago merchant?

16. A merchant owes $2000 to a wholesale firm in New York. How much will he save if, instead of sending his check, he purchases a draft in which the exchange rate is 75 cents discount?

17. A draft on San Antonio was purchased in Boston when exchange was 50 cents premium. If $10,000 was transferred, how much did the exchange and the face of the draft cost the sender?

18. A merchant in New Orleans bought a bill of goods in Chicago amounting to $12,500, on which the discounts were 15% and 10%. If he paid for the goods with a draft on which the discount was $\frac{1}{2}\%$, find the sum he remitted to Chicago.

19. In making advance payment for a bill of goods amounting to $10,600, a merchant transferred the money by telegraph. If the premium on the exchange was 75¢, and if the cost of the telegraph transfer was $22.30, how much did the merchant pay out on the whole transaction?

20. A merchant paid two bills by buying drafts. One of them called for $2700 in a city on which the exchange rate was 30 cents premium, and the other for $5600 in another city on which the exchange rate was $\frac{3}{4}\%$ discount. How much did both drafts cost? How much did he save by buying the drafts instead of using his personal checks?

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RATIO AND PROPORTION

RATIO

A Ratio is the quotient of one quantity divided by another, both being of the same kind.

Thus: The ratio of 3 to 6 is $3 \div 6$, or $\frac{3}{6}$.
The ratio of 6 to 3 is $6 \div 3$, or $\frac{6}{3}$.

The quotient of one number divided by another number of the same kind is always an abstract number, hence,

All ratios are abstract numbers.

The ratio of two numbers is often expressed by using the **Ratio Sign** (:)

Thus: The ratio of 3 to 6 may be expressed 3 : 6.

The first number of a ratio is the **Antecedent.** The second number is the **Consequent.**

Thus: In the ratio 5 : 7, which is read "Five to seven,"
The antecedent is 5, and the consequent is 7.

The Terms of a Ratio are the antecedent and consequent of that ratio.

ORAL PRACTICE

Expressing both measures in the same kind of unit, give the ratio of:

1. 1 inch to 1 foot.
2. 1 foot to 1 yard.
3. 1 foot to 1 mile.
4. 1 ounce to 1 pound.
5. 1 hour to 1 day.
6. 1 second to 1 hour.
7. 1 minute to 1 day.
8. 1 foot to 1 inch.
9. 1 mile to 1 yard.
10. 1 hour to 1 minute.
11. 1 gallon to 1 pint.
12. 1 bushel to 1 quart.
13. 1 week to 1 year.
14. 1 cu. ft. to 1 cu. yd.

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APPLICATIONS OF RATIOS

I. To change the form of a ratio, or to reduce its terms without changing their ratio.

Since a ratio may be expressed as a fraction it follows that

The terms of a ratio may be multiplied by, or may be divided by, the same number without changing the value of the ratio.

Illustration:

1. Find the ratio of 15 to 20.

$$15 : 20 = \tfrac{15}{20} = \tfrac{3}{4}.$$

That is, the ratio of 15 to 20 is the same as the ratio of 3 to 4.

Illustration:

2. Find the ratio of $1\frac{2}{3}$ to $1\frac{1}{4}$.

To change to whole numbers multiply each term of the ratio by 12, the L. C. M. of the denominators.

Then $$\frac{1\frac{2}{3}}{1\frac{1}{4}} = \frac{1\frac{2}{3}\times 12}{1\frac{1}{4}\times 12} = \frac{20}{15} = \frac{4}{3}.$$ *Result.*

That is, the ratio of $1\frac{2}{3}$ to $1\frac{1}{4}$ is the same as the ratio of 4 to 3.

BLACKBOARD PRACTICE

Give, in simplest form, the ratio of:

1.	4 to 8.	8 to 4.	6 to 12.	12 to 6.	6 to 9.
2.	8 to 12.	8 to 16.	8 to 20.	12 to 15.	12 to 18.
3.	10 to 5.	10 to 6.	12 to 8.	12 to 20.	16 to 20.
4.	16 to 4.	20 to 6.	20 to 5.	20 to 24.	20 to 30.
5.	30 to 45.	40 to 60.	45 to 60.	55 to 60.	24 to 72.
6.	$\frac{1}{2}$ to $\frac{3}{4}$.	$\frac{1}{2}$ to $\frac{1}{3}$.	$\frac{1}{4}$ to $\frac{3}{8}$.	$\frac{2}{5}$ to $\frac{4}{5}$.	$\frac{2}{3}$ to $\frac{7}{8}$.
7.	$\frac{1}{8}$ to $\frac{1}{4}$.	$\frac{2}{5}$ to $\frac{3}{10}$.	$\frac{2}{5}$ to $\frac{5}{6}$.	$\frac{3}{4}$ to $\frac{5}{8}$.	$\frac{5}{6}$ to $\frac{15}{16}$.
8.	.4 to .7.	.5 to .9.	.15 to .9.	.3 to 6.	1.5 to 7.5.
9.	1.25 to 5.	.05 to .1.	.75 to 30.	12.5 to .625.	.18 to 14.4.

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II. To separate a number into parts that shall have a ratio equal to a given ratio.

Illustration:

Divide a line 100 feet long into parts that shall have the ratio of 2 to 3.

For each 2 feet in one part there will be 3 feet in the other part.

And each portion 2 feet long plus each corresponding portion 3 feet long together make a length 5 feet long.

Hence, one part of the 100 feet will be $\frac{2}{5}$ of the whole, and the other part of the 100 feet will be $\frac{3}{5}$ of the whole.

$\frac{2}{5}$ of 100 ft. = 40 ft., one part,

$\frac{3}{5}$ of 100 ft. = 60 ft., the other part. *Result.*

The result is readily proved, for $40 : 60 = \frac{40}{60} = \frac{2}{3}$.

BLACKBOARD PRACTICE

Separate

1. 36 in the ratio of 1 to 3.
2. 45 in the ratio of 4 to 5.
3. 75 in the ratio of 7 to 8.
4. 110 in the ratio of 4 to 7.
5. 140 in the ratio of 3 to 7.
6. 165 in the ratio of 5 to 11.
7. 150 in the ratio of 6 to 19.
8. 200 in the ratio of 7 to 13.
9. 208 in the ratio of 4 to 9.
10. 264 in the ratio of 6 to 11.
11. 324 in the ratio of 10 to 19.
12. 500 in the ratio of 12 to 29

WRITTEN APPLICATIONS

1. In mixing the grades of coffee a grocer puts in 60 pounds of one kind and 40 pounds of another kind. What is the ratio of the amount of the first kind to the amount of the second kind?

2. In a certain fertilizer a farmer uses 875 pounds of acid phosphate and 175 pounds of kainit. What is the ratio of the phosphate to the kainit?

3. A contractor uses 35 bags of cement and 5 cubic yards of crushed stone for a concrete floor. If a bag of cement is equivalent to 1 cubic foot, what is the ratio of this amount to the amount of crushed stone used?

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4. A fruit grower has on hand 150 pounds of copper sulphate, and in the spray he plans to use he must have sufficient lime to make the ratio of the sulphate to the lime 2 to 7. How many pounds of lime must he use?

5. A fertilizer used for market gardening contains 2500 pounds of nitrate of soda, and the ratio of this nitrate to the acid phosphate in the mixture must be 8 to 5. How many pounds of acid phosphate must be used?

6. Two men rented a roller and paid $60 for its use. One man used it 9 days and the other man used it 21 days. How much should each man pay?

7. A farmer finds that the ratio of the butter fat to the milk produced by his herd is 1 to 25. At this rate how many pounds of butter fat are produced at one milking, if his 20 cows average 18 pounds of milk each?

8. One man invests $3000 and another $4200 in a business, and the profit of this business amounts to $4800. Divide this profit into two shares that shall be in the same ratio as the amounts each man invested.

9. An estate is so divided that the only child receives $2 for every $5 left to the widow. If the total value of the property is $37,500, find the amount each of the two heirs receives.

10. Two merchants bought a carload of coal weighing 100,000 pounds. One of them took 30 tons of it, and the other took the rest of it, each taking 2000 pounds to the ton. The freight was $.60 per ton. Find the amount of freight that each should pay, if they divide the freight in the same ratio as they divided the coal.

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PROPORTION

A Proportion is an expression of equality between two ratios.

Thus: $4:8=5:10$ is a proportion.

The same proportion may be expressed $\frac{4}{8}=\frac{5}{10}$.

This proportion is read

"The ratio of 4 to 8 is equal to the ratio of 5 to 10,"

Or, briefly,

"4 is to 8 as 5 is to 10."

The Extremes of a Proportion are the first and fourth terms.

4 and 10 are the extremes of the proportion given above.

The Means of a Proportion are the second and third terms.

8 and 5 are the means of the proportion given above.

In the proportion

$$4:8=5:10,$$

It will be observed that $4\times10=40$,

and also that $8\times5=40$. That is,

The product of the means is equal to the product of the extremes.

This principle may be proved true for any proportion.

If the pupil requires a proof the following will convince.

The proportion $4:8=5:10$

may be written $\frac{4}{8}=\frac{5}{10}$.

Multiplying each member of the equation by 8×10 we have

$$\frac{8\times10\times4}{8}=\frac{8\times10\times5}{10}.$$

Canceling "8" from the left member, and "10" from the right member, we have

$$10\times4=8\times5.$$

The pupil should go through the same process with some other proportion.

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By this principle we may find any missing term of a proportion when three terms are known.

In a proportion as, for example, $6:9=10:15$,

We have $6\times15=9\times10$.

Dividing both members by one extreme (6), $\frac{6\times15}{6}=\frac{9\times10}{6}$.

Canceling the 6 in the left member, $15=\frac{9\times10}{6}$.

Or, dividing both members by the other extreme (15),

$$\frac{6\times15}{15}=\frac{9\times10}{15}$$

Canceling the 15 in the left member, $6=\frac{9\times10}{15}$.

In general, therefore,

Either extreme of a proportion is equal to the product of the means divided by the other extreme.

By dividing both members by either mean it may be shown that

Either mean of a proportion is equal to the product of the extremes divided by the other mean.

Illustrations:

1. Given $8:12=4:x$. Find x.

$8x=48$.

$x=6$. *Result.*

2. Given $7:x=10:30$. Find x.

$10x=210$.

$x=21$. *Result.*

BLACKBOARD PRACTICE

In each of the following proportions find the value of the unknown term:

1. $x:6=8:16$.
2. $x:7=4:14$.
3. $5:x=4:12$.
4. $9:x=8:24$.
5. $x:12=10:15$.
6. $10:x=9:12$.
7. $12:18=x:15$.
8. $x:\frac{2}{3}=\frac{3}{4}:\frac{1}{2}$.
9. $\frac{3}{4}:x=\frac{3}{5}:\frac{5}{6}$.
10. $\frac{7}{8}:\frac{1}{3}=x:\frac{8}{9}$.
11. $\frac{3}{5}:\frac{5}{8}=1\frac{2}{25}:x$.
12. $0.2:x=0.15:0.03$.
13. $0.5:1.5=x:0.06$.
14. $x:2.5=0.75:0.625$.

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Practical Uses of Proportion in Business. Many problems occurring in every-day business practice are readily solved by the use of proportion.

Illustration:

If 12 barrels of flour cost \$69, how much will 21 barrels of flour cost?

We make up our proportion by (1) placing quantities of the same kind together in our ratios, and (2) by observing that a small number of barrels is the first term of one ratio, and a small number of dollars is the first term of the second ratio. That is,

small no. bbl. : large no. bbl. = small no. dollars : large no. dollars.

The proportion is, then, $12 : 21 = \$69 : \$x$.

Whence, $$x = \frac{69 \times 21}{12}.$$

Solving, $$x = 120\tfrac{3}{4}.$$

But the second ratio is an expression in dollars, hence, the result is a number of dollars. Therefore, the cost of 21 barrels will be \$120.75. *Result.*

In certain types of problems care must be taken to observe the order in which the ratios are expressed. Problems involving labor illustrate this type.

Illustration:

If 10 men can complete a task in 9 days, how many days will it take 15 men to do the same work?

Increasing the number of men will shorten the time required, hence,

large no. men : small no. men = large no. days : small no. days.

The proportion is, then, $15 : 10 = 9 : x$.

From which, $x = 6$.

That is, 15 men will do the work in 6 days. *Result.*

In all problems in proportion the words "at the same rate" are always understood. Proportion is frequently used in the work of later mathematics, and this simple treatment should be thoroughly understood.

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WRITTEN APPLICATIONS

Proportion Applied to Business

1. If 5 chains cost \$15, what will 9 chains cost?
2. If 6 books cost \$21, what will 10 books cost?
3. If 9 chairs cost \$36, what will 15 chairs cost?
4. If 12 watches cost \$96, what will 20 watches cost?
5. If 15 sheets cost \$12, what will 48 sheets cost?
6. If 18 pairs of shoes cost \$54, what will 45 pairs cost?
7. If 20 clocks cost \$110, what will 29 clocks cost?
8. If 12 stick pins cost \$30, what will 56 stick pins cost?
9. If 15 mattresses cost \$112.50, what will 21 mattresses cost?
10. A dealer paid \$412.50 for 11 rugs. How much should he pay for 17 such rugs?
11. A dealer in clothing bought 12 dresses for \$219. How much should he pay for 23 dresses of the same kind?
12. A bicycle manufacturer sold 13 bicycles for \$327.60, and later he sold 27 more at the same price apiece. What amount did he receive for the second lot?
13. A laborer was paid \$16.80 for 8 days' work. How much should he receive for working 13 days?
14. 45 M feet of lumber cost a dealer \$990, and later he bought 24 M feet more of the same kind of lumber. How much did he pay for the second lot at the same rate?
15. An automobile runs 16 miles in 30 minutes. At the same rate, how many miles will it run in one and one quarter hours?
16. A dealer in provisions paid \$5.76 freight charges on 18 cases of eggs. At this rate, how much was the freight charge on 33 cases?
17. For the cost of delivering 80 tons of coal a dealer paid \$35. At this rate how much should he charge for delivering 15 tons to another customer?
18. In 350 pounds of milk there are 13.3 pounds of butter fat. At this rate, how much butter fat should there be in 1000 pounds of milk?

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19. A machine makes 250 bolts in one and one quarter hours. How many bolts will this machine make in 55 hours?

20. A cleaning establishment charged $2.16 for cleaning a rug 9 feet long and 6 feet wide. How much will they charge, at the same rate, for cleaning a rug 10 feet long and 8 feet wide?

21. A parquet floor 16 feet long and 12 feet wide is laid at a cost of $38.40. How much will it cost, at the same rate, to lay such a floor in a room 18 feet long and 15 feet wide, and in a hallway 18 feet long and 9 feet wide?

22. A farmer finds that 80 bushels of grain is sufficient to feed his herd of cows for one week if each cow gets 4 quarts a day. If he changes his plan so as to feed each cow 5 quarts daily, how many days will the 80 bushels last?

23. A practical cattleman feeds 3 pounds of hay at a feeding for each 100 pounds of live weight of his herd. At that rate what amount should he feed to a steer weighing 1250 pounds? What amount should he feed to one weighing 900 pounds?

Proportion Applied to Problems Involving Time

24. If 12 men can complete a task in 10 days, how many days will it take 15 men to complete the same task?

25. 12 men built a section of a wall in 9 days. How many days would it take 18 men to build such a section?

26. 26 men can complete a concrete pavement in 12 days, but only 8 men are put at the work. How many days will it take these 8 men to complete it?

27. 32 women can pick all of the strawberries in a strawberry bed in 30 hours. How many women will it take to pick them in 20 hours?

28. A farmer had enough feed to supply his 27 cows for 72 days, but he sold 9 of them. How many days will this feed supply the rest of his cows?

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29. A herd of 75 head of cattle will consume a certain amount of grain in 9 days, but the owner adds 30 head to the herd. How many days will this grain last?

30. 50 men were engaged to work upon a paving job which they could finish in 25 days, but 5 of them failed to report for work. How long should it take the others to do the work?

31. 100 men were building a highway which they could finish in 12 days. If 20 of them gave up the work, how long should it take the others to complete it?

32. 120 men can complete a paving job in 15 days, but the contractor wishes to finish the work in 10 days. How many more men must he employ to do this?

33. A farmer has on hand a supply of grain that will feed 46 cows for the three months of November, December, and January. If he sells 10 cows, how many days will the feed last for those that he keeps?

34. A contractor has 100 days in which to build a wall, and he has 12 men to do the work. If he should decide to use 15 men, in how many days should he be able to complete the work?

35. A builder agrees to complete a gymnasium in 150 days, and plans to have an average of 30 men at work on it daily during that time. If he begins 10 days late, how many men should he add to his force in order to complete the work at the end of the specified time?

36. 30 linemen can build a telephone line in 9 days, but the contractor has only 4 days in which to do the work. How many additional men must be put to work in order to complete the line on time?

37. A contractor agrees to decorate the interior of a building, and to complete the work in 36 working days. He is delayed 4 days, however, before beginning. How many men must he put upon the work in order to complete it in 32 days, if 48 men could have completed it in 36 days?

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POWERS

A Power is a product obtained by using a given number a certain number of times as a factor.

Thus: $2\times2=4$. $2\times2\times2=8$. $2\times2\times2\times2=16$. Etc.
And 4, 8, and 16 are powers of 2.

Similarly: $5\times5=25$. 25 is a power of 5.
$3\times3\times3=27$. 27 is a power of 3.

A Square is a power obtained by using a number twice as a factor.

Thus: The square of 5 is obtained from the product of 5×5.
That is: $5\times5=25$. 25 is the square of 5.

A Cube is a power obtained by using a number three times as a factor.

Thus: The cube of 5 is obtained from the product of $5\times5\times5$.
That is: $5\times5\times5=125$. 125 is the cube of 5.

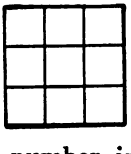
The product of 2×2 may be indicated by writing 2^2.
2^2 is read "two squared." $2^2=4$.

Similarly: $2\times2\times2$ may be indicated by writing 2^3.
2^3 is read "two cubed." $2^3=8$.
Also: $3\times3\times3$ or $3^3=27$.
5×5 or $5^2=25$.

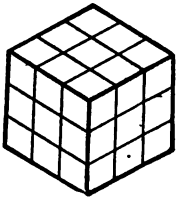
The small figure, placed at the right of and a little above the number, indicates the number of times the given number is to be used as a factor, and is called an **Exponent.**

The Relation of the Square to the Cube is readily illustrated by reviewing our knowledge of an area and also of a volume.

A square whose side is 3 units contains (3×3) square units, or 9 square units of area.



A cube whose edge is 3 contains $(3\times3\times3)$ cubic units, or 27 cubic units of volume.



The square of a number is often called the **Second Power** of the number.

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The cube of a number is often called the **Third Power** of the number.

In like manner we may speak of the fourth, the fifth, the sixth power, etc.

The following table should be memorized.

POWERS

$2^2 = 4$	$10^2 = 100$	$18^2 = 324$	$2^3 = 8$
$3^2 = 9$	$11^2 = 121$	$19^2 = 361$	$3^3 = 27$
$4^2 = 16$	$12^2 = 144$	$20^2 = 400$	$4^3 = 64$
$5^2 = 25$	$13^2 = 169$	$21^2 = 441$	$5^3 = 125$
$6^2 = 36$	$14^2 = 196$	$22^2 = 484$	$6^3 = 216$
$7^2 = 49$	$15^2 = 225$	$23^2 = 529$	$7^3 = 343$
$8^2 = 64$	$16^2 = 256$	$24^2 = 576$	$8^3 = 512$
$9^2 = 81$	$17^2 = 289$	$25^2 = 625$	$9^3 = 729$

To Find the Power of a Fraction.

Since the square of a fraction is the product of the fraction by itself we have, And for the cube

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}. \qquad \left(\frac{3}{4}\right)^3 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{3 \times 3 \times 3}{4 \times 4 \times 4} = \frac{27}{64}. \quad \text{Etc.}$$

That is, the square of a fraction equals the square of the numerator divided by the square of the denominator;

And, the cube of a fraction equals the cube of the numerator divided by the cube of the denominator.

The same principle applies to powers higher than the cube.

To Find a Power of a Decimal.

$$(2.5)^2 = 2.5 \times 2.5 = 6.25.$$
$$(1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728. \quad \text{Etc.}$$

A power of a decimal is obtained by multiplication as in decimals, care being exercised in the proper pointing-off of the product.

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BLACKBOARD PRACTICE

Find the required power in each of the following:

1. 26^2.	6. 15^2.	11. $(\frac{1}{2})^2$.	16. $(2.5)^2$.
2. 35^2.	7. 21^2.	12. $(\frac{2}{3})^2$.	17. $(3.15)^2$.
3. 42^2.	8. 25^2.	13. $(\frac{3}{4})^3$.	18. $(4.01)^3$.
4. 54^2.	9. 30^2.	14. $(\frac{2}{5})^3$.	19. $(2.15)^2$.
5. 65^2.	10. 45^2.	15. $(\frac{3}{8})^3$.	20. $(3.25)^3$.

Find the area of a square whose side is:

21. 15 in.	26. 50 ft.	31. 10 ft. 6 in.	36. 10 yd. 1 ft.
22. 24 in.	27. 50 yd.	32. 15 ft. 3 in.	37. 12 yd. 2 ft.
23. 30 ft.	28. 50 rd.	33. 18 ft. 8 in.	38. 15 yd. 1 ft.
24. 35 ft.	29. 65 yd.	34. 20 ft. 6 in.	39. 18 yd. 2 ft.
25. 42 ft.	30. 75 rd.	35. 24 ft. 3 in.	40. 20 yd. 2 ft.

Find the volume of a cube whose edge is:

41. 6 in.	44. 9 ft.	47. 1 ft. 6 in.	50. 3 ft. 6 in.
42. 10 in.	45. 12 ft.	48. 1 ft. 3 in.	51. 5 ft. 9 in.
43. 16 in.	46. 20 ft.	49. 2 ft. 6 in.	52. 8 ft. 6 in.

ROOTS

A Root of a number is one of the equal factors of that number.

A Square Root of a number is one of the two equal factors of that number.

Thus: 3 is the square root of 9. (For $9 = 3 \times 3$.)
12 is the square root of 144. (For $144 = 12 \times 12$.)

A Cube Root is one of the three equal factors of a number.

Thus: 2 is the cube root of 8. (For $8 = 2 \times 2 \times 2$.)
4 is the cube root of 64. (For $64 = 4 \times 4 \times 4$.)
5 is the cube root of 125. (For $125 = 5 \times 5 \times 5$.)

The symbol for square root is the **Radical Sign**, $\sqrt{}$.

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A small figure, called the **Index**, is written in the radical sign to show what root or factor is required. Thus:

$\sqrt[2]{25}$ means "What is the square root of 25?" 2 is the index.

$\sqrt[3]{27}$ means "What is the cube root of 27?" 3 is the index.

It is not customary to write the index when a square root is required. Thus:

$\sqrt[2]{16}$ is the same as $\sqrt{16}$.

Square Root Obtained by Factoring

By separating a number into its prime factors we are able in many cases to find its square root by inspection.

Thus: $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

In the factors we find four 2's and two 3's.

Hence, we may group the 2's in pairs and write

$$144 = (2 \times 2) \times (2 \times 2) \times (3 \times 3).$$

Since finding square root is finding one of two equal factors, we may select one factor from each pair for the square root of that particular pair.

The product of the single factors selected will be the square root of the given number, that is:

$$\sqrt{144} = 2 \times 2 \times 3 = 12. \quad \textit{Result.}$$

By a similar process we may often find the cube root of a number.

BLACKBOARD PRACTICE

By factoring find the square root of:

1. 225.	**5.** 784.	**9.** 1225.	**13.** 2304.
2. 324.	**6.** 961.	**10.** 1296.	**14.** 2500.
3. 576.	**7.** 1024.	**11.** 1764.	**15.** 3136.
4. 729.	**8.** 1089.	**12.** 2025.	**16.** 4096.

Find the number of feet in the side of the square whose area is:

17. 484 square feet.	**21.** 729 square yards.
18. 676 square feet.	**22.** 784 square yards.
19. 900 square feet.	**23.** 3025 square rods.
20. 961 square feet.	**24.** 4096 square rods.

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THE GENERAL METHOD OF FINDING SQUARE ROOT

The Relation between the number of figures in a square and the number in the corresponding square root may be shown thus:

The largest number having one figure is 9. $9^2 = 81$.
The largest number having two figures is 99. $99^2 = 9801$.
The largest number having three figures is 999. $999^2 = 998001$.

That is:

(1) *A number having one figure may have as many as two figures in its square.*

(2) *A number having two figures may have as many as four figures in its square.*

(3) *A number having three figures may have as many as six figures in its square. Etc.*

If, therefore, a number is considered as "separated into periods of two figures each," the number of such periods obtained will equal the number of figures in the square root which is sought.

For example: 998001 written thus, 99 80 01, gives three periods, and its square root, 999, has three figures.

The period at the extreme left need have but one figure.

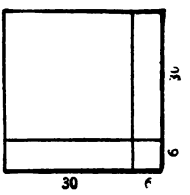
For example: $125^2 = 15625$.
15625 = written 1 56 25 gives three periods, and its square root 125 has three figures.

Separating into periods is begun at the decimal point.

The actual process of finding square root will be understood to better advantage if we first note the close relation between the process of squaring a number and the area representing the same number of square units.

Consider the number 36 and the square whose side is 36 units in length.

36	30	+6
36	30	+6
216	30×6	$+6^2$
108	30^2+	30×6
1296	$30^2+2(30\times6)+6^2 = 900+360+36 = 1296.$	



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It will be clear to the student that:

(1) The square area is made up of two squares and two rectangles; and

(2) The dimensions of the two equal rectangles are the lengths of the sides of the two squares.

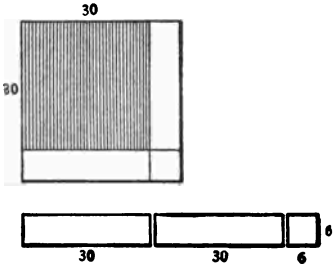
Finding the Square Root of a Given Number.

Illustration:

Find the square root of 1296.

Separate the number into periods of two figures each.

$$\begin{array}{rcl} & & 12\,96 \mid 30+6 \\ 30^2 & = & 9\,00 \\ \hline 60 & & 3\,96 \\ (60+6)6 & = & 3\,96 \\ \hline \end{array}$$



The greatest square contained in the Hundreds' period is 900.

The square root of 900 is 30. Subtract 30^2, or 900, from 1296.

Write 30 for the first part of the result.

Multiply 30 by 2.

Reference to the drawing will explain the need for multiplying 30 by 2. When the square "900" is removed, there remains an area whose length is 30+30, or 60. Hence, the remainder of 396 square units has a length of 60. 60 is the *Trial Divisor*. (2×30.)

Divide 396 by 60.

This division of an area by the length, 60, gives the other dimension, width. Hence, the quotient, 6, is the width of the remaining area.

Write 6 for the second part of the root.

Add 6 to the trial divisor, 60, making 60+6.

This addition is necessary because the length of the side of the small squares must be included in the length of the unused area, 396.

Multiply 60+6, or 66, by 6.

This product of length, 66, by width, 6, gives area. Or, 396.

Subtract this product, 396, from the remainder, 396.

We obtain 0, hence, the exact root has been found.

Therefore, the square root of 1296 is 36. *Result.*

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It will be found that division by the trial divisor is frequently an approximate division, for we cannot always find the exact quotient as in the simple case illustrated.

1. Find the square root of 18769.

Beginning at the right, separate the number into periods of two places each.	1^2	1 87 69 | 137 $=1$
The greatest square contained in the first period is 1. Subtract 1.	20	87
The next period is 87.	$(20+3)\times 3$	$=$ 69
The trial divisor is 20. $(2\times 10.)$	260	1869
87 will contain 20 at least 4 times, but we must allow for the part of the root that must be added. Hence, the approximate quotient is 3. Add 3, and multiply by 3.	$(260+7)\times 7$	$=$ 1869
The next period is 1869.		
The trial divisor is 260. $(2\times 130.)$		
1869 contains 260 at least 7 times.		
Add 7 and multiply by 7.		
The remainder is 0, and the exact root has been found.		

2. Find the square root of 5784025.

In this example the process is similar to that already illustrated except in one detail. The second remainder is 2, and on annexing the next period, 40, we find that the entire remainder, 240, will not contain the divisor, 480.	2^2	5 78 40 25 | 2405 $=$ 4
	40	1 78
	$(40+4)\times 4$	$=$ 1 76
	4800	2 40 25
	$(4800+5)\times 5=$	2 40 25

We write 0, in the root, annex the next period, and the new trial divisor is 4800. The remainder, 24025, contains 4800 at least 5 times. Add 5 to the root and to the trial divisor and proceed as in the other illustrations.

BLACKBOARD PRACTICE

Find the square root of

1. 1156.	6. 1764.	11. 3136.	16. 4761.	21. 7396.
2. 1225.	7. 2025.	12. 3481.	17. 5476.	22. 8649.
3. 1849.	8. 2601.	13. 3249.	18. 5184.	23. 9409.
4. 1521.	9. 2916.	14. 4356.	19. 6241.	24. 9216.
5. 1936.	10. 1681.	15. 4096.	20. 7744.	25. 9604.

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26. 14641.	**34.** 30625.	**42.** 95481.	**50.** 788544.
27. 18225.	**35.** 29584.	**43.** 103041.	**51.** 808201.
28. 16129.	**36.** 32761.	**44.** 124609.	**52.** 958441.
29. 15376.	**37.** 38416.	**45.** 150544.	**53.** 27144100.
30. 19044.	**38.** 41209.	**46.** 236196.	**54.** 35640900.
31. 22801.	**39.** 43264.	**47.** 332929.	**55.** 26224641.
32. 21609.	**40.** 91204.	**48.** 418609.	**56.** 64080025.
33. 24964.	**41.** 93025.	**49.** 597529.	**57.** 81270225.

Finding the Square Root of a Decimal.

We have found that the number of figures in the square root of a number depends upon the number of figures in the given square. A definite relation exists also between the number of figures in a decimal square and its square root.

$.1^2 = .01$. One decimal figure in the decimal; two in the square.

$.01^2 = .0001$. Two decimal figures in the decimal: four in the square.

$.001^2 = .000001$. Three decimal figures in the decimal: six in the square.

$.0001^2 = .00000001$. Four decimal figures in the decimal: eight in the square.

In general, therefore, there are twice as many figures in a decimal square as there are figures in its square root.

To point off, or separate a decimal square into periods:

Begin at the decimal point and point off two figures to the right for each period.

From this principle combined with the principle that governs with whole numbers we may point off numbers that are part integral and part decimal.

Thus:	14197824	is separated	14 19 78 24 sq. rt. = 3768.
	1419.7824	is separated	14 19 . 78 24 sq. rt. = 37.68.
	141978.24	is separated	14 19 78 . 24 sq. rt. = 376.8.
	.14197824	is separated	.14 19 78 24 sq. rt. = .3768.

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Instead of separating the periods by spaces many mathematicians prefer to use a small accent placed over the last figure of each period.

Thus: 14197824 may be separated 1419$\acute{7}$82$\acute{4}$.

1419.7824 may be separated 141$\acute{9}$.78$\acute{2}$4.

In finding the square root of a number wholly or partially decimal, we point off as indicated and extract the square root just as if it were a whole number.

Point off as many places at the right of the root as there are decimal periods in the given number.

Illustration:

Find the square root of 0.14197824.

		.14$\acute{1}$97$\acute{8}$2$\acute{4}$ | .3768
3^2	=	9
60		519
(60+7)×7	=	469
740		5078
(740+6)×6	=	4476
7520		60224
(7520+8)×8	=	60224

The root figures obtained are 3768.

Counting from the right of the square we have four periods in the decimal.

Therefore, for the decimal point in the root count four places from the right.

Hence, .3768. *Result.*

Find the square root of 54.804409.

		5$\acute{4}$.80$\acute{4}$40$\acute{9}$ | 7 403
		49
140		5 80
(140+4)×4	=	5 76
14800		44409
(14800+3)×3	=	44409

There are three periods in the decimal.

For the decimal point in the root point off three places from the right.

Hence, 7.403. *Result.*

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Finding the approximate square root of a number which is not a perfect square.

When a number is not a perfect square we may annex zeros and carry out the root to as many decimal places as desired.

Illustration:

Find the square root of $\frac{3}{4}$ to three decimal places.

Changing $\frac{3}{4}$ to a decimal, $\frac{3}{4} = .75$.

Since three decimal places are required in the result, and since two decimal places in the square give one decimal place in the root, we must find the square root of .750000. That is, six decimal places must be used to obtain a root which will have three places.

	.750000	.866+
8^2	= 64	
160	1100	
$(160+6)\times 6$	= 996	
1720	10400	
$(1720+6)\times 6$	= 10356	
	44	

Therefore, the square root of $\frac{3}{4}$ = .866.

When a root is not exact it is customary to obtain the number of decimal places asked for, and to indicate the approximate nature of the root by affixing the sign "+."

BLACKBOARD PRACTICE

Find the square root of:

1. 26.4196.	5. .355216.	9. .00717409.	13. .00013689.
2. 2641.96.	6. .001296.	10. .00032761.	14. .00003249.
3. .264196.	7. .000729.	11. 4943.4961.	15. 250300.09.
4. 2992.09.	8. 3003.04.	12. 96.177249.	16. 6403.2004.

Find, to three decimal places, the square root of:

17. 150.	21. 1200.	25. 124356.	29. .18763.
18. 325.	22. 17500.	26. 208070.	30. .000177.
19. 570.	23. 35410.	27. 154.118.	31. .511411.
20. 791.	24. 65117.	28. 3456.77.	32. 675.4030.

Find the square root of the following to three decimal places.

33. $\frac{2}{3}$.	35. $\frac{5}{7}$.	37. $\frac{11}{12}$.	39. $\frac{245}{311}$.
34. $\frac{3}{5}$.	36. $\frac{7}{8}$.	38. $\frac{15}{19}$.	40. $\frac{419}{567}$.

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WRITTEN APPLICATIONS OF SQUARE ROOT

1. How many feet are there in the side of a square whose area is 4624 square feet?

2. The total area of the six faces or sides of a cube is 3456 square inches. What is the length of each edge of the cube?

3. How many feet in the side of a square field containing 1 acre, if an acre is equal to 43,560 square feet?

4. A farmer built a fence around a square field known to contain 640 square rods. How many feet in the length of the fence?

5. A contractor uses 14,400 paving tiles, each 6 inches square, for covering a square courtyard. What is the length of the sides of the yard in feet?

6. A man walking at the rate of 4 miles per hour goes around a square park whose area is 160 acres. In how many hours can he go once around the park?

7. A fence is built around a square court known to contain 15625 square feet. At \$1.25 per foot, find the cost of the fence.

8. Jack and his father walked around a square field whose area was exactly $22\frac{1}{2}$ acres. How many rods did they walk in going around it?

9. A lot 160 ft. deep and 90 ft. wide on a street sold for \$40 a front foot. How much should be received for a square lot of the same area at the same price per front foot?

10. A real estate agent gave a building lot 150 feet long and 100 feet wide in exchange for a square corner lot containing the same amount of ground. Find the length of the sides of the square lot.

11. In attempting to arrange 1000 boys in a square formation at an exhibition, a gymnasium instructor finds that he has 39 boys who are not included in his square. How many boys are there on each side of the square?

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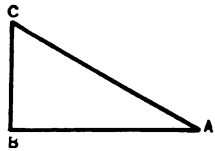
THE RIGHT TRIANGLE

A Right Triangle is a triangle that contains a right angle.

In the figure,



ABC is a right triangle.
The angle at *B* is a right angle.

The Hypotenuse of a Right Triangle is the side opposite the right angle.

AC is the hypotenuse of the right triangle *ABC*.

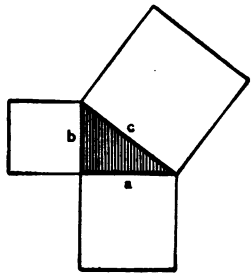
The Legs of a Right Triangle are the two sides that include the right angle.

BA and *BC* are the legs of the right triangle *ABC*.

For convenience we frequently refer to the side *BA* as the base of the triangle, and to the side *BC* as the altitude, but these terms apply only in particular cases.

By geometry it may be proved that

In any right triangle the square drawn upon the hypotenuse is equal to the sum of the squares drawn upon the legs.

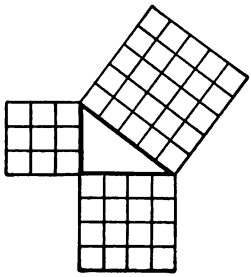


That is, in the figure,

c is the hypotenuse, *a* and *b* are the legs.

Then: The area of the square whose side is *c* is equal to the area of the square on *b* plus the area of the square on *a*. Or,

$$c^2 = a^2 + b^2.$$



In many right triangles this relation can be seen at once by dividing the three squares into unit squares.

Thus, in the figure, the length of the hypotenuse is 5, and the lengths of the legs, 4 and 3 respectively. That is,

$$5^2 = 4^2 + 3^2.$$
$$25 = 16 + 9.$$

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To find the length of the hypotenuse of a right triangle:

Since, in any right triangle,

$$c^2 = a^2 + b^2,$$

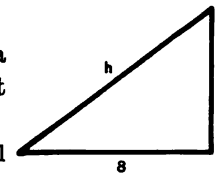
Extracting the square root of both expressions,

$$c = \sqrt{a^2 + b^2}.$$

Or, *The length of the hypotenuse of a right triangle equals the square root of the sum of the squares on the legs.*

Illustration:

1. Find the length of the hypotenuse of a right triangle whose legs are 6 feet and 8 feet respectively.



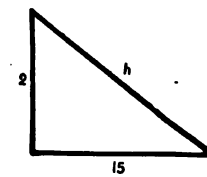
The figure will aid in the solution, and we will let the hypotenuse be represented by h.

Then: $h^2 = 6^2 + 8^2$.

Whence,

$h = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$. *Result.*

2. Find the length of the hypotenuse of the right triangle in the figure.



$h = \sqrt{12^2 + 15^2} = \sqrt{144 + 225} = \sqrt{369} = 19.2094$.

Result.

BLACKBOARD PRACTICE

Draw a right triangle to represent each of the following cases, and find the length of the hypotenuse if the length of the legs are, respectively,

1. 12 in., and 16 in.
2. 15 in., and 20 in.
3. 18 ft., and 24 ft.
4. 24 ft., and 32 ft.
5. 48 ft., and 64 ft.
6. 5 ft., and 12 ft.
7. 10 rd., and 24 rd.
8. 12 ft., and 14 ft.
9. 15 ft., and 30 ft.
10. 25 ft., and 30 ft.
11. 50 ft., and 75 ft.
12. 75 ft., and 80 ft.
13. 90 ft., and 100 ft.
14. 110 ft., and 140 ft.

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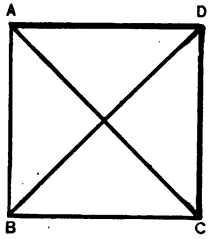
To Find the Length of the Diagonal of a Square or Rectangle.

The Vertices of a square or a rectangle are the meeting points of the sides.

In the figure the vertices are A, B, C, and D.



The Diagonals of a square or a rectangle are the two straight lines joining opposite vertices.

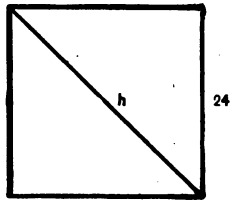
In the figure the diagonals are AC and BD.

As the angles of a square or rectangle are right angles we may form a right triangle in either by drawing one diagonal. Therefore,

The diagonal of a square is the hypotenuse of a right triangle having equal legs, and the diagonal of the rectangle ABCD is the hypotenuse of a right triangle having unequal legs.

Illustrations:

1. Find the length of the diagonal of the floor of a square room, the length of whose side is 24 feet.



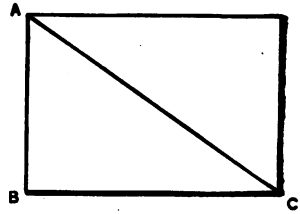
The figure represents the floor of the room, and we are to find the hypotenuse of a right triangle whose legs are each 24 feet long.

$$h^2 = 24^2 + 24^2 = 576 + 576 = 1152.$$

Then $h = \sqrt{1152} = 33.94$. Or, 33.94 + feet.

Result.

2. A man stands at the corner of a field 350 feet long and 250 feet wide. If he goes diagonally across the field to the corner opposite, how much less distance does he travel than by going along two sides?



The figure represents the field, and the man starts at A. Find the length of AC.

$$AC = \sqrt{350^2 + 250^2}$$
$$= \sqrt{122500 + 62500}$$
$$= \sqrt{185000}$$
$$= 430.11 + \text{feet.}$$

The distance around two sides $= 350 + 250 = 600$ feet.

Hence, he saves $(600 - 430.11+)$ feet $= 169.89-$ feet.

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BLACKBOARD PRACTICE

Find the diagonal of a square whose side is:

1. 10 feet. 3. 30 feet. 5. 40 rods. 7. 90 yards.
2. 15 feet. 4. 50 feet. 6. 65 rods. 8. 100 yards.

Find the diagonal of a rectangle whose length and width respectively are:

9. 15 feet, 10 feet.
10. 20 feet, 12 feet.
11. 30 yards, 12 yards.
12. 40 yards, 15 yards.
13. 50 rods, 30 rods.
14. 100 rods, 65 rods.

To find the length of one leg of a right triangle when the length of the hypotenuse and the length of the other leg are given.

We have learned that the square drawn upon the hypotenuse of a right triangle is equal to the sum of the squares drawn upon its legs.

That is, $c^2=a^2+b^2$

Writing this expression in the form $a^2+b^2=c^2$, and subtracting b^2 from both members, we have:

$$\begin{array}{r} a^2+b^2=c^2 \\ b^2=b^2 \\ \hline a^2+b^2-b^2=c^2-b^2 \end{array}$$

That is, $a^2=c^2-b^2$.

And, $a=\sqrt{c^2-b^2}$.

Or, subtracting a^2 from both members,

$$\begin{array}{r} a^2+b^2=c^2 \\ a^2 \quad =a^2 \\ \hline a^2-a^2+b^2=c^2-a^2 \end{array}$$

That is, $b^2=c^2-a^2$.

And, $b=\sqrt{c^2-a^2}$.

Either leg of a right triangle is equal to the square root of the difference of the square on the hypotenuse and the square on the other leg.

BLACKBOARD PRACTICE

Find the length of one leg of a right triangle when the length of the hypotenuse and the length of the other leg are, respectively:

1. 12 feet, 9 feet. 4. 15 yards, 10 yards. 7. 75 feet, 60 feet.
2. 20 feet, 15 feet. 5. 24 rods, 20 rods. 8. 90 feet, 80 feet.
3. 45 feet, 30 feet. 6. 45 yards, 36 yards. 9. 100 rods, 60 rods.

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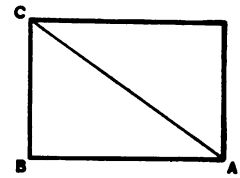
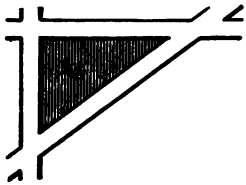
WRITTEN APPLICATIONS OF THE RIGHT TRIANGLE

1. One of the most common sizes in manufactured rugs is the "9 by 12," which is 9 feet wide and 12 feet long. What is the length of its diagonal? What is the length of the diagonal of a "6 by 9" rug?

2. Measure the length and the width of the floor of any rectangular room, and calculate the length of the diagonal in feet and inches, to the nearest inch. Then measure the diagonal carefully, and compare the measurement with your calculation.

3. A father and son during their daily walk came to a corner of a field at A. The father walked around the field, going 90 rods along AB, and 67.5 rods along BC; but the boy went straight across the field from A to C. Find the number of rods that each walked.

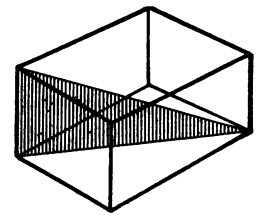




4. A small park at the intersection of three streets is shaped like a right triangle, and the sides which include the right angle are 160 feet and 120 feet respectively. Find the cost of inclosing the plot with a concrete curb costing \$1.10 a running foot.

5. At a point on a river bank 180 feet from the opposite bank a swimmer starts to swim straight across, but the current carries him downstream so that he lands at a point 80 feet down the river and on a line at right angles to his intended line of crossing. How far does he actually swim?

6. A room is 24 feet long, 18 feet wide, and 10 feet high from the floor to the ceiling. What is the distance in feet and inches from any one of the lower corners of the room to the opposite upper corner?



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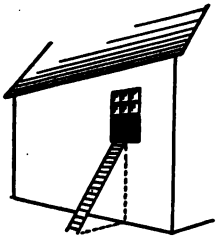
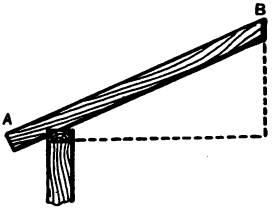
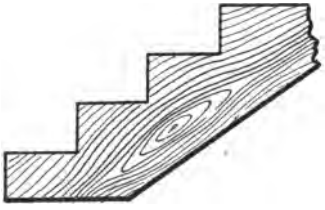


7. A window sill in a house is 16 feet from the ground, which is level, and a ladder 20 feet long is so placed that the upper end of it rests upon the sill. How far is it from the foot of the ladder to the point in the wall directly under the window?

8. Jones and Brown start at the same time and at the same point, Jones going straight east for 7 hours, while Brown goes straight south for 7 hours. If Jones travels 20 miles an hour, and Brown travels 28 miles an hour, how many miles apart will they be at the end of seven hours?

9. The height of a roof at the peak is 6 feet above the height at the eaves, and the width of the building is 30 feet. Allowing 1 foot for the "overhang," how long must a carpenter cut the "rafter" AB in the figure?





10. A stairway is made up of 12 steps, each "tread" being 12 inches wide, and each "riser" 9 inches high. Calculate the length of the "stringer" necessary for the stairs.

11. A pole is supported on two opposite sides by wires, one of which stretches from the top of the pole to the ground level with the pole, while the other stretches from the top of the pole to the top of a wall 20 feet high. If the pole is 65 feet high, the long wire 90 feet in length, and the short wire 60 feet in length, find the distance from the foot of the wall to the foot of the long wire. (Draw a figure to illustrate the problem.)

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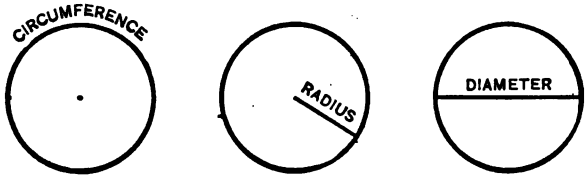
THE CIRCLE

A Circle is a figure bounded by a curved line, all points of which are equally distant from a point within called the **Center**



Because of the needs of later mathematics, the circle *may be defined as a curved line*, all points of which are equally distant from a point within called the center.

The Circumference is the length of the bounding line of a circle.

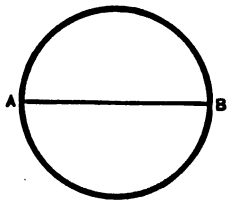
The Radius is the distance from the center to any point on the circumference. (The plural of radius is *radii*.)

The Diameter of a circle is the distance from any point on the circumference measured through the center and ending in the circumference.

The circumference, radius, and diameter are indicated in the illustrations above.

THE LENGTH OF THE CIRCUMFERENCE OF A CIRCLE

The length of the circumference of a circle has been found to be very nearly 3.1416 times the length of the diameter. That is, in the figure, the distance from A around the circumference to A is nearly 3.1416 times the distance from A through the center to B.



Since the decimal .1416 is a little too large, we express the relation of circumference to diameter more accurately if we say that the circumference is approximately 3.1416– times the diameter.

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In the case of small circles we may use $3\frac{1}{7}$, or $\frac{22}{7}$, instead of 3.1416–, and our results will be reasonably accurate.

In the following exercise the pupil should draw in each case a circle to represent the problem, placing upon this circle the given lines. After finding the required result, write it upon the line whose length is sought.

BLACKBOARD PRACTICE

Using 3.1416–, find, approximately, the circumference of a circle whose diameter is:

1. 15 in.	**6.** 2.7 in.	**11.** 21.4 ft.	**16.** 5.12 yd.
2. 20 in.	**7.** 3.7 ft.	**12.** 13.5 yd.	**17.** 3.08 rd.
3. 42 in.	**8.** 5.8 ft.	**13.** 20.5 yd.	**18.** 1.75 yd.
4. 34 ft.	**9.** 9.2 yd.	**14.** 31.4 rd.	**19.** 50.9 rd.
5. 27 rd.	**10.** 8.5 rd.	**15.** 5.25 rd.	**20.** 150.5 yd.

Using 3.1416–, find, approximately, the circumference of a circle whose radius is:

21. 5 ft.	**25.** 3 ft. 6 in.	**29.** 15.75 yd.
22. 7.5 ft.	**26.** 4 ft. 10 in.	**30.** 125.75 yd.
23. 9.25 ft.	**27.** 3 yd. 2 ft.	**31.** 131.05 rd.
24. 10.5 ft.	**28.** 5 rd. 3 yd.	**32.** 250.5 ft.

Using $3\frac{1}{7}$, find, approximately, the circumference of a circle whose diameter is:

33. 14 in.	**36.** 42.5 in.	**39.** 142 ft.	**42.** .434 ft.
34. 28 in.	**37.** 25.1 ft.	**40.** 253 yd.	**43.** 1.04 yd.
35. 35 in.	**38.** 37.8 yd.	**41.** 45.5 yd.	**44.** 11.42 rd.

To find the diameter of a circle when the circumference is given, divide the given circumference by 3.1416. (The result will be slightly larger than the true diameter.)

THE AREA OF A CIRCLE

If we think of a circle as composed of a very large number of little triangles, we can imagine these triangles laid out in a straight line as in the figure.

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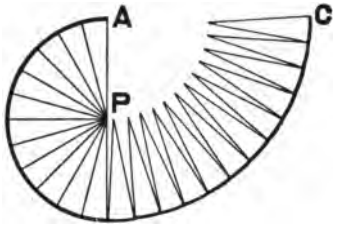
In the figure

AC is the length of the circumference.

Also

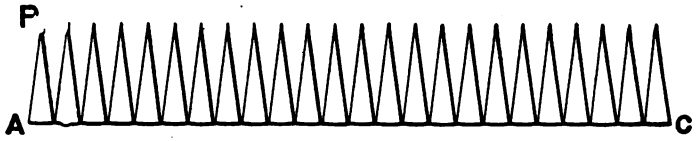
AC is the sum of the bases of all the triangles.



Now each little triangle has an altitude approximately equal to the radius of the circle, and if the triangles were small enough so as to be exact triangles, their total area would be

$\frac{1}{2}$ altitude $\times$ sum of all the bases,

Or, $\frac{1}{2}$ radius $\times$ circumference of the circle.



That is,

The Area of the Circle $= \frac{1}{2}$ product of radius $\times$ circumference.

But the circumference $= 3.1416- \times$ diameter, or, $3.1416- \times$ (twice the radius).

Or, using letters, Area $= \frac{1}{2} R \times 3.1416- \times 2R$

$$= \frac{R \times 3.1416- \times 2 \times R}{2}$$

$$= 3.1416- \times R^2.$$

That is,

The area of a circle equals 3.1416– times the square of the radius.

Illustrations:

Find the area of a circle whose radius is 12 inches.

Squaring the radius, $12^2 = 144$.

Then, Area $= 3.1416- \times 144$ square inches

$= 452.3904$ square inches. *Result.*

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Find the area of a circle whose diameter is 17 feet.

The radius	$17 \div 2 = 8.5$ feet.
Squaring the radius,	$8.5^2 = 72.25$ square feet.
Then,	Area $= 3.1416- \times 72.25$ square feet.

BLACKBOARD PRACTICE

Using $3\frac{1}{7}$, find approximately the area of a circle whose radius is:

1. 7 in.	6. 7.7 in.	11. .14 ft.	16. 2.5 ft.
2. 10 in.	7. 5.6 in.	12. 2.8 ft.	17. 5.3 ft.
3. 14 in.	8. 4.3 in.	13. 9.1 ft.	18. 6.1 ft.
4. 21 in.	9. 128 in.	14. 14.7 in.	19. 8.2 ft.
5. 30 in.	10. 149 in.	15. 210 in.	20. 9.8 ft.

Using 3.1416−, find the area of a circle

Whose radius is:		Whose diameter is:	
21. 12 in.	25. 31 ft.	29. 12 in.	33. 7.8 ft.
22. 16 in.	26. 4.2 ft.	30. 18 in.	34. 5.7 ft.
23. 28 in.	27. 5.1 ft.	31. 14 ft.	35. 61 yd.
24. 32 in.	28. 7.3 ft.	32. 19 ft.	36. 8.4 yd.

37. What is the area of the end of a circular stone column whose diameter is 15 inches?

38. What is the area of the bottom of a circular tin pail whose diameter is 10 inches?

39. What is the area of the bottom of a circular catch-basin whose diameter is 4.2 feet?

40. A circular pond whose diameter is 45 feet has a concrete bottom. What is the area of the concrete surface in the bottom of the pond?

41. The piston-head in the cylinder of a locomotive is 20 inches in diameter. How many square feet are there in the area of this piston-head?

42. A concrete paving at the base of a fountain is shaped as in the figure. Find the number of square feet in its area.

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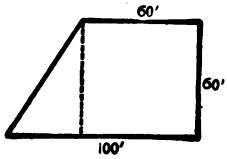
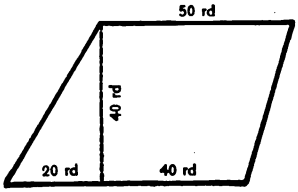
WRITTEN APPLICATIONS

Problems Involving Combinations of the Rectangle, the Triangle, and the Circle in a Single Area.

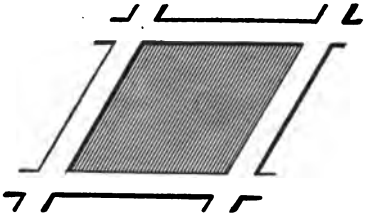
1. What is the area of a plot of ground shaped as in the figure? What is the value of the plot at \$240 an acre? (1 A. = 43,560 sq. ft.)

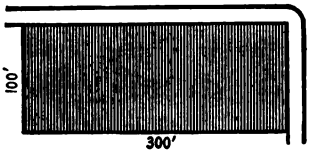




2. A city building lot is shaped as in the figure. At the rate of \$4000 an acre, what is the value of the lot?

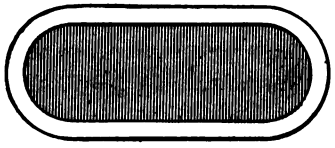
3. Four city streets intersect as in the figure, and the inclosed plot, which is 400 feet long and 300 feet wide, was sold at the rate of \$16,000 an acre. How much did the plot sell for?





4. A builder lays a concrete walk along two sides of a new corner property. At \$1.40 a square yard, find the cost of the walk, if it is 6 feet wide.

5. A school athletic association constructed a cinder track shaped as in the figure, at a cost of \$.80 a square yard. If the width of the track was 12 feet, the straightaway length was 500 feet, and the radii of the ends 50 feet, find the cost of the track.



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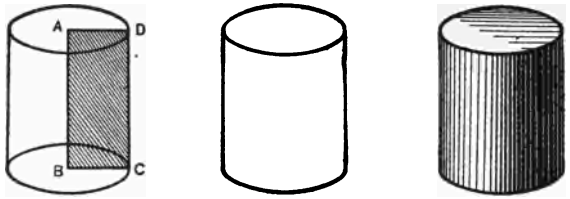
THE CYLINDER



If a rectangle is revolved about one of its sides as an axis, the solid formed is called a **Cylinder.**

If AB, in the figure at the left, remains fixed and the rectangle $BCDA$ revolves about AB, the rectangle in its revolution forms a cylinder.

The two circles that are formed by AD and BC are called the **Bases of the Cylinder.**

The distance between the bases is called the **Altitude of the Cylinder.**

The curved surface made by the line CD is called the **Lateral Surface.**

THE VOLUME OF A CYLINDER

We have found that the volume of a rectangular solid equals the product of the number representing the square units in the area of its base by the number representing the units in its height.

In like manner, therefore:

The volume of a cylinder equals the product of the number representing the square units in the area of the base by the number representing the units in the height. Or, briefly,

Volume of a cylinder = The number of units in the area of the base times the number of units in the altitude.

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Illustrations:

1. Find the volume of a cylinder the area of whose base is 6 square inches, and whose altitude is 10 inches.

Volume = area of base times altitude
= (6 × 10) cubic inches.
= 60 cubic inches. *Result.*

2. Find the volume of a cylinder whose base is 10 inches in diameter, and whose altitude is 20 inches.

Using the rule for finding the area of the circular base,
Radius of base = 5 in.
Area of base = $3.1416 \times 5^2 = 3.1416 \times 25 = 78.54$ sq. in.
Volume = area of base times altitude
= (78.54 × 20) cu. in.
= 157.08 cu. in. *Result.*

BLACKBOARD PRACTICE

Find the volume of a cylinder whose altitude and base area respectively are:

1. 10 in., 8 sq. in.
2. 12 in., 10 sq. in.
3. 12 in., 12 sq. in.
4. 16 in., 8 sq. in.
5. 24 in., 30 sq. in.
6. 48 in., 35 sq. in.
7. 66 in., 24 sq. in.
8. 2 ft. 6 in., 15 sq. in.
9. 3 ft. 4 in., 4.2 sq. in.
10. 5 ft. 9 in., 6.75 sq. in.
11. 8 ft. 4 in., 4 sq. ft.
12. 9 ft. 6 in., 5 sq. ft. 24 sq. in.
13. 10 ft. 8 in., 6 sq. ft. 72 sq. in.
14. 14 ft. 10 in., 7 sq. ft. 96 sq. in.

Find the volume of a cylinder whose altitude and the radius of whose base respectively are:

15. 14 in., 5 in.
16. 16 in., 9 in.
17. 24 in., 4 in.
18. 12 ft., 2 ft.
19. 20 ft., 3 ft.
20. 22 ft., 3 ft.
21. 3 ft. 6 in., 1 ft. 4 in.
22. 4 ft. 3 in., 2 ft. 3 in.
23. 6 ft. 4 in., 2 ft. 8 in.
24. 7 ft. 6 in., 3 ft. 1 in.
25. 8 ft. 4 in., 4 ft. 6 in.
26. 10 ft. 6 in., 5 ft. 9 in.

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Capacity of Silos. The capacity of a silo may be estimated in tons or in cubic feet. The silo is used for storing silage to be used in the winter. While the practice varies under different conditions and in different places, for our discussion we will assume that

1 cubic foot of silage weighs 40 pounds.

Then, 50 cubic feet of silage = 1 ton.

The amount of silage fed daily to a cow varies in different sections, but for this book it is assumed that a recognized average daily feeding is $1\frac{1}{2}$ cubic feet of silage. As the feeding season is about six months, a cow will eat in that time

$6 \times 30 \times 1\frac{1}{2} = 270$, the number of cubic feet of silage.

Or, allowing for waste, 300 cubic feet of silage per cow per year.

Silos are usually built cylindrical in shape, and are generally constructed of wood, brick, tile, or concrete.

Illustrations:

1. How many tons of silage can be placed in a cylindrical silo 10′ in diameter and 18′ high?

Area of the base $= 3.1416 \times 5^2 = 78.54$ sq. ft.
Volume of cylinder $= 78.54 \times 18 = 1413.72$ cu. ft.
Since 1 ton = 50 cu. ft. of silage, $1413.72 \div 50 = 28.274$ tons. *Result.*

2. Allowing 1.5 cu. ft. of silage per cow per day, how many days' feed for 30 cows can be stored in a silo 12 ft. in diameter and 25 ft. high?

Capacity of silo $= 3.1416 \times 6^2 \times 25 = 2827.44$ cu. ft.
Daily consumption $= 30 \times 1.5 = 45$ cu. ft.
Then, $2827.44 \div 45 = 62.8$ days. *Result.*

BLACKBOARD PRACTICE

Find the number of cubic feet capacity of a cylindrical silo:

1. 10′ in diameter, 20′ high.
2. 10′ in diameter, 24′ high.
3. 12′ in diameter, 24′ high.
4. 12′ in diameter, 30′ high.
5. 15′ in diameter, 24′ high.
6. 16′ in diameter, 30′ high.

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Allowing 300 cu. ft. of silage per cow for a season, how many tons will be required to provide feed for:

7. 12 cows? **9.** 20 cows? **11.** 35 cows? **13.** 50 cows?
8. 15 cows? **10.** 25 cows? **12.** 45 cows? **14.** 65 cows?

Allowing $1\frac{1}{2}$ cubic feet of silage per cow per day, for how many days will a silo:

15. 11 ft. in diameter with silage 21 ft. high provide feed for 8 cows?

16. 12 ft. in diameter with silage 20 ft. high provide feed for 10 cows?

17. 15 ft. in diameter with silage 24 ft. high provide feed for 16 cows?

18. 15 ft. in diameter with silage 25 ft. high provide feed for 20 cows?

19. 16 ft. in diameter with silage 27 ft. high provide feed for 28 cows?

20. A farmer keeps 36 cows and requires storage for enough silage to feed $1\frac{1}{4}$ cu. ft. daily to each cow from November 1 to April 15. How many tons of silage must he provide to supply this feed?

Capacity of Tanks and Cisterns. In measuring the capacity of tanks we use three different units of measure, the cubic foot, the gallon, and the barrel. It is often convenient to use in such measurements the weight of a unit quantity of a liquid.

EQUIVALENTS

1 cubic foot of water	=	$62\frac{1}{2}$ pounds.	(Approx.)
1 cubic foot of any liquid	=	$7\frac{1}{2}$ gallons.	(Approx.)
1 gallon of water	=	$8\frac{1}{3}$ pounds.	(Approx.)
1 gallon of water	=	231 cubic inches.	(Exact)
1 barrel of water	=	$4\frac{1}{5}$ cubic feet.	(Approx.)
1 bushel of small grain	=	2150.42 cubic inches.	(Exact)

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BLACKBOARD PRACTICE

Change to cubic inches:

1. 12 gal.
2. 19 gal.
3. 35 gal.
4. 25 bu.
5. 32 bu.
6. 45 bu.

Change to cubic feet:

7. 45 bbl.
8. 64 bbl.
9. 120 bbl.
10. 110 gal.
11. 165 gal.
12. 210 gal.

Change to gallons:

13. 45 cu. ft.
14. 70 cu. ft.
15. 91 cu. ft.
16. 1155 cu. in.
17. 1386 cu. in.
18. 16,170 cu. in.

Change to pounds:

19. 15 cu. ft. of water.
20. 15 gal. of water.
21. 12 bbl. of water.

Find, in bushels, the capacity of a rectangular bin:

22. $8' \times 5' \times 4'$.
23. $10' \times 5' \times 3'$.
24. $10' \times 6' \times 5'$.
25. $12' \times 8' \times 6'$.
26. $12' \times 10' \times 7'$.
27. $16' \times 12' \times 10'$.

Find, in gallons, the capacity of a rectangular tank:

28. $6' \times 5' \times 4'$.
29. $10' \times 6' \times 4'$.
30. $12' \times 8' \times 6'$.
31. $16' \times 6' \times 4'$.
32. $16' \times 12' \times 7'$.
33. $20' \times 8' \times 11'$.

Find, in barrels, the capacity of a rectangular tank:

34. $8' \times 6' \times 4'$.
35. $10' \times 7' \times 5'$.
36. $12' \times 7' \times 6'$.
37. $14' \times 8' \times 6'$.
38. $18' \times 12' \times 8'$.
39. $24' \times 16' \times 12'$.

Find, in cubic feet, the capacity of a box that will hold:

40. 12 bu.
41. 28 bu.
42. 45 bu.
43. 65 bu.
44. 72 bu.
45. 100 bu.
46. 120 bu.
47. 250 bu.
48. 360 bu.
49. 525 bu.

Find the weight of the water that fills a tank:

50. 8′ 6″ long, 5′ 3″ wide, 2′ 6″ deep.
51. 10′ 4″ long, 6′ 5″ wide, 3′ 7″ deep.
52. 16′ long, 10′ 6″ wide, 6′ 8″ deep.
53. 24′ 6″ long, 15′ 9″ wide, 5′ 10″ deep.

Find, in gallons, the capacity of a cylindrical tank:

54. 3′ in diameter, 6′ long.
55. 4′ in diameter, 7′ long.
56. 5′ in diameter, 8′ long.
57. 4′ in diameter, 9′ long.
58. 10′ in diameter, 12′ high.
59. 12′ in diameter, 14′ high.
60. 18′ in diameter, 15′ high.
61. 20′ in diameter, 16′ high.

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BUSINESS FORMS

I. ACCOUNTS

An Account, in business practice, is a record of transactions.

A Daybook is a book used for recording the business of a day, each transaction being set down as it occurs.

EXAMPLE: A merchant sells an article to a customer who has the privilege of paying for purchases at the end of each month. The purchase is delivered to the customer, and immediately there is written upon the daybook a statement that goods to a certain amount have been bought.

A Ledger is a book in which all of the dealings with an individual, or all of the dealings of a particular class, are kept on one sheet.

This is accomplished by transferring the different items on the daybook to individual accounts, and this work is done daily or weekly according to the amount of business done. The process enables a merchant to find at any given time a complete record of the transactions with an individual, and by using the ledger the merchant saves the trouble and annoyance of searching through a mass of transactions in his daybook.

Debits in a ledger account are the amounts charged against an individual or firm.

In practice the left side of a record is used for debits.

Credits in a ledger account are the amounts received from an individual or firm.

In practice the right side of a record is used for credits.

The Balance in a ledger account is the difference between the sum of the debits and the sum of the credits.

Balancing the Books is a process of finding at regular intervals the exact condition of all the accounts on the books of a firm.

In business practice it is a common custom to send out each month a record of transactions for the last thirty days. Such a process is a form of balancing an account, for it indicates the condition in which the account stands.

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Abbreviations Used in Keeping Accounts

Account	acc't.	Credit	Cr.	Paid	Pd.
Balance	bal.	Debit	Dr.	Payment	Pay't.
Company	Co.	Merchandise	mdse.	Received	Rec'd.

The symbol # has two uses. 5# means " 5 pounds." #5 means " number 5."

I. The Daybook. The following form illustrates the method of entering daily transactions in the daybook.

247

June 16, 1918

45	John M. French	Dr.					
	5# Sugar @ 6¢		.30				
	16 yd cloth @ 18½¢		2.96			3	26
91	W. R. Shepard	Dr.					
	1 bbl Flour					11	25
18	F. H. Robbins	Cr.					
	Cash			12	75		
164	Mrs. J. A. Phelps	Dr.					
	12 yd cotton @ 9¢		1.08				
	25# Sugar @ 9¢		2.25			3	33

The numbers at the left indicate the page in the ledger on which is recorded the individual account. (See Ledger, p. 177.)

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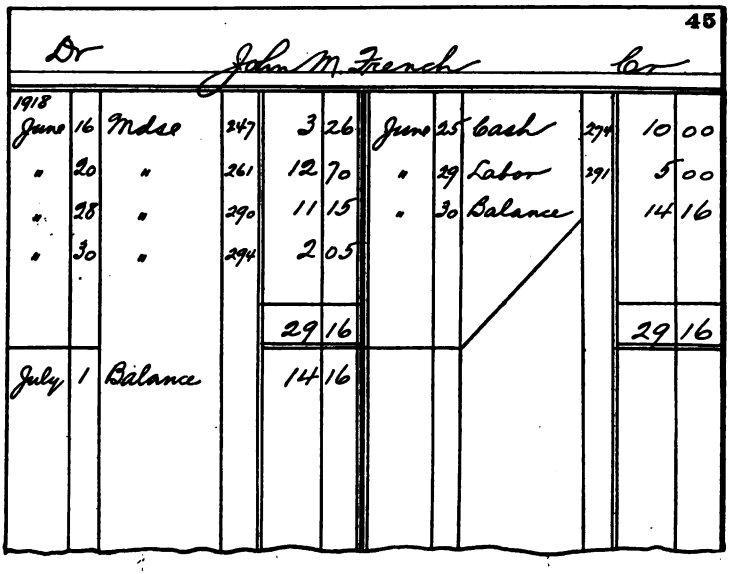
It will be observed by consulting the sample ledger page in the next illustration that this same item is recorded only in total, and that the page reference is "247" or the daybook page indicated above.

II. The Ledger. The following form illustrates the method of carrying to, or "posting" transactions in the ledger. The illustration assumes that the charge in John M. French's name is the first one on his account, or, that he has just "opened" a new account with the merchant.



45

Dr John M. French Cr

1918						1918					
June	16	Mdse	247	3	26	June	25	Cash	274	10	00
"	20	"	261	12	70	"	29	Labor	291	5	00
"	28	"	290	11	15	"	30	Balance		14	16
"	30	"	294	2	05						
				29	16					29	16
July	1	Balance		14	16						

The account above has been balanced on the last day of the month of June. It includes other purchases made during the month, and on the credit side it shows a cash credit of $10, and a labor credit of $5. Upon adding the amounts on the debit side and those on the credit side, we find that the debits amount to $29.16 and that the credits amount to $15. The excess of the debits over the credits is $29.16 − $15, or $14.16.

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This amount is the balance due, and to make the totals on the two sides equal, the $14.16 is usually entered in red ink on the proper side. This same balance is entered under date of July 1 on the debit side, indicating that the account of Mr. French starts the month of July with $14.16 owing the merchant.

WRITTEN APPLICATIONS

Make a ruled ledger form for each of the following accounts, supplying your own daybook page numbers. Balance each account on the day indicated, using red ink for the ruling and for the entry of the balance item.

1. Account with W. R. Shepard.

June 16, Dr., Mdse. as per daybook item in illustration, $11.25; June 18, Dr., Mdse., $1.90; June 19, Dr., Mdse., $4.30; June 20, Dr., Mdse., $7.91; June 22, Cr., Cash, $10; June 23, Dr., Mdse., $3.75; June 24, Cr., Labor, $4.50; June 25, Dr., Mdse., $6.15; June 25, Cr., Produce, $5.08; June 26, Dr., Mdse., $16; June 27, Cr., Cash, $5; June 29, Dr., Mdse., $11.75; June 30, Cr., Cash, $12.

Show that Mr. Shepard owes a balance of $26.43 on June 30, and carry this balance to the next month under date of July 1.

2. Account with F. H. Robbins.

June 5, Dr., Mdse., $5.11; June 7, Dr., Mdse., $11.10; June 8, Dr., Mdse., $57; June 11, Dr., Mdse., $21.20; June 15, Cr., Cash, $50; June 16, Cr , Cash, $12.75 (see daybook item in illustration); June 19, Dr., Mdse., $32.75; June 27, Cr., Cash, $40; June 30, Dr., Mdse., $17.75; July 3, Dr., Mdse., $35.30; July 9, Dr., Mdse., $54.50; July 12, Cr., Cash, $75; July 14, Dr., Mdse., $19.

Balance the account on June 30, and carry balance to proper side under date of July 1. Mr. Robbins left town on July 15, and made a full settlement with the merchant on that day. Balance the account on that date and find the amount Mr. Robbins paid to the merchant.

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3. Account with Mrs. J. A. Phelps.

June 4, Dr., Mdse., $1.09; June 5, Dr., Mdse., $2.11; June 8, Dr., Mdse., $0.98; June 10, Dr., Mdse., $3.05; June 11, Dr., Mdse., $1; June 14, Dr., Mdse., $1.10; June 16, Dr., Mdse., $3.33 (see daybook item in illustration); June 21, Dr., Mdse., $1.74; June 25, Dr., Mdse., $2.50; June 29, Dr., Mdse., $2.06; July 3, Dr., Mdse., $1.19; July 10, Dr., Mdse., $6.17; July 14, Dr., Mdse., $0.65; July 16, Dr., Mdse., $2.47.

On July 1 the merchant sent Mrs. Phelps a statement showing the balance due him. What was the amount of that balance? What was the balance on July 16? On July 17 Mrs. Phelps paid $20 on account and on the 25th she paid the balance. As no purchases were made after July 16, what amount was due on the 25th?

II. BILLS

A Bill is a detailed statement of indebtedness for goods purchased or for services rendered.

Bills are usually presented on blanks printed for the purpose, giving the name of the firm presenting the bill and the nature of its business. The bill form is filled in with all the details of the purchases, giving the date of each purchase, the name of each article, the unit price of each, the total cost of each lot, and the total amount of the several lots.

As a rule merchants present a bill immediately following a purchase, and follow up the bill, if still unpaid at the end of the current month, by a brief statement that gives only the total of the whole bill or bills purchased during that month. Some follow the practice, however, of presenting bills only at the end of calendar months, and use statements only in cases where their bills have run for some time.

It will add to the interest in this work if sample bill forms obtained from your local merchants are brought in for study and discussion.

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A Bill is Receipted when, upon payment, the seller or merchant writes the words, "Received Payment," and signs his name at the bottom of the bill. If some one connected with the firm and authorized to sign the firm name receipts a bill, it is customary to indicate this fact by writing under the firm name the signature or the initials of the person who receipted it. In the illustration following the bill is receipted by an agent of the firm.

TERMS: CASH PHILADELPHIA, Dec 19 1918

PILLSBURY FURNITURE CO.

DEALERS IN

FINE FURNITURE, RUGS AND DRAPERIES

Sold to Eliot and Hadley

Denver, Colorado

12 Brass Beds #716 @ $15.			$180 00
24 " " #951 @ 18.			432 00
10 Bookcases #1001 @ 24.			240 00
			$852 00
Received Payment, 1/10/19 Pillsbury Furniture Co. per F. M. J.		60	

Using names suggested by the local business in your city make out bills for each of the following applications. In each case draw a billhead, find the total of each bill, calculate the discounts where they are given, and receipt each bill.

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WRITTEN APPLICATIONS

1. 10 lb. sugar at 9¢; 15 lb. rice at 12¢; and 5 lb. butter at 58¢.

2. 12 lb. coffee at 35¢; 50 lb. sugar at $8\frac{1}{2}$¢; and 10 lb. starch at 5¢.

3. 20 yd. cotton at 10¢; 10 yd. gingham at 20¢; and 40 yd. denim at 18¢.

4. 18 oranges at 3¢; 48 lemons at $2\frac{1}{2}$¢; and 60 grapefruit at $12\frac{1}{2}$¢.

5. 36 bu. wheat at $1.75; 45 bu. oats at 90¢; and 45 bu. barley at 85¢.

6. 34 cans peas at 11¢; 50 cans corn at $12\frac{1}{2}$¢; and 60 cans tomatoes at 16¢.

7. 25 doz. clothespins at 4¢; 12 yd. toweling at 15¢; and 10 yd. muslin at 9¢.

8. 25 hammers at 95¢; 30 chisels at 60¢; 12 rip saws at $2.50; 15 back saws at $2.10; and 18 screwdrivers at 35¢.

9. 24 blockplanes at 95¢; 30 jackplanes at $2.25; 24 try squares at $1.00; 36 two-foot rules at $1.60 per dozen; and 60 nail sets at $1.25 per dozen.

10. 25 yd. carpet at $1.10; 36 yd. carpet at $1.75; 48 yd. linoleum at $0.75; 48 yd. matting at 35¢; 45 yd. filling at 50¢. Discounted at 5% for cash. Find the net amount of the bill.

11. A retail coal dealer bought 110 tons of coal at $5.10. Make out a bill if sold by the Lehigh Coal Company to this dealer, allowing a discount of 5% for cash, and showing a receipted bill.

12. Make out to yourself a bill from your leading furniture dealer for the following: 1 dining room table, $45; 6 dining room chairs at $5.75 each; 1 bureau, $65; 1 dresser, $55; 1 brass bed, $27.50; 1 box spring, $14.50; 1 Wilton rug, $52.50; 1 Wilton rug, $42.40; and 30 window shades at $0.75 each. Discount the bill at 5% and receipt same.

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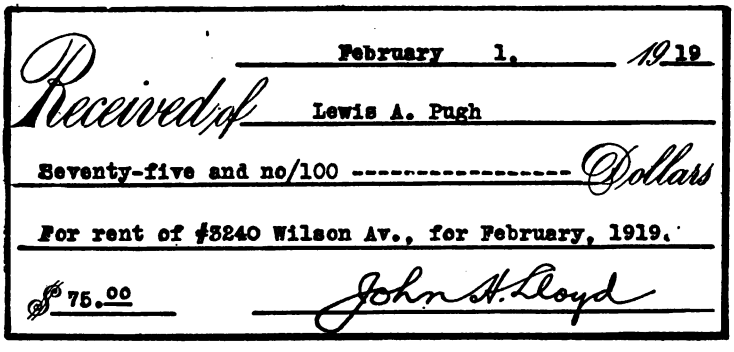
RECEIPTS

In many activities not strictly commercial there is a constant need for the giving of receipts, and printed blank forms for the use of real estate agents, professional men, and also for home use, are readily obtained. A receipt should show the name of the person or the firm to whom it is given, the amount received, the date that it was received, and the signature of the person or firm who received the payment. In addition some receipts provide space in which the nature of the transaction may be written. Three forms are:

(1) *The Cash Receipt.* This is given either for cash received on account, or in part payment of a debt; or "in full payment" when the whole of a debt is paid.

(2) *The Rent Receipt.* This is given for the payment of rent at fixed periods of time.

(3) *The Receipt for Service.* Usually given for work.



A variety of other forms might be enumerated. The illustration shows a receipt given for the payment of house rent.

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WRITTEN APPLICATIONS

Draw a neat blank form and fill out a receipt for:

1. $25 on account.
2. $3 for class dues.
3. $25 for service.
4. $40 for rent.
5. $20 on account.
6. $50 for a bicycle.
7. $75 in full.
8. $15 for rent.
9. $35 for service.

10. Make a receipt indicating that you have paid $20 to your grocer in full of all indebtedness to him.

11. Make a receipt indicating that you have paid $10 to your butcher to be applied on your account.

12. Make a receipt showing that you have received $75 from a tenant in payment of house rent.

13. Write a receipt indicating that you have made a deposit of $10 with a tailor, in part payment for a suit.

14. Mrs. Thomas agreed to pay $5 each week on a player-piano. Write a receipt indicating that the Brunswick Piano Company received her tenth installment on the debt.

THE PARCEL POST

Under the provisions of the Parcel Post it is possible to send a great variety of articles by mail between any two points in the United States. The rates for this service are governed by the weight of the article and by the distance it is carried; the government has fixed certain limits of distance called Zones and has placed definite rates per pound for each zone.

The principal restrictions governing the service are:

(1) Parcel Post matter must not be of a kind likely to injure any postal employee, or to damage the mail equipment or other mail matter; nor must it be of a character perishable within the period reasonably required for transportation and delivery.

(2) A Parcel Post package must not exceed 84 inches in the length and the girth combined.

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Zones and Rates:

Parcels mailed at any office for local delivery from that office are rated as local. For such parcels the rate is 5 cents for the first pound and 1 cent for each additional two pounds, or fractional part of two pounds. Parcels mailed at any office for delivery in the first or in the second zone are charged for at the rate of 5 cents for the first pound and 1 cent for each additional pound, or fractional part of one pound.

TABLE OF ZONES AND RATES

Number of Zone	Distance in Miles	Rate in Cents	
		First Pound	Additional Pounds
1 & 2	1 to 150	5	1
3	150 to 300	6	2
4	300 to 600	7	4
5	600 to 1000	8	6
6	1000 to 1400	9	8
7	1400 to 1800	11	10
8	over 1800	12	12

The Limit of Weight for local delivery and for the first three zones is 70 pounds. For all other zones the limit is 50 pounds.

WRITTEN APPLICATIONS

1. Find the cost of mailing a package weighing 2 pounds from your home to any point in the fifth zone.

2. Find the cost of mailing a package weighing $12\frac{1}{2}$ pounds to a point 425 miles distant from your home.

3. The express rate on a package weighing 5 pounds was 30 cents for a distance of 175 miles. Would it cost more or less, and how much, to send the same package by parcel post?

4. A wholesaler sends out 10 packages weighing 3 pounds each to a town 450 miles distant, and 8 packages weighing $5\frac{1}{2}$ pounds each to a town 200 miles distant. Find the total cost of the postage on the 18 packages.

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INSURANCE

Insurance is an agreement made by one party to pay for loss sustained by another party.

An Insurance Policy is the written agreement between the two parties.

The Premium is the amount paid for the insurance.

The Face of the Policy is the sum payable by the company according to contract in case of loss.

There are several kinds of insurance, the most common and important kinds being Fire Insurance and Life Insurance. Other forms are Accident, Health, Marine, Automobile, Liability, Burglar, and Plate Glass Insurance.

PROPERTY INSURANCE

Fire Insurance covers loss or damage to property by fire, water, smoke, and chemicals used in extinguishers.

The premium required for a certain amount of insurance is determined by an application of percentage.

The amount of insurance which can be carried on any given property is always some fractional part of the actual value of the property.

Illustration of Fire Insurance.

Brown & Co. own an office building worth $50,000.

They insure the building against loss or damage by fire, making the contract with the Franklin Fire Insurance Company.

The insurance company agrees to pay a sum not to exceed $35,000 in case of total loss.

Brown & Co. pay the insurance company a premium of $\frac{3}{4}\%$.

That is, the insurance costs them annually $.00\frac{3}{4} \times \$35{,}000$, or $252.50.

The problem of computing the premium when the face of the policy and the rate are given is a simple application of percentage.

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Illustrations:

1. Find the premium paid for insurance on a property valued at \$4500 at two thirds of the valuation, and at a rate of $1\frac{1}{2}\%$.

The phrase, "at two thirds of the valuation," means that the company will return no more than two thirds of the actual value in case of loss, even though the loss may be total.

$$.015 \times \tfrac{2}{3} \times \$4500 = \$45. \quad \textit{Result.}$$

2. A frame house is insured for \$4500 at a 3-year rate of .45 per \$100. Find the premium.

The face of the policy being \$4500, or "45 hundreds," the premium equals

$$45 \times \$.45 = \$20.25. \quad \textit{Result.}$$

BLACKBOARD PRACTICE

Find the premium when the face of the policy and the rate are, respectively:

1. \$1600, 1%.
2. \$1800, $1\frac{1}{2}\%$.
3. \$2000, $1\frac{1}{3}\%$.
4. \$2500, $1\frac{1}{4}\%$.
5. \$4500, $1\frac{3}{4}\%$.
6. \$7500, $2\frac{1}{4}\%$.
7. \$10,000, $1\frac{1}{4}\%$.
8. \$12,000, $2\frac{1}{2}\%$.
9. \$20,000, $2\frac{1}{2}\%$.
10. \$35,000, $2\frac{3}{4}\%$.
11. \$40,000, $2\frac{1}{2}\%$.
12. \$56,000, $1\frac{3}{4}\%$.
13. \$4500, 1.5%.
14. \$12,250, 1.7%.
15. \$15,000, 3.1%.
16. \$20,000, 2.5%.
17. \$30,000, 1.6%.
18. \$35,000, 2.5%.

Find the premium when the face of the policy and the rate per \$100 are,

19. \$8000, \$.40.
20. \$9000, \$.60.
21. \$9400, \$.75.
22. \$9600, \$.90.
23. \$10,000, \$1.10.
24. \$12,500, \$1.12.
25. \$20,000, \$1.50.
26. \$24,000, \$1.35.
27. \$21,000, \$1.05.
28. \$25,000, \$1.10.
29. \$30,000, \$1.20.
30. \$37,500, \$2.25.

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WRITTEN APPLICATIONS OF PROPERTY INSURANCE

1. Find the cost of insuring a house for $3000, for 3 years, at 60 cents per $100 for that time.

2. A frame house with a shingle roof is insured for $2500, for 3 years, at a rate of 60 cents for that time. Find the premium paid.

3. A similar frame house with a slate roof is insured for $2500, for 3 years, at a rate of 45 cents for that time. Find the premium paid.

4. Compare in examples 2 and 3 the cost of insuring the two houses, one of which had a better roof protection than the other had. What is the increase in per cent in the premium because of the poorer roof?

5. A merchant pays an annual premium of $240 for insurance on his stock of goods, and the rate is $2 per $100. What is the amount of the insurance he carried?

6. A dwelling-house is insured for $\frac{2}{3}$ of its value, for 3 years, and the rate is 45 cents per $100 for that time. The value of the house is $12,000. Find the annual cost of the insurance.

7. A wooden house is insured for $4000 at a rate of $1\frac{3}{4}\%$, and a brick house is insured for the same sum but at a rate of $1\frac{1}{4}\%$. Find the difference between the annual premiums paid for the insurance.

8. A factory valued at $20,000 was insured for $\frac{3}{4}$ of its value at a 2% rate. The insurance had run five years when the building was totally destroyed by fire. What was the difference between the total of the premium collected and the amount paid to the insured?

9. The insurance on a new high school building valued at $250,000 was written at the rate of $10.35 per $1000 for a term of five years. The risk was divided equally among five different companies. What premium did each company receive for the term?

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LIFE INSURANCE

Life Insurance is an agreement between an individual and a company, whereby the individual pays a certain amount to the company at stated periods and the company agrees to pay a stipulated sum to him or to his heirs, or to a beneficiary named in the policy.

The Policy is the written agreement provided by the company.

The Face of the Policy is the amount the company agrees to pay.

The Insured is the individual with whom the company makes the agreement.

The Premium is the amount paid by the insured to the company.

The Beneficiary is the person who receives the face of the policy under the conditions named in it.

The Ordinary Life Policy provides that the insured shall pay a fixed premium to the company annually, semi-annually, or quarterly, during the whole of his life. In return the company pays the face of the policy to the beneficiary at the death of the insured. The beneficiary is named in the policy, and may be his estate or a person or persons named by the insured.

The Endowment Policy provides that the insured shall pay a stated premium as in the case of the ordinary life policy, but these premiums are limited to a certain number of years. If the insured lives to pay all of the premiums required, the policy is said to mature, and the company will return to him the face of the policy, together with an additional amount which has accumulated through interest during the term of payment. This form of policy usually gives the insured two options at maturity. He may accept the face with interest, as already explained, or he may permit the policy to remain in force under an agreement that the face plus interest and a share of the company's profits shall be paid to his estate at his death, or to the beneficiary named in the policy.

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The Limited-payment Life Policy provides that the insured shall pay a stated premium for a certain number of years, after which the face of the policy remains in the possession of the company until paid to the beneficiary at the death of the insured. The phrase, " 15-payment life," means that the insured must make fifteen consecutive annual payments, and that his obligation to the company is filled upon making the fifteenth annual payment.

The premiums charged by insurance companies for life insurance are based upon accurate estimates of the time a healthy person of a given age may be expected to live. These estimates have been reached by investigating hundreds of thousands of actual cases, and by men especially trained for this kind of investigation. The insurance companies provide their agents with tables of premiums charged for various kinds of policies at different ages, and the premiums given in these tables are the charges per $1000 of insurance. The following table, supplied by a prominent company, gives the premiums on different kinds of policies at certain ages.

ANNUAL COST OF LIFE INSURANCE

AGE	ORDINARY LIFE	20-PAYMENT LIFE	20-YEAR ENDOWMENT
20	$18.01	$27.78	$47.54
21	18.40	28.21	47.67
22	18.80	28.65	47.72
23	19.23	29.10	47.81
24	19.68	29.59	47.91
25	20.14	30.07	48.03
26	20.64	30.58	48.14
27	21.15	31.12	48.27
28	21.69	31.87	48.41
29	22.26	32.23	48.55
30	22.85	32.83	48.71
35	28.35	36.17	49.75
40	30.94	40.34	51.39
45	37.09	45.69	54.15

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Using the Table.

Illustration:

At the age of 24 a young man took a policy for $3000, at the ordinary life rate. How much must he pay annually for the policy?

From the table, under "Ordinary Life" and opposite the age line "24," we find the rate per $1000, or $19.68

Hence, he must pay: $3 \times \$19.68 = \$59.04$. *Result.*

WRITTEN APPLICATIONS

Find the annual premium on each of the following policies; the kind of policy; its face; and the age of the applicant being:

1. Ordinary Life: $2000, 25 years.
2. 20-year Endowment: $3000, 25 years.
3. Ordinary Life: $5000, 30 years.
4. 20-payment Life: $8000, 25 years.
5. 20-year Endowment: $10,000, 30 years.
6. 20-payment Life: $5000, 35 years.
7. 20-year Endowment: $2000, 20 years.
8. Ordinary Life: $3000, 40 years.
9. Ordinary Life: $3000, 30 years.
10. 20-year Endowment: $3000, 30 years.
11. 20-payment Life: $10,000, 45 years.
12. 20-payment Life: $15,000, 40 years.

(In the following exercises use the table of rates unless rates are given.)

13. A premium of $30.94 annually was paid for a 15-payment life policy of $1000. At the end of the term what amount had the insured paid for his protection?

14. After paying 12 annual premiums of $44.70 on an insurance policy for $2000, the insured died. What amount had he paid for the $2000 turned over to his family by the company at his death?

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15. A man took out a policy for \$2000 on his 30th birthday anniversary, paying \$76.40 premium for a 15-payment life policy. If the man died in his 65th year, show approximately the profit to the insurance company through holding the premiums for the long period of 35 years, figuring interest at 4%.

16. At the age of 25 a man insured his life for \$3000, taking a 20-year endowment policy at the rate given. At the age of 45 the company paid him \$4157, the increase over the face of the policy representing accumulated profits. Disregarding interest, find the excess amount received by the insured over the total that he paid the company.

17. At the age of 30 Mr. Wilson took out a 20-year endowment policy for \$3000, and at the age of 35 he took out a 20-year endowment policy for \$5000. At maturity, both policies were paid to him by the companies issuing them. What excess did he receive over the total amount he had paid them in premiums? If accumulated profits in dividends had been permitted to remain with the companies during the endowment periods, amounting in all to \$2350, what was the approximate profit gained by him on his investment in insurance?

18. At the age of 36 a gentleman insured his life for \$10,000, taking out a 15-payment life policy at a cost of \$41.60 annually per \$1000. He died at 50 years of age, leaving this \$10,000 of insurance to be divided equally between two daughters. Compare the amount the two daughters received from the company with the amount which would have been left them had the father invested annually at 4% simple interest the sum paid in premiums. (Observe that the invested premiums would have been on interest for periods respectively 14, 13, 12, 11, ··· to 1 year. These periods added give the total time at which the several premiums would have been at interest.)

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TAXES

The Federal Government must meet the expense of salaries paid to its officers, of army and navy maintenance, of pensions, forestry, irrigation projects, etc.; the State must provide for its official salaries, for general education, and for various institutions; and Cities and Towns must provide for schools, for police and fire protection, and for sewers and other necessary improvements. And, to meet these financial needs, property owners are compelled to contribute annually, or at stated intervals, a certain percentage of their property valuation. This sum is called a tax. Hence, the definition,

A Tax is a sum demanded by a government for its support, for public purposes, or for improvements.

A Direct Tax is a tax levied upon a person or upon the value of his property or his business.

An Indirect Tax is a tax levied upon imported goods, upon liquors, or upon the manufactured products of tobacco, etc.

A tax upon imports is called a **Tariff**.

A tax upon manufactures is called **Internal Revenue**.

Real Estate is fixed property, like land and the structures built upon it.

Personal Property is movable property, like money, bonds, mortgages, live stock, merchandise, etc.

A Property Tax is a tax levied on property of any kind, real or personal.

An Assessor is an officer who estimates and records the value of property which is to be taxed.

The Assessed Value of a property is the estimate of its value made by an assessor, or by two or more assessors working together.

Property is usually assessed for taxation at a figure considerably below its actual value. No rules govern the process and in different localities there may be a wide variation in the valuation of properties which have the same actual value.

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The Rate of Taxation is nearly always expressed as a certain number of mills on each dollar of valuation.

The actual process is illustrated as follows:

Suppose that a property is assessed at $5000.

If the rate of taxation is 1.4 mills, this expression means that a tax of 1.4 mills is placed upon each dollar of valuation, or, that $.14 is the tax required on each $100 of value.

Hence, the tax on a valuation of $5000, at a rate of 1.4 mills, is

$$50 \times \$.14 = \$7.00. \quad \textit{Result.}$$

A Tax Collector is an officer legally authorized to collect taxes. He usually receives a commission on the total amount he collects.

A Poll Tax is a tax levied upon a person.

This tax is no longer collected in some states, while others still demand it.

Special Forms of Taxation. Other forms of taxation which come under the head of licenses are the privilege of running an automobile, of selling liquor, and of keeping a dog.

APPLICATION OF THE PRINCIPLES GOVERNING TAXATION

I. To Find the Amount of a Tax when the Assessed Valuation and the Tax Rate are Known.

Illustration:

Find the tax on property assessed at $3000, if the rate is 4 mills on $1.00.

4 mills on $1 = $.004 on each dollar.

Hence $3000 \times \$.004 = \$12.00.$ *Result.*

BLACKBOARD PRACTICE

Find the amount of the tax on an assessed valuation of:

1. $1200 at 3 mills.
2. $1800 at $3\frac{1}{2}$ mills.
3. $2400 at 4 mills.
4. $3500 at 4 mills.
5. $4000 at 3 mills.
6. $5000 at $1\frac{1}{4}$ mills.
7. $7500 at $1\frac{1}{2}$ mills.
8. $12,000 at $2\frac{1}{2}$ mills.
9. $15,000 at $3\frac{1}{4}$ mills.
10. $20,000 at $4\frac{1}{2}$ mills.

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II. To Find the Tax Rate When the Assessed Valuation and the Amount of the Tax are Known.

Illustration:

Find the tax rate when property assessed at $3500 is taxed for $8.75.

$8.75 is what per cent of $3500?

$8.75 ÷ $3500 = .002½. Hence, the rate is 2½ mills. *Result.*

BLACKBOARD PRACTICE

Find the tax rate when the amount of the tax on:

1. $1000 is $1.50.
2. $1500 is $3.00.
3. $1800 is $4.16.
4. $2000 is $6.00.
5. $2750 is $8.25.
6. $5000 is $12.50.
7. $8000 is $30.00.
8. $12,000 is $50.40.
9. $18,500 is $74.00.
10. $20,000 is $66.00.

III. To Find the Assessed Valuation when the Amount of the Tax and the Tax Rate are Known.

Illustration:

Find the assessed valuation of a property upon which a tax of $18 is paid when the tax rate is 3 mills.

3 mills expressed as a percentage, .3 of 1% = .003.

Then, $18.00 is .003 of the assessed valuation.

$18.00 ÷ .003 = $6000.

And, $6000 is the assessed valuation. *Result.*

BLACKBOARD PRACTICE

Find the assessed valuation when the tax and the corresponding tax rate are, respectively:

1. $12.00 and 3 mills.
2. $15.00 and 3 mills.
3. $20.00 and 2 mills.
4. $18.75 and 2½ mills.
5. $22.50 and 5 mills.
6. $110.00 and 2.2 mills.
7. $125.00 and 2.5 mills.
8. $137.40 and 3 mills.
9. $162.50 and 3.8 mills.
10. $171.04 and 3.2 mills.

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IV. Calculating the Amount of a Tax by the Tax Table.

In order to reduce the labor of calculating a large number of tax bills, assessors often make use of a table constructed with the tax rate as a basis. By using such a table the labor is reduced to simple addition.

TAX TABLE—BASED UPON A TAX RATE OF 1.65 MILLS

VALUATION	TAX	VALUATION	TAX	VALUATION	TAX	VALUATION	TAX
$1	$.002	$10	$.017	$100	$ .165	$1000	$ 1.65
2	.003	20	.033	200	.330	2000	3.30
3	.005	30	.050	300	.495	3000	4.95
4	.007	40	.066	400	.660	4000	6.60
5	.009	50	.083	500	.825	5000	8.25
6	.010	60	.099	600	.990	6000	9.90
7	.012	70	.116	700	1.155	7000	11.55
8	.014	80	.132	800	1.320	8000	13.20
9	.015	90	.149	900	1.485	9000	14.85

Illustration:

Using the table and the rate 1.65, find the tax on a property assessed at $3750.

Tax on $3000 = $4.95 (3d line, last two columns.)
Tax on 700 = 1.155 (7th line, 5th and 6th columns.)
Tax on 50 = .083 (5th line, 3d and 4th columns.)
Adding, Tax on $3750 = $6.188

Therefore, the tax bill would be for $6.19. *Result.*

BLACKBOARD PRACTICE

Using the table and the rate 1.65 mills on the dollar, find the tax on property assessed at

1. $750.
2. $825.
3. $860.
4. $945.
5. $985.
6. $1200.
7. $1450.
8. $1890.
9. $2100.
10. $2400.
11. $3500.
12. $3900.
13. $4250.
14. $4600.
15. $4825.
16. $5600.
17. $6250.
18. $7500.
19. $8725.
20. $9450.

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V. The Method of Determining the Tax Rate which will Raise a Required Amount on a Given Assessed Valuation.

Illustration:

A town whose real estate is assessed at a total valuation of $2,950,000 needs to raise $32,450 for the coming year's expenses and to build a new road. Find rate of taxation.

$32,450 is what per cent of $2,950,000?

$$\$32{,}450 \div \$2{,}950{,}000 = .011, \text{ or } 1.1\%.$$

Hence, the tax rate must be 11 mills on $1.00, or $11 on $1000.

If a poll tax is collected in the town or if the town gets a share of the State tax, the conditions would be changed in the foregoing problem.

Suppose that this town has 1500 male voters, from each of whom a poll tax of $1 is collected; and suppose, furthermore, that the State turns over $12,500 of the State tax raised for the new road, a portion of which passes through this town. The calculation of the tax rate will be as follows:

Amount of tax to be raised		$32,450
Received from 1500 polls at $1 each	$ 1500	
Received from State	12,500	
Total amount received	$14,000	14,000
Total amount to be raised by tax		$18,450

Then $18,450 is what per cent of $2,950,000?

$$\$18{,}450 \div \$2{,}950{,}000 = .00625. \quad \textit{Result.}$$

Hence, the tax raised must be 6.25 mills on $1.

In such a case the officials would probably raise a tax of 7 mills, and the balance left in the town treasury after paying the year's running expenses and the improvements would be carried over to the next year.

WRITTEN APPLICATIONS

1. A town levied a tax of $3500 on an assessed valuation of $1,400,000. What was the tax rate?

2. A village needs $4500 for the expense of its school system for one year. If the assessed valuation is $1,350,000, what tax rate is needed to raise the $4500?

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3. A town requires \$16,000 for a year's expenses and its property valuation is \$2,700,000. If the State turns over \$4000, what tax rate is required to raise the balance?

4. In a town that levies a poll tax of \$1 each on 950 voters the officials decide to raise \$6750. With an assessed valuation of \$1,450,000 what tax rate will raise the balance required?

5. Mr. Johnson, a citizen in the town in example 4, has property valued at \$15,000. What is the amount of the tax he pays? What would he pay if no poll tax were demanded in the town?

6. The valuation of all the property in a town is \$2,157,500, the tax to be raised amounts to \$28,500, and a taxpayer, Mr. Raymond, owns property assessed at \$12,500. Find the tax rate and the amount of the tax Mr. Raymond pays.

7. If the town in example 6 collected a poll tax from each of its 1250 males, and if the funds received from the State amounted to \$7250, what would be the tax levied upon the property owned by Mr. Raymond?

8. Find from your own town officials or from printed reports the amount of the assessed valuation in your home town. Find also the amount of the poll tax, if that tax is required, and the amount of State funds received. Then calculate the tax on a property valued at \$10,000.

9. If a man owns property in a town but does not reside there, he pays the property tax but no poll tax. Mr. Swift owns property worth \$15,000 in a town in which he resides, and in which he pays a poll tax of \$2.00. He also owns property worth \$11,500 in a city where the assessed valuation is \$1,750,000, and where \$77,000 is raised to build a new school. In each town the tax rate is $3\frac{1}{2}$ mills. Find his total tax.

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10. Some states levy a tax on property inherited from estates, the rate of taxation depending upon the nearness of the relationship of the heirs to the person leaving the property. If the tax on a widow's share is 2%, and on a son's or daughter's share 1%, what amount does the state receive from an estate worth $300,000 if the widow receives $175,000, and the son $50,000, and if the balance is divided between two daughters?

MORTGAGES

A Legal Title to a property is the right of possession given by law.

Illustration:

A man pays $5000 for a house. He receives a written document called a *deed* from the former owner, giving him full possession of the property. This deed is recognized by law as a *title* to the property.

A Mortgage is an agreement through which the party loaning money is given a legal claim on a property as a security for the loan.

Illustration:

John Farnham had $2500, but wished to buy a house costing $4000. Robert Barber loaned Farnham $1500, and as security for the loan Farnham gave Barber a conditional conveyance, or transfer of his house, until the loan should be paid. This *conditional title* is called a mortgage.

Interest on Mortgages is usually paid semi-annually.

Failure to make an interest payment when due gives the holder of the mortgage the right to take the legal steps that will compel the sale of the property. In this way the holder of the mortgage collects the amount of money loaned and the interest due upon it.

Illustration:

If the interest on the mortgage given Barber is at the rate of 6%, Farnham must pay Barber annually 6% of $1500, or $90. If Farnham should fail to pay the interest, Barber might have the property sold by the sheriff, whereupon $1500 of the sale price, together with the interest due him, would be paid to Barber. The balance of the sum received by the sheriff, less all costs, would be paid over to Farnham.

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WRITTEN APPLICATIONS

1. A young man saved $2700 and bought a house for $4000. If he placed a mortgage on the property for the difference between his savings and the purchase price of the house, and if he paid annually 6% interest on the loan, how much was his annual interest payment?

2. A young man who was paying $360 annually for rent, bought a house for $3500, paying $3000 cash and borrowing the balance at 5%. If his own money had been earning 4% in bank, and if his expenses for taxes and repairs were $100 annually, how much did he gain each year by investing in the house?

BUILDING AND LOAN ASSOCIATIONS

A Building and Loan Association is an organization formed by a number of individuals for the purpose of saving. Such associations are incorporated under the laws of the state in which they do business, and are protected by the Banking laws of that state.

To become a member of a building and loan association a person agrees to save a certain number of dollars each month. $1.00 per month buys a share in the association, and this share is worth $200 at maturity.

The savings of the members of a building and loan association, added together each month, are invested by the association in loans made to the members of that association; these loans being secured by mortgages held by the association.

The profits of a building and loan association consist of the interest on the investment, and the reinvestment, of the savings of its members.

These profits are divided equally among the members, each member's share being in proportion to the amount of money he has saved in the association, and depending upon the time that he has been a member of it.

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When the savings of a member, with the interest that they have accumulated, amount to $200, for each share he holds, the stock is said to *mature*.

Under the Pennsylvania plan the time necessary for a share to mature is about 11½ years. As a rule, the payment of $1.00 per month for 138 months accumulates earnings of $62.

If a member has *free stock*, that is, if he is in the association merely for the purpose of saving, he gets $200 at the end of about 11½ years, for each $1.00 per month that he has invested. Similarly, a monthly saving of $5.00 for the same time would amount to $1000.

If a member is *a borrower*, that is, if he gives the association a mortgage on a property he owns or is buying, he pays $1.00 each month as *dues for each* $200 he borrows, and also $1.00 each month as interest on each $200, assuming the rate to be 6%. After he has continued these payments for about 11½ years, the loan is paid by his savings.

The advantages of the building and loan association as a means of saving are:

1. It affords almost absolute security for the investment of money in the locality in which the investor resides.
2. It affords the borrower the opportunity to buy a home at any period during the life of his stock.
3. It pays a higher rate of interest, consistent with safety, than any other form of investment.
4. It gives an investor an opportunity to borrow money in case of sickness or misfortune.
5. It provides for the return of the investor's savings, with his share of the profits of the association, at any time during the life of his stock.

Because of the wide variation in methods, it is difficult to give practical applications that will be helpful. You should be able to obtain from a near-by Building and Loan Association abundant material for good problems.

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STOCKS AND BONDS

A Corporation is a company, or group of persons, authorized by law to transact business of a stated kind.

A Stock Company is an incorporated company whose capital is represented by shares held by different persons.

The Organization of a Stock Company or Corporation is brought about in the following manner.

1. The persons forming the corporation agree to pay a certain amount for which they receive shares of stock.
2. The members, or stockholders, elect from their number a Board of Directors to have immediate charge of the business.
3. The Board of Directors elects officers of the corporation, including, as a rule, a President, a Secretary, and a Treasurer.
4. Application for a charter is made to a State Government.

The Capital Stock of a corporation is the amount represented by the total face value of all of the shares.

A Share of Stock is one of the equal parts into which the capital stock of a corporation is divided. Thus:

A capital of $250,000 might be divided into 2500 shares of $100 each, or into 5000 shares of $50 each.

A Stock Certificate is a statement issued by the corporation to the stockholders showing the number of shares owned by him, the face value of each, and how the stock may be transferred.

A Dividend is a sum paid to the shareholders of a company or corporation out of the earnings or the surplus of its business.

Preferred Shares are the shares upon which a corporation agrees to pay a certain rate of interest. Such interest may be paid annually, semi-annually, or quarterly, as the directors may decide.

Common Shares are the ordinary shares of a company or corporation, and carry no guarantee of dividends.

An Assessment is a demand for cash made upon the shareholders by a corporation when poor business or misfortune brings a loss instead of a profit.

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The Par Value of a share of stock is the face value at which the share is issued and originally sold by the corporation.

Thus: If a capital of $500,000 is issued in 500 shares, each share has a value of $100, and this is called the par value. If this stock had consisted of 10,000 shares, each share would have had a par value of $50.

The Market Value of a Share of stock is the amount which can be obtained from the public sale of the share at any given time.

Thus: If a share of stock originally issued at par value of $100 is sold during a period of prosperous business it may bring $110, and this value is the market value of the share. But during a period of poor business this same stock might be earning merely a fractional part of its earnings during prosperous times, and in the opinion of investors its market value might not be more than $75 per share. In general, the market price of a share is constantly changing.

BUYING AND SELLING STOCK

A Stock Broker is a person whose business is the buying and selling of stock.

A Stock Exchange is a business institution which provides a trading room and the necessary clerical system for the buying and selling of stock.

The Stock Exchange provides a natural channel through which brokers publicly offer to buy and to sell stock certificates. Its transactions are governed by fixed and rigid rules, and the prices established are recognized as fair expressions of the demand for, and the value of, a stock. For example, if a corporation is known to be doing a large and profitable business its shares are looked upon as an excellent investment. Consequently the demand for them naturally grows and the "bidding" by brokers results in increased prices, just as any increased demand for an article usually brings an advance in the price. On the other hand, a period of poor business and decreasing earnings makes a stock of less value to investors, and this condition is reflected by a smaller demand on the stock exchange, or by a general desire to sell, which inevitably lowers the market price.

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Stock Exchanges are located in all of the larger cities, but the New York Stock Exchange is the principal market for securities in the United States.

The Commission, or Brokerage, charged for buying or selling a stock is

$7\frac{1}{2}$	cents	per	share	on	stock	quoted	below	$10.
15	"	"	"	"	"	"	between	$10 and $125.
20	"	"	"	"	"	"	above	$125.

Stocks are quoted in dollars and fractional parts of a dollar based upon $\frac{1}{8}$ of a dollar. Thus, a stock rising from 75 to 76 may be quoted at each of the fractional parts of a dollar represented in eighths, and the successive quotations would be 75, $75\frac{1}{8}$, $75\frac{1}{4}$, $75\frac{3}{8}$, $75\frac{1}{2}$, $75\frac{5}{8}$, $75\frac{3}{4}$, $75\frac{7}{8}$, and 76.

To buy 100 shares of a stock quoted at $85, the purchaser must pay:

For the stock	$100 \times \$85$	= $8500
Plus the commission	$100 \times \$.15$	= 15
Total cost to the purchaser		= $8515.

If this stock is sold sometime later at $130 per share:

For the stock	$100 \times \$130$	= $13000
Less the commission	$100 \times \$.20$	= 20.
Total received by the seller		= $12980.

Stock Quotations. The stock exchanges are open on every business day of the year, and at the close of each session the prices at which stocks have been bought and sold are furnished to the newspapers for publication. The late editions of the afternoon papers publish them, and they are also published in the morning papers on the following day.

The only fractions used in stock quotations are eighths, quarters, and halves, as already illustrated. Large speculators buy shares in lots numbering from one hundred shares up to several thousands, and on many active days the total of the transactions reaches well above a million shares.

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Stocks usually bear names which indicate their corporate business, but brokers shorten the names for convenience. The phrase " United States Steel Common " is rarely heard in stock exchange circles, but the briefer form " Steel common," or " Steel," is constantly used.

Less important stocks bear full names in some instances, but all are frequently abbreviated. When a broker speaks of buying 100 shares of Erie Railroad Common Stock he speaks of "100 Erie."

I. To Find the Cost of a Number of Shares of Stock at a Given Market Value.

Illustration:

Find the cost of 100 Steel at $75\frac{1}{2}$.

The quotation $75\frac{1}{2}$ means \$75.50.
The commission on 100 shares = $100 \times \$.15 = 100 \times \$0.15 = \$15$.
The cost of 100 shares at \$75.50 = \$7550.
Hence, the total cost to the purchaser = \$7550 + \$15 = \$7565. *Result.*

BLACKBOARD PRACTICE

Adding the broker's commission, find the cost of:

1. 100 Erie at 45.
2. 100 Long Island at 24.
3. 200 Atchinson at 95.
4. 200 D. & H. at 150.
5. 200 Reading at $82\frac{1}{2}$.
6. 250 Am. Sugar at 118.
7. 100 Pennsylvania at $59\frac{1}{2}$.
8. 200 Woolworth at $112\frac{1}{4}$.
9. 300 Tenn. Copper at $57\frac{3}{4}$.
10. 250 Gen. Elec. at $175\frac{3}{8}$.
11. 400 Lehigh Valley at $82\frac{1}{8}$.
12. 300 N. Y. Central at $101\frac{1}{2}$.
13. 250 O. & W. at $29\frac{1}{4}$.
14. 300 Studebaker at $152\frac{3}{4}$.
15. 300 Beth. Steel at $345\frac{1}{4}$.
16. 350 C. F. & I. at $47\frac{7}{8}$.
17. 350 Elec. Storage at $60\frac{1}{8}$.
18. 400 Am. Tobacco at $211\frac{3}{4}$.
19. 450 Beet Sugar at $66\frac{3}{8}$.
20. 500 Westinghouse at $67\frac{7}{8}$.
21. 600 Ill. Central at $106\frac{1}{4}$.
22. 1000 Erie at $44\frac{5}{8}$.
23. 1000 Can. Pacific at $181\frac{3}{8}$.
24. 2000 Am. Loco. at $70\frac{7}{8}$.

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II. To Find the Amount Received from the Sale of a Number of Shares of Stock at a Given Market Value.

Illustration:

Find the amount received by the owner from the sale of 100 Erie at $45\frac{3}{4}$.

The quotation $45\frac{3}{4}$ means \$45.75.

The commission on 100 shares = $100 \times \$.15 = 100 \times \$.15 = \$15$.

The broker sells 100 shares and receives $100 \times \$45.75 = \$4575.00$.

Hence, the net amount received by the owner = $\$4575.00 - \$15 = \$4560$. *Result.*

BLACKBOARD PRACTICE

Deducting the broker's commission, find the net amount received by the owner from the sale of:

1. 100 Reading at 81.
2. 100 Pullman at 167.
3. 100 Seaboard at $18\frac{1}{2}$.
4. 100 Can. Pacific at 185.
5. 200 N. Y. O. & W. at $30\frac{7}{8}$.
6. 200 Pittsburg Coal at $35\frac{1}{4}$.
7. 150 Westinghouse at $68\frac{1}{2}$.
8. 200 C. F. & I. at $46\frac{1}{2}$.
9. 250 Long Island at $23\frac{5}{8}$.
10. 200 Studebaker at $150\frac{1}{2}$.
11. 300 Reading at $81\frac{3}{4}$.
12. 350 New Haven at $72\frac{1}{2}$.
13. 400 Union Pac. at $140\frac{1}{2}$.
14. 400 Texas Pac. at $15\frac{7}{8}$.
15. 500 Wabash at $13\frac{1}{4}$.
16. 450 Erie at $42\frac{1}{2}$.
17. 550 Utah Copper at 80.
18. 600 N. Y. C. at $101\frac{1}{4}$.
19. 700 Atchison at $106\frac{1}{2}$.
20. 800 Pennsylvania at $59\frac{3}{8}$.
21. 1000 C. F. & I. at $52\frac{1}{4}$.
22. 2000 Reading at $80\frac{1}{4}$.
23. 3000 Am. Sugar at $116\frac{1}{4}$.
24. 2500 D. & H. at $150\frac{3}{8}$.

The following list of stock quotations was published at the close of business on the New York Stock Exchange on August 5, 1915. (This list includes only a small part of the entire list in which trading is carried on. In an active business day purchases and sales are made in from one hundred twenty-five to one hundred fifty different stocks.)

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Stock Quotations — August 5, 1915

American Ice	24	New York Central	90⅜
Baldwin Locomotive	80¾	Pacific Mail	36¼
Brooklyn Rapid Transit	86¼	Pennsylvania	107⅛
Canadian Pacific	145½	Pullman	152
Central Leather	43	Reading	149⅞
Delaware & Hudson	150¼	Texas & Pacific	14¼
Erie	27½	Third Avenue	51¼
Lehigh Valley	143½	United States Steel	70⅜
Louisville & Nashville	112	Westinghouse	112½
Missouri Pacific	2¾	Woolworth	104½

BLACKBOARD PRACTICE

Referring to the quotations in the table, and including the broker's commission,

Find the cost of:

1. 10 Am. Ice.
2. 20 B. R. T.
3. 30 D. & H
4. 50 Erie.
5. 75 Woolworth.
6. 80 U. S. Steel.
7. 100 Can. Pac.
8. 100 Westinghouse.
9. 100 Leh. Val.
10. 100 Penn.
11. 100 Tex. & Pac
12. 150 Reading.
13. 200 Pullman.

Find the amount received for:

14. 20 Cent. Lea.
15. 40 N. Y. C.
16. 50 D. & H.
17. 75 L. & N.
18. 100 Mo. Pac.
19. 100 Woolworth.
20. 100 N. Y. C.
21. 200 Erie.
22. 300 Reading.
23. 500 N. Y. C.
24. 1000 U. S. Steel.
25. 1000 Baldwin.
26. 1500 Third Ave.

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III. Calculating Profit or Loss on Stock Transactions.

Stock purchased at a certain price and sold later when the market price is higher brings a profit, and, in like manner, stock sold at a point below the purchase price brings a loss. On the preceding page a brief list of stock quotations is given, and quotations for the same stock more than two months later are given below. The pupil can readily find the actual changes in market price of these several stocks during the interval between August 5 and November 10, 1915.

Stock Quotations — November 10, 1915

American Ice	25¾	New York Central	102¼
Baldwin Locomotive	120	Pacific Mail	33
Brooklyn Rapid Transit	89½	Pennsylvania	121⅜
Canadian Pacific	185	Pullman	160½
Central Leather	58⅜	Reading	82¾
Delaware & Hudson	141½	Texas & Pacific	16
Erie	42½	Third Avenue	62⅜
Lehigh Valley	80½	United States Steel	85½
Louisville & Nashville	120⅞	Westinghouse	68¾
Missouri Pacific	8¼	Woolworth	113

BLACKBOARD PRACTICE

Referring to the quotations given for August 5 and for November 10, and assuming that each lot of stock was purchased on the first date and sold on the second date, find the profit or loss, deducting the broker's commission.

1. 50 Penn.
2. 50 Steel.
3. 100 Reading.
4. 100 Ice.
5. 100 Penn.
6. 100 L. V.
7. 200 D. & H.
8. 100 Westinghouse.
9. 300 N. Y. C.
10. 300 B. R. T.
11. 500 Pullman.
12. 800 Pac. Mail.
13. 900 Steel.
14. 1000 Erie.
15. 1000 L. & N.
16. 1000 Baldwin.

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A Bond is a formal written promise by which a person or a corporation is bound to pay a certain sum at a specified time. Bonds are issued by corporations to provide money for improvements or other business needs, and they bear interest that must be met at stated times. They must be paid when due, and the laws provide that failure to pay interest when due gives the owner the right to sell the property of the corporation issuing them.

Corporate Bonds are bonds issued by a corporation.

Municipal Bonds are bonds issued by a city or a town.

Government Bonds are bonds issued by the United States, or by other countries.

Municipal and government bonds are not secured by mortgage, that is, by a provision that permits sale of property.

Registered Bonds are bonds that are recorded by number and in the name of the purchaser, and their ownership cannot be transferred from one person to another without registering the fact of transfer on the books of the corporation or the government which issued them.

Coupon Bonds are bonds that bear small interest certificates, each one of which is a promissory note to pay a certain sum at a stated time. Banks make a practice of cashing these coupons just as they cash checks, the bank in turn collecting the amount from the corporation issuing the bond.

Bonds are usually named in a brief form that indicates the corporation that issued them and the interest rate they pay.

Thus: United States Government Bonds bearing 4% interest are referred to as "U. S. 4s."

New York City Bonds bearing 4% interest are called "N. Y. City 4s."

The student will observe that a bond is usually a better investment than a stock. The bonds of a corporation have first claim upon the profits of the business, and moreover, the bonds are usually secured by a mortgage on the property of the corporation issuing them. Interest on a bond is promised at a fixed rate, while dividends on a stock depend upon business prosperity, good management, and such elements as may change at any time and without notice.

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IV. To Find the Number of Shares or Bonds a Given Sum Will Buy.

Illustration:

When U. S. Steel is selling at $72\frac{1}{8}$ how many shares can be bought with $4337?

Market price of one share = $72.125
Broker's commission = .15
Total cost of one share = $72.28
Then, $4337 ÷ $72.28 = 60, the number of shares. *Result.*

BLACKBOARD PRACTICE

Using the list of quotations given on page 207 find the number of shares that can be bought when the buyer invests:

1. $6050 in L. & N.
2. $57,175 in Am. Ice.
3. $6275 in Third Ave.
4. $17,125 in Steel.
5. $41,000 in N. Y. C.
6. $12,000 in Erie.
7. $12,000 in D. & H.
8. $14,500 in Reading.
9. $16,000 in Westinghouse.
10. $20,000 in Woolworth.

11. How many U. S. 4s selling at $104\frac{1}{8}$ can be purchased with $21,000?

12. How many Midvale Steel 5s selling at $78\frac{3}{8}$ can be purchased with $15,700?

13. How many U. S. Steel 4s selling at $105\frac{1}{8}$ can be purchased with the proceeds from 300 shares of Erie that were sold for $35\frac{5}{8}$ less commission?

V. To Find the Rate of Income to be Derived from Investments.

Dividends paid on stock and interest paid on bonds are paid at a certain rate per cent of the par value.

I. Stock bought below par pays a net income greater than the fixed dividend rate.

Suppose a stock is bought at $74\frac{1}{2}$, and pays 4% annually.
The cost plus brokerage is $74.50 + $.15 = $74.65.
The actual cash income per share is $4.00, annually.
The actual rate of income is $4.00 ÷ $74.65 = 5.35%. *Result.*

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II. Stock bought above par pays a net income less than the fixed dividend rate.

Suppose a stock is bought at \$110, and pays annually 4%.
The actual cost plus brokerage is \$110.00 + \$.15 = \$110.15.
The actual cash income per share is \$4.00, annually.
The actual rate of income is \$4.00 ÷ \$110.15 = 3.63%.

In the case of bonds which run for a definite length of time and, at maturity, are paid at par, the net annual income is affected by the difference between the par value and the purchase price.

Suppose a 5% bond cost \$103⅜, and had 5 years to run.
The cost plus brokerage was \$103⅜ + .15 = \$103.53.

Each year's interest return was	\$ 5.00
The total interest received in 5 years was	25.00
The amount received for the bond at maturity was	100.00
Therefore the purchaser received in all	125.00
His actual cash profit was \$125.00 − \$103.53, or	21.47
His actual annual cash profit on each bond was	4.30

Hence, his annual interest return was

\$4.30 ÷ \$103.53 = 4.15%.

These three cases illustrate the general problem of determining the rate of income derived annually from a given investment.

Tables showing the different rates of income derived from investments made above and below par values are obtainable at most brokers'.

WRITTEN APPLICATIONS

Find the rate of income on the investment when the dividend rate and the market price, respectively, are:

1. 6%, \$90.
2. 6%, \$120.
3. 6%, \$150.
4. 6%, \$125.
5. 6%, \$105.
6. 5%, \$90.
7. 4½%, \$90.
8. 5½%, \$110.
9. 5⅛%, \$125.
10. 6%, \$121.50.
11. 7%, \$140.
12. 7%, \$125.
13. 4%, \$84.50.
14. 5%, \$115.50.
15. 4½%, \$121.75.

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16. From which investment will the buyer receive the greater income, a 5% stock bought at \$140 a share, or a 4% stock bought at \$120?

17. Which investment will pay the greater return, a 5% stock bought at \$125 a share, or a 4% stock bought at \$100 a share?

18. 120 shares of stock paying 5% were sold at par of \$100, and the money was reinvested in a 4% stock selling at \$75. What was the annual gain by the change?

19. 200 shares of stock paying 4% were sold at \$10 below par of \$100. If the money was reinvested in a 5% stock selling at \$75, find the gain in annual income.

20. Find the total annual income from 150 shares of a 7% stock, bought at 90, and 200 shares of a 4% stock bought at 85. Which of the two investments pays the greater cash sum? Which of them pays the higher rate of interest?

21. 100 shares of U. S. Steel common, par value, \$100 a share, were bought at \$50 a share. If the stock paid 4%, find the actual return on the investment, and the per cent return on the money invested.

22. A man sold 500 shares of 4% stock that cost him \$85 a share, and reinvested the proceeds in a 5% stock that cost him \$100 per share. Find the amount of the commissions on both transactions. Find, also, the gain or loss in the rate per cent.

23. An investor has an opportunity to buy a 5% stock that is selling at a discount of 10%, or to buy a 6% stock in another company at a premium of \$25 per share. Which investment will pay the greater return? Omitting the brokerage charges, find what actual difference there will be in the return if the total amount invested is \$22,500.

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I. PROBLEMS IN THE HOME

I. Keeping House Accounts.

Many thrifty and systematic persons keep a careful record of the expenses incurred in keeping up the home. The cost of food, the cost of fuel, and the cost of supplies for the kitchen are among the most important of the home expenses; and the illustration shows a practical method for recording these particular expense items.

January, 1917

	Groceries	Meat	Milk	Ice	Laundry	Fuel	Gas and Elec.	Help	Totals
1	85	40	30						1 55
2	90	70	30						1 90
3	1 10	51	30						1 91
4	1 45	1 67	30		1 04			6 00	
5			45						
6	1 21		30				3 80		
7	97	80	30						
8	1 04	49	30		66				
9	78	90	30						
10	1 12	67	30			11 50			
11	2 09	1 35	30		1 27			6 00	
12			45						
13	64		30						

Books providing the columns necessary for accounts of this character may be obtained at nearly all bookstores, and the number of columns needed for each page will be determined by the number of items you wish to record.

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WRITTEN APPLICATIONS

1. Find the total amount spent for each of the different items during the week from January 1 to January 7 in the illustration.

2. Find the total expense for each of the seven days of that week, and the total amount for the week.

3. Check the result in the preceding example by referring to the totals found in example 1, and adding those totals.

4. Make a sheet ruled as in the illustration, and upon your sheet tabulate a series of expenditures of your own. Find the total expenses for each day in your record.

5. Check your own problem by taking the total of the expenses for each item and the total of these sums for the entire week. This total should equal that obtained by adding the daily totals from example 4.

II. Division of Income for Living Expenses and Savings.

Thrifty persons are careful to regulate their expenditures so that a portion of the family income is saved after necessary expenses have been provided for. Many suggestions have been published in books and in prominent journals, and in nearly all of them we find that a fixed sum should be devoted to such necessary expenses as rent, food, clothing, and insurance; that another sum should be provided for the expenses of physician, dentist, periodicals, and amusements; and also that another definite sum should be saved. This plan of devoting certain fixed portions or sums to definite items of expense is called *Arranging a Budget.*

Careful students of this plan have suggested the following table as a practicable plan for a family of four persons. At the extreme left the table gives the earnings or income of the family, and the other columns show the percentages of that income that may reasonably be spent on living expenses. It will be observed that the item of "rent" varies but little, investigators having determined that in families of small or moderate income the percentage for that item is practically fixed.

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ESTIMATED EXPENSE — FAMILY OF FOUR

ANNUAL INCOME	RENT	FOOD	CLOTHING	MISCELLANEOUS	SAVINGS
$1200	20%	35%	15%	25%	5%
1500	20%	35%	15%	25%	5%
1800	20%	30%	15%	25%	10%
2400	20%	30%	15%	20%	15%
3000	20%	25%	20%	15%	20%

From this table we may obtain with reasonable accuracy the amounts a family with a given income may safely spend on the different items of living. Such a table would fail in application to large incomes, for rent and food expense would not continue in the same ratio, and saving of greater percentages would be possible.

Using the Table.

Illustrations:

A man's annual salary is $1200. What amounts can he afford to spend upon rent, food, clothing, and miscellaneous expenses, and what amount should he save?

Referring to the table under salary of $1200,

For rent, 20% of $1200 = $240.
For food, 35% of $1200 = $420.
For clothing, 15% of $1200 = $180.
For miscellaneous expense, 25% of $1200 = $300.
Amount saved, 5% of $1200 = $60.

(The work is checked by adding the total expenses and the savings. $1200.)

In using the table for cases in which the salary does not exactly correspond to the given figures take the nearest lower salary. Thus, in working with a given salary of $2000 use the figures for the salary of $1800.

Tables which extend beyond that given are readily obtained, but the relative per cents change as the annual income increases.

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WRITTEN APPLICATIONS

Based upon the percentages in the table, find the amounts which may reasonably be spent on the different items when the family income is:

1. $1200.	5. $1300.	9. $2000.	13. $2700.
2. $1500.	6. $1400.	10. $2200.	14. $2800.
3. $1600.	7. $1600.	11. $2400.	15. $3000.
4. $1800.	8. $1700.	12. $2500.	16. $3500.

III. The Cost of Owning a House and the Cost of Renting One.

The relative cost of owning or renting a house is a common problem. As a great many considerations enter into both sides of such a question, it is impossible to give a general statement which will apply in all cases. However, we may illustrate the process by which a man may determine the expense of either plan under given conditions; and by following the steps in the illustration the student can readily make a reasonably accurate solution of a given problem.

Illustration:

A young man can buy a house for $4000, or rent it for $30 a month. The annual expense of owning it will include, if he pays for it in full, taxes, $30; insurance, $8; water, $18; and repairs, $25; total, $81.

For this illustration we will assume that the young man has $4000 in cash invested in a savings fund which pays him 4%.

Case 1. Suppose he pays cash for the property.

To own the house and "carry" it 1 year will cost:

Interest on his money invested in the house . . .	$160
Carrying expense	81
Total annual expense	$241

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Case 2. Suppose he pays $2500 cash, and gives a mortgage for $1500 at 6%.

To own the house and "carry" it 1 year will cost:

Interest on his own money invested in the house	$100
Interest on borrowed money	90
Carrying expense	81
Total annual expense	$271

Case 3. Suppose he pays $1000 cash, and gives a mortgage for $3000 at 6%.

To own the house and carry it 1 year will cost:

Interest on his own money invested in the house	$ 40
Interest on borrowed money	180
Carrying expense	81
Total annual expense	$301

If he rents the house the annual cost will be $12 \times \$30 = \$360.00$.

We may conclude, therefore, that

The saving if owned clear of debt is	$\$360 - \$241 = \$119$.
The saving if owned on a small mortgage is	$\$360 - \$271 = \$89$.
The saving if owned on a large mortgage is	$\$360 - \$230 = \$59$.

It is difficult, if not impossible, to reach exact conclusions in such comparisons, but it appears that owning a small property clear of debt or with a small mortgage is a means of considerable saving. The following problems should be discussed by the class, and much interest may be created if the pupils bring actual cases for illustration.

WRITTEN APPLICATIONS

1. Find the annual cost of owning a house valued at $5000, if money is worth 4% and if the cost of maintaining the house amounts to $110.

2. Find the annual cost of owning a house valued at $5000, and upon which there is a mortgage of $2000 at 5%; the owner's money being worth $3\frac{1}{2}$%, and the cost for taxes, insurance, and repairs being $90.

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3. A family rented a house for a year at $40 monthly, and then bought it for $5600. The money invested could have been loaned at 5%, and the cost of maintaining the house was $120. What amount was saved in one year by the change?

4. A man owns a home costing $5600, and has on it a mortgage of $2600. His annual carrying charges are $280. If his money now invested was in a bank at 5%, and if he rented the house for $40 a month, how much would he save in one year?

5. A family rented a city house for $50 monthly, and finally moved into a new house in a suburb on which they paid $2500 cash and placed a $3500 mortgage at 5%. The annual taxes were $80; the water rent, $24; and for four years no repairs were needed. Railroad fares for the family averaged $100 annually for the four years they occupied the house. What amount did they save annually by the change?

IV. The Cost of Furnishing a House.

The cost of furnishing a house may be divided between two main items, 1st, the expense of putting the house or apartment into good condition; and 2d, the expense of the furniture and general furnishings.

WRITTEN APPLICATIONS

1. A young man rented a small house, agreeing to pay for all necessary repairs, and also to pay $20 per month rent. How much rent did he pay annually?

2. He painted the walls of the kitchen at a cost of 20¢ per square yard. The room being 12 feet long and 12 feet wide and 9 feet high, how much did the painting cost, no allowance being made for openings?

3. He papered four rooms, each 15 feet long, 12 feet wide, and 9 feet high, using paper that cost $0.25 per single roll. How much did he pay for the paper, if 18 strips were deducted for the openings?

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4. He finished the floor of each of the five rooms at a cost of $0.25 per square yard, and kalsomined the ceilings of all five rooms at a cost of $0.10 per square yard. What was the total cost of finishing the floors and kalsomining the ceilings?

5. The furnishings of the kitchen were: gas stove, $18.75; table, $2.25; tinware and utensils, $21.19; linoleum, 9 sq. yd. at $1.25 per yard; and shades for 3 windows at 69¢ each. What was the total cost of the kitchen furnishings?

6. The furnishings for the dining room were a dining table costing $28; 5 chairs at $5.75 each; 1 buffet at $42.50; 1 serving table at $21; glassware at $9.25; 4 tablecloths at $4.25 each; 3 dozen napkins at $2.75 per dozen; 2 shades at $0.90 each; and a rug at $18.75. Find the total cost of these furnishings.

7. Two bedrooms were furnished at exactly the same cost, one in birch and the other in maple. Each had a chiffonnier at $28; a bureau at $30; a brass bed at $21, with mattress at $13, and spring at $7; 2 chairs at $3.25 each; 2 window shades at 69¢ each; and a rug at $16.75. Find the total cost of the furnishings for both bedrooms.

8. The living room was furnished with a center table at $25; 3 chairs at $7.50 each; 1 chair at $11.25; a couch at $22.50; a rug at $21.50; 3 window shades at $.90 each, and 3 pairs of curtains at $1.75 each. Find the total cost of furnishing the living room.

9. Make out a complete statement showing the total amount spent on repairs for each room; the total spent on repairs for all the rooms; also the total spent in the furnishing of each room, and a total showing the whole amount spent for the furnishing. Give also the total amount spent on both repairs and furnishing. How much did the young man pay during the first year for rent and for repairs and furniture?

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V. The Cost of Maintaining a Home.

(a) Cost of Gas Lighting.

Measuring Gas. A gas meter records on three dials the number of cubic feet of gas which passes into the pipes of a house during a given period of time. Most gas companies read the meters once each month, and the difference between two consecutive readings is the amount of gas consumed during the month.

Reading a Gas Meter. In reading a gas meter observe that

1. Each division of the dial at the right represents 100 cubic feet.
One complete revolution of the hand on this dial represents 1000 cubic feet.

2. Each division on the middle dial represents 1000 cubic feet.
One complete revolution of the hand on this dial represents 10,000 cubic feet.

3. Each division on the dial at the left represents 10,000 cubic feet.
One complete revolution of the hand on this dial represents 100,000 cubic feet.

To read the meter in the illustration, therefore, we note that

The hand between 1 and 2 on the left dial means "10,000 and more."
The hand between 3 and 4 on the middle dial means "3000 and more."
The hand exactly on 5 on the right dial means "500 exactly."
Combining the three readings we have $10,000+3000+500=13,500$, the reading.

BLACKBOARD PRACTICE

In each of the following examples let the student draw upon the board three dials to represent the dials of a gas meter, and let him indicate the positions of the hands for the following readings.

1. 5600 cu. ft.	**5.** 1000 cu. ft.	**9.** 21,300 cu. ft.
2. 8500 cu. ft.	**6.** 11,900 cu. ft.	**10.** 22,500 cu. ft.
3. 9200 cu. ft.	**7.** 12,500 cu. ft.	**11.** 27,600 cu. ft.
4. 9700 cu. ft.	**8.** 18,700 cu. ft.	**12.** 30,600 cu. ft.

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13. On July 1 a gas meter registered 12,400 cubic feet, and on the 1st of August 14,500 cubic feet. At \$1.10 per thousand cubic feet, what was the cost of the gas consumed between these dates?

14. A gas company presents bills at the rate of \$1.10 per thousand cubic feet, and then makes a reduction of \$0.10 per thousand if the bill is paid within a certain number of days. How much would this reduction take from the monthly gas bill in the preceding problem?

15. The six consecutive monthly readings of a gas meter were 23,100; 26,800; 29,400; 31,600; 32,500; and 33,800 cubic feet respectively. Find the amount of each one of the five bills when the cost of gas was \$0.80 per thousand cubic feet, and also find the total amount of the five bills.

16. Find local rate and solve problems 14 and 15.

(*b*) **Cost of Electric Lighting.**

Electric current for lighting purposes is sold by the thousand watts, the "watt" being the unit of electrical measurement for electric power.

Electric meters are installed on the premises of each consumer, and they are read in the same way as the gas meter.

Electric bulbs are made in a great variety of sizes, the amount of current consumed and the amount of light given depending upon the size. The bulbs are classed according to the amount of current they consume in one hour. Thus, the "50-watt light" consumes 50 watts of current in 1 hour.

The common sizes of lights in domestic use are the bulbs that use 10, 15, 25, 45, and 60 watts of current, respectively.

The Amount of Current Used depends upon the size of the bulb and the length of time it is used.

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Illustration:

How much current is consumed if a room is lighted with 2 45-watt lights and 4 15-watt lights for five hours; and what is the cost of lighting at 10 cents per thousand watts?

In 5 hr. 2 bulbs consuming 45 watts will use $5 \times 2 \times 45 = 450$ watts.
In 5 hr. 4 bulbs consuming 15 watts will use $5 \times 4 \times 15 = 300$ watts.
Therefore, the six lights consume in all 750 watts.
Result.

The cost of electric current is usually based upon a stated charge for each 1000 watts used.

In this case: 750 watts $= \frac{750}{1000}$ or .75 of 1000 watts.
Hence, at 10¢ per 1000 watts,
The cost of 750 watts $= .75 \times \$.10 = \$.075$, *Result.*

It is common practice to call 1000 watts a "kilo-watt," the prefix "kilo" being derived from the Greek for "one thousand."

BLACKBOARD PRACTICE

Find the number of watts consumed by:

1. 1 25-watt lamp in 3 hours.
2. 2 15-watt lamps in 4 hours.
3. 3 25-watt lamps in 6 hours.
4. 5 45-watt lamps in 5 hours.
5. 2 60-watt, 3 25-watt, and 5 10-watt lamps in 4 hours.
6. 5 25-watt, 8 15-watt, and 12 10-watt lamps in 3 hours.
7. If the lights in problem 3 are used in a grocery store, what is the cost of lighting the store on each of the 24 evenings in a month, at \$.10 per kilowatt-hour?
8. The lights in problem 6 are used in a school gymnasium three afternoons a week for ten weeks, at an average of 3 hours each afternoon. What is the cost of lighting this gymnasium at \$.10 per kilowatt-hour?
9. A family averages during the month of January the following light consumption In the kitchen, 1 45-watt lamp for 3 hours; in the dining room, 4 25-watt lamps for 1 hour; in the living room, 1 60-watt lamp and 2 25-watt lamps for 3 hours; and in each of three bedrooms, 2 15-watt lamps for one and one half hours each. At \$.10 per kilowatt-hour, how much should the light bill for this family amount to in that month?

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(*c*) **Kitchen Measures and Costs.** Many housekeepers calculate from time to time the daily, monthly, or yearly expense of some article of food. Such calculations enable them to estimate in advance the amount which may be spent on a particular thing, and the "budget plan" is often worked out in this way.

WRITTEN APPLICATIONS

1. A family uses daily 2 quarts of milk at 12 cents a quart, and $\frac{1}{4}$ pint of cream at 72¢ a quart. How much does this family spend for both in the month of April?

2. How many pounds of butter will a family use in 90 days, 3 meals to each day, if they average a pound for every eight meals?

3. A farmer's family uses, on the average, $2\frac{1}{2}$ pounds of potatoes at each of two meals daily from September 1 to June 30 following. How many bushels of potatoes should the farmer raise for his own use?

4. A cookbook directs that a turkey be roasted 20 minutes for each pound of its weight. At what hour should a turkey weighing $9\frac{1}{2}$ pounds be placed in the oven to be ready for dinner at half-past six o'clock?

5. A family of four persons uses two pounds of butter a week for 1 year. If the average price of butter during that time is 45¢, find the total amount spent for butter by this family during the year.

6. A family used a gas stove and a gas water-heater for six months at an average expense of $3.25 per month. They then changed to a coal stove and in six months used 4 tons at $8 a ton. Which was the more economical?

7. A housekeeper preserved 60 cans of strawberries, using for each can 1 quart of berries and 1 pound of sugar. The berries cost $12\frac{1}{2}$¢ a quart, the sugar $8\frac{1}{2}$¢ a pound, the cans 75¢ a dozen, and the gas burner consumed 8 cu. ft. of gas per hour for 5 hours at a cost of 90¢ per thousand cubic feet. Find the total cost of the fruit when preserved and the cost per can.

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(*d*) **Economy in Buying.** Shrewd housekeepers are careful to purchase such staple articles as flour, sugar, soap, etc., in quantities, and many stores make a practice of offering reductions in such goods on "bargain days." The saving appears small at first, but if the practice is carried on over any great length of time, it is possible to save a large amount.

WRITTEN APPLICATIONS

1. A large store offers 4 pounds of the regular 35-cent cocoa for $1.10. If a family uses regularly 12 pounds of cocoa in a year, how much would be saved by purchasing a year's supply at this bargain rate?

2. At a grocery sale 100 cakes of laundry soap are offered for $3.25, the regular price of the soap being 5 cents per cake. How much would be saved by purchasing 100 cakes at this reduced rate?

3. Butter can be purchased in 20-pound tubs at 42 cents a pound, and print butter at 45 cents. If a family uses 3 pounds of butter a week, how much would be saved in one year by buying the tub butter?

4. A grocer sold potatoes at $1.50 a bushel, or at 40 cents a peck. How much could be saved by buying 3 bushels at the first price instead of purchasing them a peck at a time?

5. A dry goods store offers a bolt of cloth measuring 45 yards for $7\frac{1}{2}¢$ a yard if the whole bolt is purchased. How much would a woman save by buying it all instead of purchasing it in small pieces at $8\frac{1}{4}¢$ a yard?

6. A man bought 3 tons of coal in April, paying $6.50 a ton, 4 tons in September at $7 a ton, and 5 tons in December at $7.75 a ton. How much would he have saved if he had bought it all in April?

7. A barrel of flour could have been purchased for $15, but a housewife bought it in $\frac{1}{8}$-barrel sacks at $1.90 per sack whenever she needed it. How much would she have saved by buying the whole barrel in the beginning?

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8. A housewife purchased 100 bars of soap at $3.15, 2 dozen cans of corn at $2.95, and 6 pounds of coffee at $.33 a pound. The regular price of the soap was 5¢ a bar; of the corn, 16¢ a can; and of the coffee, 38¢ a pound. Find the total amount she saved by buying in quantity.

(*e*) **Menus and Their Costs.** Authorities on domestic science have suggested various menus which provide all of the necessary food values, and yet provide them at a minimum of expense.

WRITTEN APPLICATION

1. What is the cost of soup for each of four persons if a can of celery costs $.10 and a pint of milk costs 8 cents?

2. A breakfast for four persons consists of grapes costing $.06, oatmeal costing $.06, griddle cakes costing $.20, sirup costing $.10, butter costing $.18, and coffee costing $.08 and cream and sugar costing $.12. Find the cost per person.

3. Which of the following breakfasts is the cheaper for a family of four:

No. 1: Oatmeal, 5¢; 8 eggs at 30¢ a dozen; ½ lb. bacon at 36¢; pan of rolls at 5¢; coffee and sugar at 6¢; butter at 12¢; cream at 8¢.

No. 2: Bananas and cream at 15¢; buttered toast at 10¢; baked beans at 20¢; and brown bread at 10¢; butter at 12¢.

4. A woman served a luncheon for eight persons at the following cost:

2 chickens	$1.87	Biscuit	.30
Celery	.25	Butter	.15
Salad:		Cocoa	.20
Lettuce	.08	Olives	.35
Eggs	.20	Ice cream	.70
Oil	.05	Cake	.50

Find the total cost of the luncheon and the cost per person.

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5. A woman served cocoa and cake to twelve friends, using 2 quarts of milk at 8¢; 1 cup of sugar at 6¢ per pound; and 12 tablespoonfuls of cocoa at 35¢ per pound. If 1 cup of sugar weighs ½ pound, and if 1 tablespoonful of cocoa weighs 1 ounce, find the cost of the cocoa; and if the cake cost 50¢, find the cost of the luncheon per person.

6. A housewife served dinner to her family of six persons with the following menu: steak, $0.65; potatoes, $0.10; string beans, $0.15; bread, $0.05; butter, $0.12; coffee, $0.06; and dessert consisting of milk, $0.12; eggs, $0.18; butter, $0.12; lemons, $0.10; and sugar, 5¢. What was the total cost of the dinner?

7. A school kitchen served a luncheon to sixty children, charging them 15¢ each. The menu consisted of tomato soup, bread, and butter, and tapioca cream.

The soup cost:

15 cans soup stock at 18¢	$2.70
30 tablespoonfuls butter (15 oz. at 48¢ per pound)	.45
8 quarts milk at 8¢	.64
Total cost	$3.79

The tapioca cost:

2 packages tapioca at 10¢	$.20
8 quarts milk at 8¢	.64
1½ dozen eggs at 32¢	.48
5 cupfuls sugar at 6¢ per pound (1 cupful = ½ pound)	.15
Vanilla	.05
Total cost	$1.52

The bread and butter cost:

10 loaves bread at 5¢	$0.50
1½ lb. butter at 56¢	.84
Total cost	$1.34

What was the amount of profit made by the school?

8. From a newspaper advertisement, or from a local grocer, obtain a list of prices of various foods. Plan the breakfast, luncheon, and dinner menus for a family of six persons, and estimate the cost of all three meals.

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II. PROBLEMS IN MANUAL TRAINING

Estimating Lumber Bills from Detail Drawings. We have learned under Practical Measurements that a detail drawing shows the dimensions of the material used in making an article. In the manual training room we have constant need for calculating the amount of material needed for making a thing, and this list of stock required is called the Bill of Lumber.

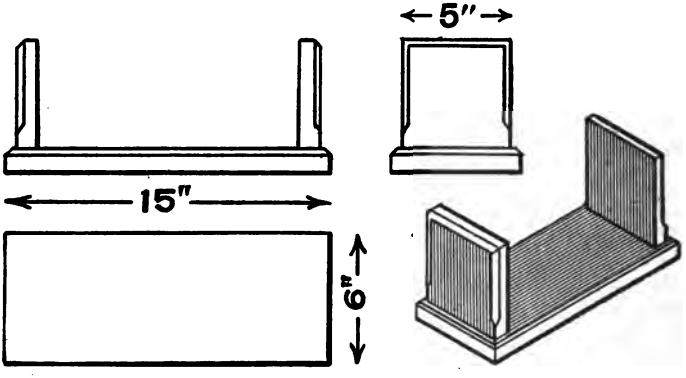

Illustration:

To estimate the bill of lumber needed for the book rack shown in the drawing, and to give, in addition, the list of material for "assembling" or putting the parts together, and the material required for "finishing."

LUMBER BILL

1 piece $\frac{7}{8}''\times6''\times15''$ (Bottom)
2 pieces $\frac{7}{8}''\times5''\times5''$ (Ends)

To allow for squaring and finishing the rack will require one board $\frac{7}{8}''\times5\frac{1}{2}''\times27''$.

Other material: 4 wood screws $1\frac{1}{2}''$ long. Stain. Wax.

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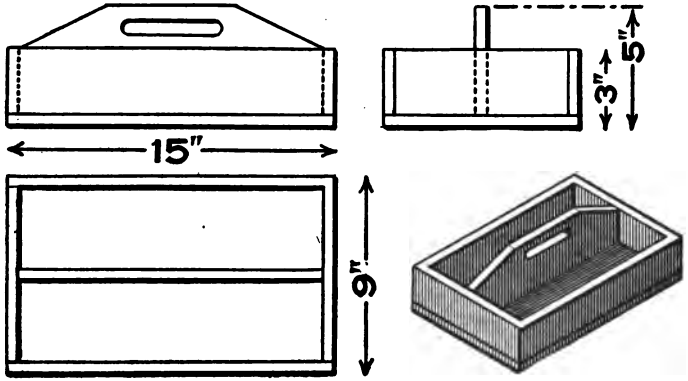
WRITTEN APPLICATIONS

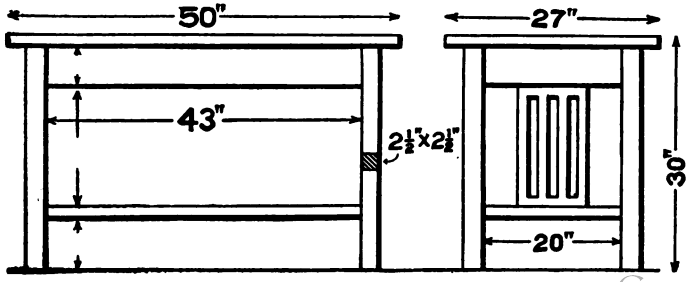
1. Find bill of lumber and estimate all material necessary for the knife and fork tray in the detail drawing.

1 piece $\frac{1}{2}''\times9''\times15''$ (Bottom) 2 pieces $\frac{1}{2}''\times3''\times15''$ (Sides)
1 piece $\frac{1}{2}''\times5''\times14''$ (Partition) 2 pieces $\frac{1}{2}''\times3''\times8''$ (Ends)

Estimate the lumber needed, as well as the nails, the stain, and the wax for finishing, and estimate the total cost of the tray at your local prices.



2. The drawings show some of the details of a table. Plan the other details and make an estimate of the lumber bill based on your local prices.



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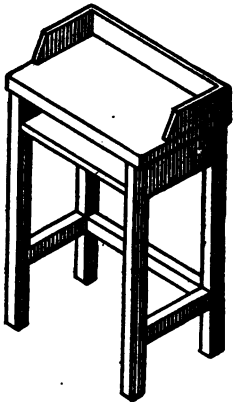
3. The estimate of a lumber bill for a chair must provide for the pieces given in the list below. Calculate the cost of the lumber if the chair is made from white oak costing $90 per thousand.

LUMBER BILL FOR CHAIR

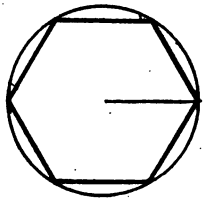
Front Legs.	2 pieces.	$2'' \times 2'' \times 1'\ 6''$
Back Legs.	2 pieces.	$2'' \times 2'' \times 2'\ 6''$
Side Braces.	6 pieces.	$\frac{3}{4}'' \times 2'' \times 14''$
Front Braces.	3 pieces.	$\frac{3}{4}'' \times 2'' \times 15''$
Rear Braces.	2 pieces.	$\frac{3}{4}'' \times 1\frac{1}{2}'' \times 15''$
Rear Brace.	1 piece.	$\frac{3}{4}'' \times 2\frac{1}{2}'' \times 15''$
Arms.	2 pieces.	$\frac{3}{4}'' \times 4'' \times 16\frac{1}{2}''$



4. Plan the proportions and make up the lumber bill for the telephone stand shown in the drawing. Allow 10% for waste by adding this allowance to the size of the finished pieces.

The Construction of Shapes Used in Manual Training Designs. Many of the shapes used in the designing of furniture may be readily laid out or drawn on wood by simple applications of geometry.

(1) **The Hexagon.** One of the common shapes in use is the hexagon, a figure having six sides. To lay out a hexagon which shall have six equal sides of a desired length we proceed as follows:



1. Draw a circle with a radius equal to the side of the desired hexagon.
2. Beginning at a point on the circle, lay off the radius six times as in the figure.
3. Join the six points.

(2) **The Octagon.** If we know the diameter of the required octagon, or figure of eight sides, the octagon may be constructed as follows:

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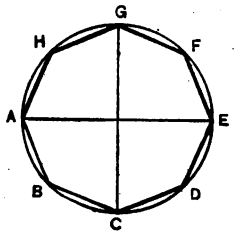


1. Draw a circle whose diameter is equal to the length of the diameter of the required octagon.

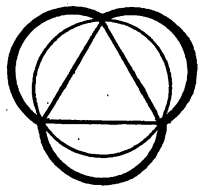
2. Draw two diameters making right angles to each other (*AE* and *GC*).

3. Find the middle point of the arc *AC* (*B*); the middle point of the arc *CE* (*D*); the middle point of the arc *EG* (*F*); and the middle point of the arc *GA* (*H*).

4. Join the eight points of division on the circle, and the figure *ABCDEFGH* is the required octagon.

WRITTEN APPLICATIONS

1. Construct a hexagon the length of whose sides shall be 10 inches.



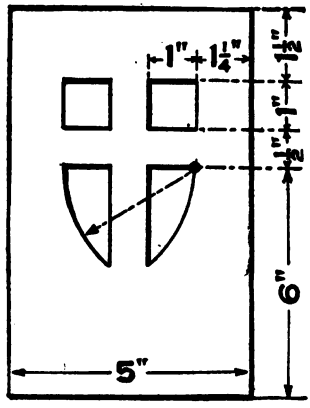
2. Construct a hexagon in a circle whose radius is 8 inches.

3. Construct a hexagon in a circle whose diameter is 12 inches.

4. Construct a triangle, all of whose sides shall be equal, in a circle whose diameter is 10″. (Hint: Study the figure obtained in problem 1.)

5. Construct a triangle, all of whose sides shall be equal, in a circle whose radius is 6 inches.

6. The ornamental figure at the right is often used in furniture design. Observe the method of construction and then make a similar figure by starting with a circle whose diameter is 5 inches.



7. Study the method by which the design at the right is laid out, and construct a similar design for one of the legs of a taboret. Assume the width of the legs to be 8 inches, and allow a margin of 1 inch on each side of the design.

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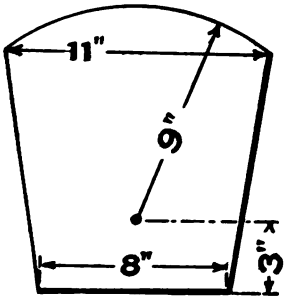
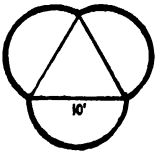


8. The figure at the left shows the dimensions of one of the four sides of a waste-basket. If the bottom of the basket is to be square and if two of the sides are to be set in between the other two, find the dimensions of the two smaller sides. Calculate the lumber bill for the basket, using half-inch stock, and estimate the cost at $75 per thousand for the lumber.

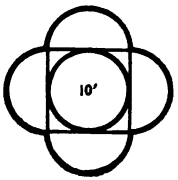
(Study the figure at the left in order to determine the shapes needed.)

9. Calculate the lumber bill necessary for a taboret whose top is hexagonal shape 12 inches in diameter, and whose three legs are made of inch stock and 15 inches high. For the 3 braces use inch stock 2 inches wide.

Review. Find the total area of each of the following.



1.



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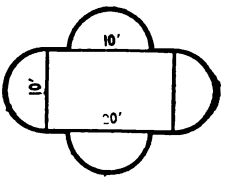
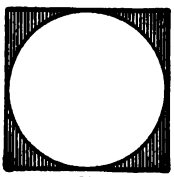


3.

Find the area of the shaded portion of each of the following figures:



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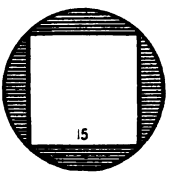

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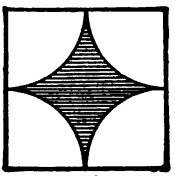

6.

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ADDITIONAL TOPICS FOR STUDY AND REFERENCE

MEASUREMENT OF SOLIDS

A Solid has three dimensions, length, breadth, and thickness.

A Polygon is a plane figure bounded by straight lines.

Polygons are named according to the number of sides.

The triangle is a polygon having three sides.

The square is a polygon having four equal sides and four right angles.

The rectangle is a polygon having right angles and its opposite sides equal.

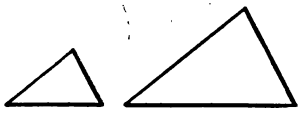
These three polygons, the triangle, the square, and the rectangle, are special and exceedingly common forms of polygons.

A pentagon is a polygon having five sides.

A hexagon is a polygon having six sides.

An octagon is a polygon having eight sides.

Similar Polygons are polygons that have exactly the same shape.

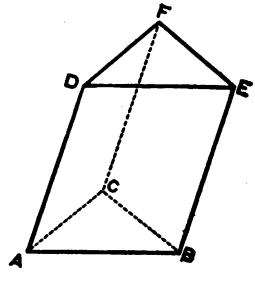


Similar polygons are not necessarily equal in size.

The two triangles in the figure are similar for the shape is the same in both.

A Prism is a solid whose ends are equal and similar polygons in parallel planes, and whose faces are parallelograms.

The solid at the right has two equal and similar polygons *ABC* and *DEF* for bases, these bases being parallel; and the faces *ADFC*, *CFEB*, and *BEDA* are parallelograms. Prisms are named from their bases.



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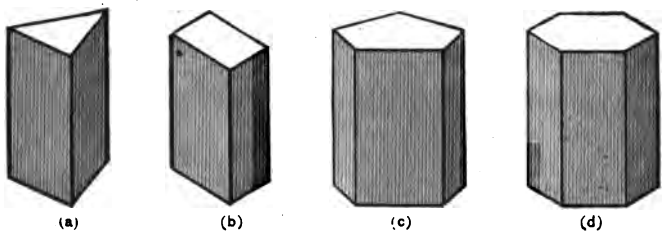


Types of Prisms

(*a*) is a triangular prism.
(*b*) is a rectangular prism.
(*c*) is a pentagonal prism.
(*d*) is a hexagonal prism.

The Altitude of a Prism is the perpendicular distance between the bases.

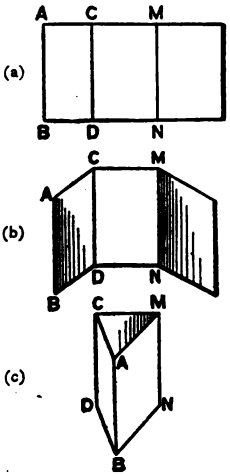
The Lateral Area of a Prism is the area of the surface formed by the parallelograms.

The lateral area of a prism is easily illustrated in the figures at the right.

In (*a*) a rectangular area is divided into three parts by the lines CD and MN.

In (*b*) the original area is partly folded upon the lines CD and MN.

In (*c*) the folding is completed and the extreme edges of the original area are brought together on the edge AB.



Thus the parallelograms have inclosed a solid, and the **lateral area** of this solid is the sum of the areas of these parallelograms.

The Total Area of a Prism is the sum of the lateral area and the areas of the two bases.

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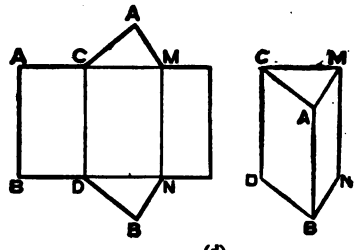
If two triangles ACM and DBN, in which two sides equal respectively the ends of the outside rectangles, are folded into the solid as in (*d*), the total area of the solid will be the sum of these triangles and the lateral area.



(*d*)

BLACKBOARD PRACTICE

Find the lateral area of a rectangular prism when the perimeter of the base and the altitude are, respectively:

1. 5 in., 7 in.
2. 10 in., 12 in.
3. 12 in., 15 in.
4. 16 in., 20 in.
5. 18 in., 21 in.
6. 2 ft. 9 in., 3 ft.
7. 3 ft. 6 in., 4 ft.
8. 3 ft. 8 in., 5 ft.
9. 5 ft. 2 in., 7 ft.
10. 8 ft. 5 in., 9 ft.
11. 12 ft. 6 in., 6 ft.
12. 15 ft. 4 in., 6 ft.
13. 18 ft. 8 in., 4 ft. 6 in.
14. 18 ft. 10 in., 5 ft. 8 in.
15. 21 ft. 11 in., 9 ft. 3 in.

Find the lateral area of a square prism, when one base edge and the altitude are, respectively:

16. 7 in., 10 in.
17. 9 in., 15 in.
18. 12 in., 18 in.
19. 15 in., 20 in.
20. 1 ft. 9 in., 20 in.
21. 2 ft., 2 ft. 5 in.
22. 3 ft., 4 ft. 6 in.
23. 5 ft. 8 in., 8 ft.
24. 4 ft., 15 ft. 6 in.
25. 6 ft., 18 ft. 10 in.
26. 7 ft. 6 in., 21 ft.
27. 8 ft. 4 in., 35 ft.

Find the total area of a rectangular prism, when the dimensions of the base and the altitude are, respectively:

28. 9 in. by 10 in., 3 ft.
29. 10 in. by 12 in., 4 ft.
30. 1 ft. by 1 ft. 3 in., 3 ft.
31. 3 ft. by 2 ft. 4 in., 4 ft.
32. 4 ft. 6 in. by 2 ft., 7 ft.
33. 4 ft. 10 in. by 8 in., 6 ft.
34. 5 ft. by 3 ft. 2 in., 4 ft.
35. 2 ft. 3 in. by 3 ft. 1 in., 4 ft. 6 in.
36. 3 ft. 8 in. by 2 ft. 7 in., 5 ft. 9 in.
37. 4 ft. 7 in. by 3 ft. 1 in., 4 ft. 4 in.
38. 6 ft. 6 in. by 4 ft. 4 in., 9 ft. 6 in.
39. 6 ft. 9 in. by 5 ft. 1 in., 10 ft. 4 in.
40. 8 ft. 3 in. by 4 ft. 8 in., 11 ft. 9 in.
41. 9 ft. 8 in. by 3 ft. 11 in., 10 ft. 3 in.

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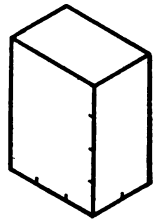
The Volume of a Prism is the measure of the prism in cubic units. We found that the volume of a rectangular solid equals the continued product of its length, breadth, and thickness, all expressed in the same units of measure. In the figure the base is three units long and two units wide, and the solid is four units high. Therefore, its volume is made up of as many cubic units as there are in the product of $3 \times 2 \times 4$, or 24 cubic units.



In general, therefore,

The volume of a prism is equal to the product of its altitude and the area of its base.

BLACKBOARD PRACTICE

Find the volume of a prism whose base area, and altitude, respectively, are:

1. 10 sq. in., 5 in.	**5.** 18 sq. in., 25 in.	**9.** 10 sq. ft., 16 ft.
2. 15 sq. in., 7 in.	**6.** 24 sq. in., 4 ft.	**10.** 12 sq. ft., 18 ft.
3. 16 sq. in., 9 in.	**7.** 27 sq. in., 6 ft.	**11.** 15 sq. ft., 20 ft.
4. 20 sq. in., 12 in.	**8.** 3 sq. ft., 21 in.	**12.** 18 sq. ft., 24 ft.

Find the volume of a square prism, the side of the base and the altitude being, respectively:

13. 10 in., 12 in.	**17.** 2 ft., 5 ft.	**21.** 3 ft. 6 in., 7 ft.
14. 15 in., 18 in.	**18.** 3 ft., 42 in.	**22.** 5 ft. 8 in., 12 ft.
15. 16 in., 24 in.	**19.** 3 ft., 45 in.	**23.** 6 ft. 4 in., 15 ft.
16. 18 in., 25 in.	**20.** 4 ft., 6 ft.	**24.** 8 ft. 10 in., 20 ft.

Find the volume of a triangular prism whose base area, and altitude, respectively, are:

25. 24 sq. in., 15 in.	**29.** 10 sq. ft., 12 ft.	**33.** 6 sq. ft., 3 ft. 6 in.
26. 30 sq. in., 18 in.	**30.** 10 sq. ft., 15 ft.	**34.** 8 sq. ft., 2 ft. 9 in.
27. 40 sq. in., 20 in.	**31.** 12 sq. ft., 12 ft.	**35.** 8 sq. ft., 3 ft. 8 in.
28. 42 sq. in., 35 in.	**32.** 15 sq. ft., 10 ft.	**36.** 9 sq. ft., 5 ft. 3 in.

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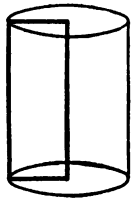
A Circular Cylinder is a solid bounded by a uniformly curved surface, and whose ends are equal circles in parallel planes.

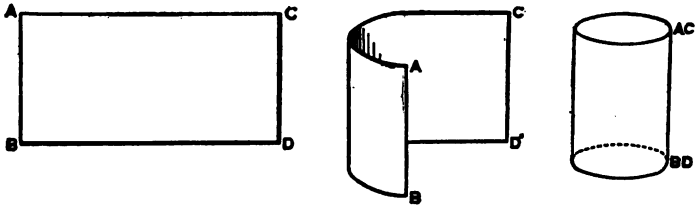
A circular cylinder is formed when a rectangle is revolved about one of its sides. The figure at the right illustrates the formation of a circular cylinder by revolution of a rectangle.



The Altitude of a Cylinder is the perpendicular distance between the bases.

The Lateral Area of a Cylinder is the area of the curved surface of the cylinder.

The lateral area of a cylinder is illustrated in the three figures below.

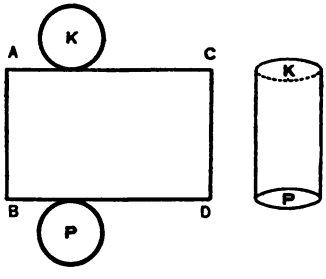


If the rectangle $ABDC$ is rolled up until the edge AB falls upon the edge CD and the lines AC and BD of the given rectangle form equal circles, the cylinder at the right results. In general,

The lateral area of a cylinder is equal to the product of the circumference of the base by the altitude.

The Total Area of a Cylinder is the sum of the lateral area and the area of the two bases.

If two circles P and K, whose circumferences are equal respectively to AB and CD, are folded into the cylinder, the total area of the solid is made up of the lateral area plus the combined area of the two circles.



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BLACKBOARD PRACTICE

Find the lateral area of a circular cylinder the circumference of the base and the altitude being, respectively:

1. 15 in., 10 in.
2. 18 in., 24 in.
3. 20 in., 30 in.
4. 24 in., 35 in.
5. 2 ft. 3 in., 10 in.
6. 3 ft. 9 in., 12 in.
7. 4 ft. 6 in., 18 in.
8. 5 ft. 8 in., 24 in.
9. 3.1416 ft., 4 ft.
10. 6.2832 ft., 6 ft.
11. 5.1112 ft., 5 ft.
12. 9.4248 ft., 6 ft.

Find the total area of a circular cylinder the radius of the base and the altitude being, respectively:

13. 6 in., 15 in.
14. 8 in., 18 in.
15. 10 in., 25 in.
16. 12 in., 30 in.
17. 1 ft. 3 in., 5 ft.
18. 2 ft. 6 in., 6 ft.

The Volume of a Circular Cylinder is obtained through the same principle that we applied in finding the volume of a prism. A cylinder may be considered as a prism with a great number of sides, these sides being so narrow that they form the unbroken curved surface of the cylinder. Considering, therefore, that the cylinder is a prism with a very great number of sides, we may find its volume in the same way that we would find the volume of a prism. That is,

The volume of a circular cylinder is equal to the product of its altitude and the area of its base.

BLACKBOARD PRACTICE

Find the volume of a circular cylinder, if the area of the base and the altitude are, respectively:

1. 10 sq. in., 15 in.
2. 12 sq. in., 18 in.
3. 15 sq. in., 20 in.
4. 16 sq. in., 20 in.
5. 18 sq. in., 24 in.
6. 2 sq. ft., 3 ft.
7. 2 sq. ft., 5 ft.
8. 3 sq. ft., 4 ft.
9. 4 sq. ft., 5 ft.
10. 5 sq. ft., 6 ft.
11. 1.5 sq. ft., 3 ft. 6 in.
12. 2.4 sq. ft., 4 ft. 3 in.
13. 3.1 sq. ft., 5 ft. 8 in.
14. 4.5 sq. ft., 4 ft. 6 in.
15. 5.6 sq. ft., 8 ft. 10 in.

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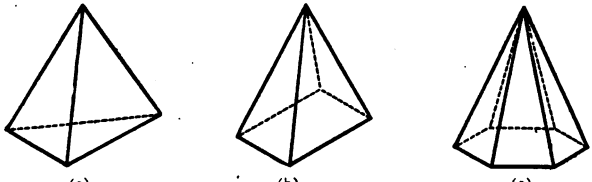
Find the volume of a circular cylinder, if the radius of the base and the altitude are, respectively:

16. 4 in., 10 in.	**21.** 2 ft., 3 ft. 6 in.	**26.** 1 ft. 3 in., 3 ft. 6 in.
17. 3 in., 15 in.	**22.** 1 ft., 3 ft. 8 in.	**27.** 1 ft. 6 in., 3 ft. 8 in.
18. 5 in., 12 in.	**23.** 2 ft., 4 ft. 3 in.	**28.** 1 ft. 9 in., 4 ft. 6 in.
19. 6 in., 15 in.	**24.** 1 ft. 3 in., 2 ft.	**29.** 2 ft. 8 in., 5 ft. 2 in.
20. 9 in., 18 in.	**25.** 1 ft. 6 in., 5 ft.	**30.** 3 ft. 6 in., 6 ft. 10 in.

A Pyramid is a solid whose base is a regular polygon and whose faces are triangles that meet in a point.



Pyramids are named from their bases.

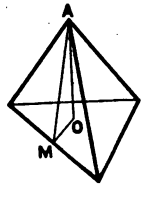
(*a*) is a triangular pyramid. (*b*) is a rectangular pyramid.

(*c*) is a hexagonal pyramid.

The Vertex of a Pyramid is the meeting-point of the triangular faces.

The Altitude of a Pyramid is the perpendicular distance from the vertex to the base.

The Slant Height of a Pyramid is the altitude of any one of the equal triangles that form the faces of the pyramid.



In the figure at the right
The vertex is A.
The altitude is AO.
The slant height is AM.

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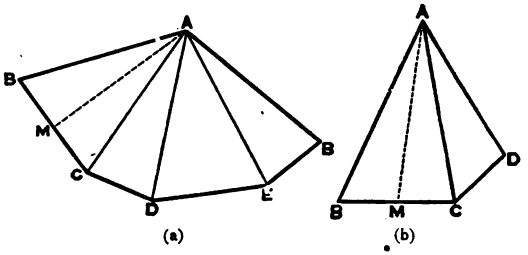
The Lateral Area of a Pyramid is the combined areas of the triangles that form the faces of the pyramid.

If the triangles in (*a*) at the right are folded on AC, AD, and AE, so that AB falls on AB, the lateral area of a solid, or pyramid, is formed as shown in (*b*). The area of any one of the four equal faces, as ABC, is the area of the triangle ABC, in which the base is BC and the altitude AM. That is, area of triangle $ABC = \frac{1}{2}(BC \times AM)$. Therefore, the area of all four triangles equals one half the product of the sum of all the bases by the slant height. Or, in general,



The lateral area of a pyramid equals one half the product of the slant height by the perimeter of the base.

The Total Area of a Pyramid is the sum of the lateral area and the area of the base.

BLACKBOARD PRACTICE

Find the lateral area of a pyramid if the perimeter of the base and the slant height are, respectively:

1. 15 in., 10 in.	**6.** 2 ft., 16 in.	**11.** 2 ft. 6 in., 3 ft. 6 in.
2. 16 in., 12 in.	**7.** 3 ft., 24 in.	**12.** 3 ft. 8 in., 1 ft. 4 in.
3. 18 in., 15 in.	**8.** 4 ft., 30 in.	**13.** 4 ft. 4 in., 2 ft. 10 in.
4. 20 in., 17 in.	**9.** 30 in., 5 ft.	**14.** 4 ft. 6 in., 5 ft. 8 in.
5. 24 in., 18 in.	**10.** 36 in., 6 ft.	**15.** 5 ft. 3 in., 6 ft. 8 in.

Find the lateral area of a square pyramid when the length of one base edge and the slant height are, respectively:

16. 8 in., 15 in.	**18.** 9 in., 20 in.	**20.** 12 in., 36 in.
17. 9 in., 16 in.	**19.** 10 in., 24 in.	**21.** 2 ft., 5 ft.

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22. 2 ft., 27 in.	**25.** 18 in., 4 ft.	**28.** 2 ft. 6 in., 3 ft. 8 in.
23. 13 in., 2 ft.	**26.** 1 ft. 6 in., 2 ft. 3 in.	**29.** 3 ft. 7 in., 4 ft. 11 in.
24. 14 in., 5 ft.	**27.** 1 ft. 8 in., 2 ft. 9 in.	**30.** 4 ft. 5 in., 5 ft. 3 in.

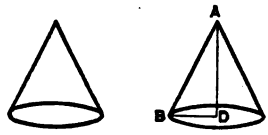
A Cone is a solid whose base is a circle and whose curved surface tapers uniformly to a point.

The Vertex of a Cone is the point to which the curved surface tapers.

The Altitude of a Cone is the perpendicular distance from the vertex to the base of the cone.

The Slant Height of a Cone is the distance from the vertex to any point on the circumference of the base.

In the figure at the right
The base is the circle whose center is D.
The vertex is the point A.
The altitude is the distance AD.
The slant height is the distance AB.
The radius of the base is the distance DB.



The Lateral Surface of a Cone is the curved surface between the vertex and the circumference of the base.

The lateral surface of a cone may be considered as made up of a great number of triangles, the altitude of each triangle being the slant height of the cone, and the sum of the bases of all the triangles being the circumference of the base. Therefore, the lateral area of a cone may be found by use of the principle through which we found the lateral surface of a pyramid. That is,

The lateral area of a cone is equal to one half the product of the slant height by the circumference of the base.

The Total Area of a Cone is the sum of the lateral area and the area of the base.

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BLACKBOARD PRACTICE

Find the lateral area of a cone if the circumference of the base and the slant height are, respectively:

1. 8 in., 6 in.	6. 2 ft., 10 in.	11. 2 ft. 4 in., 2 ft. 6 in.
2. 8 in., 7 in.	7. 2 ft., 14 in.	12. 2 ft. 9 in., 3 ft. 4 in.
3. 9 in., 5 in.	8. 2 ft., 19 in.	13. 3 ft. 6 in., 4 ft. 8 in.
4. 9 in., 6 in.	9. 18 in., 2 ft.	14. 3 ft. 11 in., 3 ft. 5 in.
5. 10 in., 8 in.	10. 19 in., 3 ft.	15. 4 ft. 10 in., 5 ft. 11 in.

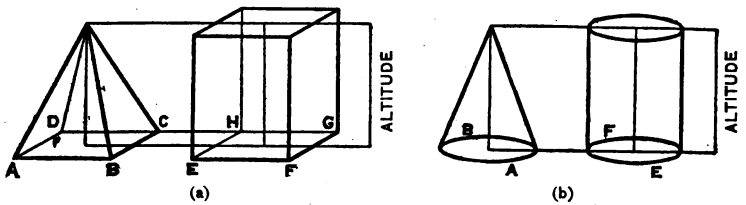
Find the lateral area of a cone if the radius of the base and the slant height, respectively, are:

16. 1 in., 6 in.	21. 1 ft., 21 in.	26. 1 ft. 2 in., 2 ft. 1 in.
17. 2 in., 6 in.	22. 1 ft., 25 in.	27. 1 ft. 7 in., 3 ft. 4 in.
18. 2 in., 9 in.	23. 11 in., 3 ft.	28. 2 ft. 1 in., 4 ft. 6 in.
19. 3 in., 15 in.	24. 15 in., 2 ft.	29. 3 ft. 5 in., 5 ft. 5 in.
20. 3 in., 19 in.	25. 18 in., 5 ft.	30. 3 ft. 6 in., 6 ft. 8 in.

The Volume of a Pyramid or Cone.

(1) A fixed relation exists between the volume of a prism and the volume of a pyramid having the same base and altitude. And,

(2) A similar relation exists between the volume of a cylinder and the volume of a cone having the same base and altitude.



In (*a*) the base of the prism equals the base of the pyramid, and the altitude of the prism equals the altitude of the pyramid.

In (*b*) the base of the cylinder equals the base of the cone, and the altitude of the cylinder equals the altitude of the cone.

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Then, by geometry:

The volume of the pyramid equals one third the volume of the prism.

And,

The volume of the cone equals one third the volume of the cylinder.

A simple experiment will serve to establish these truths in a general way, and will *illustrate* the principle without proving it. If a hollow prism has the same base as a hollow pyramid, and if each has the same altitude, the amount of sand that just fills the prism will fill the pyramid just *three times*. That is, the volume of the pyramid is one third the volume of the prism. The same practical experiment will illustrate the fact that if a cylinder and a cone have the same base and altitude, the amount of sand that exactly fills the cylinder will fill the cone *three times*.

BLACKBOARD PRACTICE

Find the volume of a square pyramid if the edge of the base and the altitude are, respectively:

1. 6 in., 15 in.
2. 9 in., 20 in.
3. 9 in., 28 in.
4. 1 ft., 2 ft.
5. 2 ft., 8 ft.
6. 30 in., 9 ft.
7. 1 ft. 3 in., 6 ft.
8. 1 ft. 6 in., 9 ft. 3 in.
9. 1 ft. 10 in., 14 ft. 6 in.

Find the volume of a cone if the diameter of the base and the altitude are, respectively:

10. 12 in., 15 in.
11. 16 in., 20 in.
12. 18 in., 30 in.
13. 2 ft., 30 in.
14. 3 ft., 39 in.
15. 27 in., 4 ft.
16. 1 ft. 3 in., 4 ft. 6 in.
17. 1 ft. 6 in., 5 ft. 8 in.
18. 1 ft. 4 in., 5 ft. 9 in.
19. 1 ft. 2 in., 5 ft. 6 in.
20. 2 ft. 3 in., 6 ft. 1 in.
21. 2 ft. 4 in., 4 ft. 6 in.

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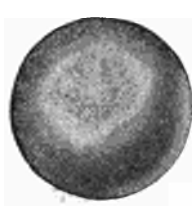
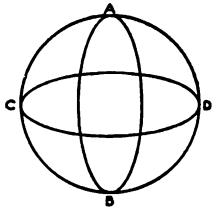


(a)



(b)

(c)

A Sphere is a solid or volume bounded by a surface every point of which is equally distant from a point within called the center.

(*a*) illustrates a sphere.

A Diameter of a Sphere is a straight line passing through the center of the sphere and terminating in its surface.

If in (*b*) *A* and *B* are joined by a straight line, *AB* is a diameter. Similarly, *CD* would be a diameter.

A Radius of a Sphere is the distance from the center to any point on the surface of the sphere.

All diameters pass through the center of the sphere. Any straight line from their intersection to the surface of the sphere is a radius of the sphere.

A Great Circle of a Sphere is a circle whose plane passes through the center of the sphere, and whose radius equals the radius of the sphere.

In (*b*) the circles through *A*, *B*, *C*, and *D* are great circles.

The Circumference of a Sphere is the circumference of a great circle of the sphere.

The circumference of a sphere is the greatest distance around it.

A Hemisphere is one of the two solids into which a sphere is divided by a great circle.

(*c*) illustrates two hemispheres.

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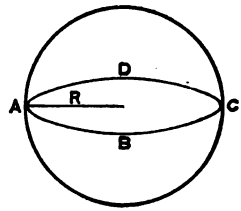
The Surface of a Sphere. In later mathematics it is proved that the surface of a sphere is equal to **the area of four great circles** of that sphere. That is,



If $ABCD$ is a great circle of the sphere whose radius is R, the surface of the sphere equals 4 times the area of $ABCD$, or $4\pi R^2$.

A simple experiment will illustrate the principle without proving it. If a waxed cord just long enough to cover the surface of the hemisphere is coiled on the surface of the great circle of the hemisphere, it will be found that the cord covers the great circle just twice. Hence, the area of the surface of the hemisphere is equal to the area of **two great circles** of that hemisphere; and it follows, therefore, that the area of the whole surface of the sphere is equal to the area of **four great circles** of the sphere.

BLACKBOARD PRACTICE

Using 3.1416– for π, find the surface of the sphere whose

Radius is:		Diameter is:	
1. 2 in.	**7.** 1 ft.	**13.** 10 in.	**19.** 1 ft. 2 in.
2. 3 in.	**8.** 1.1 ft.	**14.** 14 in.	**20.** 1 ft. 3 in.
3. 4 in.	**9.** 1.5 ft.	**15.** 15 in.	**21.** 1 ft. 4 in.
4. 5 in.	**10.** 1.6 ft.	**16.** 18 in.	**22.** 1 ft. 7 in.
5. 6 in.	**11.** 1.8 ft.	**17.** 20 in.	**23.** 1 ft. 9 in.
6. 8 in.	**12.** 1.25 ft.	**18.** 23 in.	**24.** 1 ft. 10 in.

25. How many square feet are there in the surface of a hemispherical dome whose diameter is 24 feet?

26. The diameter of the planet Jupiter is 11 times the diameter of the earth. If the earth's diameter is taken as 8000 miles, find the number of square miles in the earth's surface and the number in Jupiter's surface.

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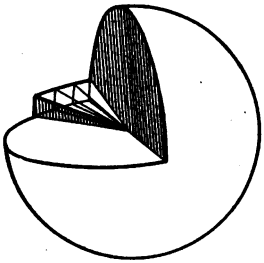
The Volume of a Sphere. In later mathematics it is proved that the volume of a sphere is equal to **one third the product of its radius by its surface.**

The principle may be illustrated by the accompanying figure. It is evident that a sphere may be considered as made up of a great number of pyramids, each pyramid having an altitude equal to the radius of the sphere, and the total area of the bases of the pyramids making up the total area of the sphere. Therefore, the volume of a sphere is made up of the total volume of all the pyramids, and we may find the volume of the sphere by applying the rule for the volume of a pyramid.



The volume of a pyramid equals one third the product of the base and altitude.

The altitude of each pyramid is the radius of the sphere.

The sum of the bases of the pyramids is the surface of the sphere. Therefore,

$$\text{Volume of a Sphere} = \tfrac{1}{3} \times R \times 4\pi R^2 = \tfrac{4}{3}\pi R^3.$$

BLACKBOARD PRACTICE

Find the volume of a sphere whose

Radius is:		Diameter is:	
1. 1 in.	6. 1 ft.	11. 5 in.	16. 1 ft. 3 in.
2. 2 in.	7. 1.2 ft.	12. 8 in.	17. 1 ft. 4 in.
3. 3 in.	8. 1.5 ft.	13. 9 in.	18. 1 ft. 8 in.
4. 4 in.	9. 1.8 ft.	14. 11 in.	19. 2 ft. 3 in.
5. 5 in.	10. 1.25 ft.	15. 13 in.	20. 2 ft. 6 in.

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PROMISSORY NOTES

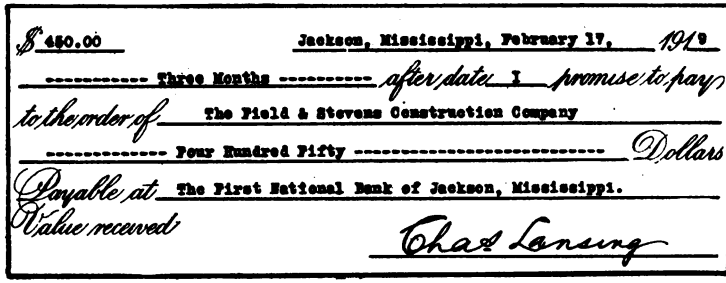
A Promissory Note is a written promise to pay a sum of money on demand or at a specified time.

A promissory note is usually called a **Note**.

The Maker is the person who signs the note.

The Payee is the person to whom the note is payable.

The Face of the note is the sum whose payment is promised.


$450.00 Jackson, Mississippi, February 17, 1919

Three Months after date I promise to pay

to the order of The Field & Stevens Construction Company

Four Hundred Fifty Dollars

Payable at The First National Bank of Jackson, Mississippi.

Value received Chas Lansing

In the note illustrated

The **maker** is Chas. Lansing.

The **payee** is The Field & Stevens Construction Company.

The **face** of the note is $450.

A note may be made payable in three ways.

1st: **On demand**, when payment may be asked for at any time.

2d: **At a specified time after date**, at which time the note is due.

3d: **At a specified date**, upon which date it is due.

The Day of Maturity is the date at which a note specifying time is due.

The day of maturity in the above note is the date, March 17, 1919, that is, the date at the end of the time specified in the note.

A note bears interest in accordance with its conditions.

(1) If written, "with interest," and with the rate indicated, a note bears interest at that rate for the time reckoned from the date of the note to the day of maturity.

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(2) If written, "with interest," but without indicating a rate, a note bears interest from the date of the note to the day of maturity with interest at the legal rate in the state in which it is payable.

(3) If no mention of interest is written in the note, no interest can be collected for the time between the date of the note and the day of maturity; but if the note is not paid at maturity, it begins to draw interest from that date at the legal rate.

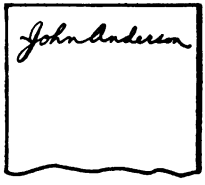
A Negotiable Note is a note made payable to the payee, or order.

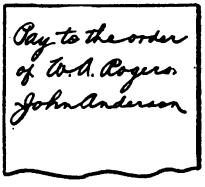
By Indorsement a negotiable note may be made payable to another person.

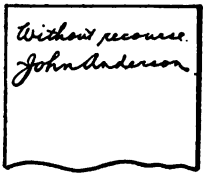
Illustration:

John Nicholson gives his note to John Anderson.

The illustrations show three common forms of indorsement.







The payee, John Anderson, can make the note payable **to any other person** who may subsequently hold it by writing his name on the back as illustrated at the left.

This is called **Indorsement in Blank**, and it indicates that Mr. Anderson has actually sold the note and indicates by his **indorsement** that he has received value for it.

If the payee should sell the note to W. A. Rogers, and should prefer to guard against the possible loss and the presentation by some other person, he indorses it with the words, "Pay to the order of W. A. Rogers," and then signs his own name. This is called an **Indorsement in Full**.

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If Mr. Anderson wishes to release himself from any future responsibility for the payment of the note, he may write the words, " without recourse," over his signature.

This is called a **Qualified Indorsement.**

A Non-negotiable Note is a note written without the words, " or order," and is payable only to the payee.

WRITTEN APPLICATIONS

Supplying names for the maker and the payee, write negotiable notes for each of the following conditions:

1. Face $200; date April 17, 1919; time 4 mo.; rate 6%.

2. Face $500; date January 10, 1919; time 6 mo.; rate 6%.

3. Face $750; dated Dec. 20, 1918; time 9 mo.; rate 5%.

4. Face $1000; date May 9, 1919; time 60 da.; rate 6%.

5. Face $2500; date October 10, 1919; time 90 da.; rate 5½%.

6. Face $4000; date July 5, 1918; time 30 da.; rate 5%.

7. Indorse three of the above notes in blank, and the other three in full.

8. Write a non-negotiable note, without interest, with yourself as payee and Frederick Thompson as maker, the face to be $150, the date January 10, 1919, and the time 60 days.

9. Write a demand note for one hundred dollars payable to yourself, the maker being Franklin Spencer, and the note to bear interest at 5%.

10. Write a 90-day note for two hundred dollars, payable to Edward Rutledge and made by yourself. Put three different indorsements on it.

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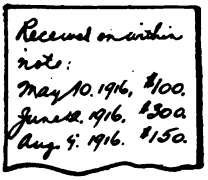
PARTIAL PAYMENTS

The maker of a note is often able to pay a part of the face before the whole is due.

Payments made upon a note during the time it runs are called **Partial Payments.**

When partial payments are made the amount paid is written upon the back of the note, together with the date of the payment. An indorsement of this kind is a legal receipt. The illustration at the right shows the manner of indorsing three different partial payments made on a 9-months note for $1000.



The United States Rule

The United States Rule is the most widely used rule for partial payments.

It has been decided by the Supreme Court of the United States that in the settlement of a note on which one or more partial payments have been made, no interest may be allowed on a payment, or on interest due on the note. Hence, the United States Rule is based upon the following principles:

(1) *Any payment must be applied to the paying of interest due.*

(2) *If any payment exceeds the interest due, the balance of that payment must reduce the principal.*

(3) *Interest must not be charged upon interest.*

Illustration:

A note for one thousand dollars is dated January 10, 1919, and bears interest at 6%. The following payments are indorsed: March 1, 1919, $200; May 10, 1919, $300; July 12, 1919, $200. Settlement is made on August 5, 1919. Find the balance due on that date.

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Principal, January 10, 1919	$1000.00	
Interest to date of 1st payment January 10, 1919 to March 1, 1919: 1 mo. 21 da.	8.50	
Amount due	$1008.50	
First payment	200.00	
New Principal, March 1, 1919	$808.50	
Interest to date of 2d payment March 1, 1919 to May 10, 1919: 2 mo. 10 da.	9.43	
Amount due	$817.93	
Second payment	300.00	
New Principal, May 10, 1919	$517.93	
Interest to date of 3d payment May 10, 1919 to July 12, 1919: 2 mo. 2 da.	5.35	
Amount due	$523.28	
Third payment	200.00	
New Principal, July 12, 1919	$323.28	
Interest to date of settlement July 12, 1919 to August 5, 1919: 24 da.	1.30	
Amount due at settlement	$324.58	*Result.*

The process in the illustration follows the

Rule: *Find the amount of the principal to the date of the first payment.*

Subtract the first payment from this amount.

With the remainder as a **New Principal** *repeat the process, finding the amount of the new principal to the date of the second payment.*

Subtract the second payment and proceed as before.

If, at any time, a payment does not equal or exceed the interest due, find the interest to the time when two or more payments equal or exceed the interest.

The use of the method when interest due exceeds a payment is illustrated in the following example. It will be observed that a payment of $50 is made at a date when the interest due exceeds that payment. Note what is done in this case.

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A promissory note is dated June 1, 1917, and is to run 2 years. The face of the note is $2000, the rate 6%, and the indorsements are: January 16, 1918, $100; July 16, 1918, $50; January 1, 1919, $500. What was due at maturity?

Principal, June 1, 1917	$2000.00	
Interest to date of 1st payment		
June 1, 1917 to January 16, 1918: 7 mo. 15 da.	75.00	
Amount due	$2075.00	
First payment	100.00	
New Principal, January 16, 1918	$1975.00	
Interest to date of 2d payment		
January 16, 1918 to July 16, 1918: 6 mo.	59.25	
Second payment, less than interest due, $50.00		
Interest on $1975.00		
July 16, 1918 to January 1, 1919: 5 mo. 15 da.	54.31	
Amount due	$2088.56	
Third payment, $500 + second payment, $50	550.00	
New Principal, January 1, 1919	$1538.56	
Interest to date of settlement		
January 1, 1919 to June 1, 1919: 6 mo.	46.16	
Amount due at settlement	$1584.72	*Result.*

The student will observe and remember that

The United States Rule applies partial payments *first to pay interest due;* and second, if any part of the payment is left, *to reduce the principal.*

WRITTEN APPLICATIONS

1. A note for $500 is dated Jan. 1, 1916, and bears interest at 6%. An indorsement of $200 was made March 15, 1916. What amount was due at settlement, Sept. 1, 1916?

2. A note for $250.60 is dated July 7, 1917, and bears interest at 7%. Indorsements are as follows: September 20, 1917, $80; January 1, 1918, $50; March 13, 1918, $50. What amount was due at settlement, April 15, 1918?

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3. A note for $800 is dated June 4, 1917, and bears interest at 6%. Indorsements are: May 1, 1918, $144; October 1, 1918, $90; January 1, 1919, $400; and February 4, 1919, $100. What amount was due January 1, 1920?

4. A note for $6255 is dated October 1, 1917, and bears interest at 6%. The indorsements are: January 1, 1918, $2000; November 1, 1918, $200; and January 1, 1919, $3000. What amount was due when the settlement is made on May 1, 1919?

5. A note for $750 bears interest at 6%, is dated April 7, 1916, and has the following indorsements: January 17, 1917, $200; March 13, 1918, $25; February 19, 1919, $30; August 3, 1919, $200; January 1, 1920, $150. What amount was due on the note at settlement on August 14, 1920?

6. On a note for $1000 bearing interest at 6% there are made four payments of $100, $200, $300, and $400, on the dates, April 1, July 1, October 1, and January 1, 1914, respectively. The date of the note was January 1, 1913. What amount was due at settlement on January 1, 1918?

The Mercantile Rule

The **Mercantile Rule** is used by bankers and by business men on *short-time* notes, that is, notes that are to run only a few months.

The rule is based upon two principles:

(1) *The face of the note shall draw interest for the entire time that the note runs.*

(2) *Each payment shall draw interest from its date to the date of settlement.*

By this rule, therefore, the maker of the note is credited with interest on any partial payment for the time he advances it on the note.

The amount due at settlement is the difference between the amount of the face at that date and the total amounts of payments and interest thereon during the time the note runs.

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Illustration:

A note for $1000 is dated January 10, 1919, and bears interest at 6%. The following payments are indorsed: March 1, 1919, $200; May 10, 1919, $300; July 12, 1919, $200. Find the balance due on August 5, 1919.

Jan. 10 to Aug. 5: 6 mo., 25 da.	Amt. of $1000, for 6 mo., 25 da. =	$1034.17
Mch. 1 to Aug. 5: 5 mo., 4 da.	Amt. of $200, for 5 mo., 4 da. =	$205.07
May 10 to Aug. 5: 2 mo., 25 da.	Amt. of $300, for 2 mo., 25 da. =	304.25
July 12 to Aug. 5: 24 da.	Amt. of $200, for 24 da. =	200.80
	Total amount of payments =	$710.12

$1034.17 − $710.12 = $324.05, Balance due at settlement, Result.

The process in the illustration follows this

Rule: *Find the amount of the principal at the date of settlement.*

Find the amount of each payment from its date to the date of settlement.

From the amount of the principal subtract the sum of the amounts of the payments.

WRITTEN APPLICATIONS

1. A note bearing interest at 6% is dated February 4, 1916. The face is $700, and the indorsements are: December 18, 1916, $164; June 24, 1917, $200; September 11, 1917, $120; July 5, 1918, $60. Find the amount due at settlement November 28, 1918.

2. A note for $172.80 is dated January 1, 1918, and bears interest at 6%. The indorsements are March 1, $30; May 16, $15; September 1, $27; and December 11, $13.50, all in the same year, 1918. What was due on December 16 of that year?

3. A note for $500 bears interest at 6% and is dated July 8, 1917. The indorsements are: September 23, 1917, $60; November 20, 1917, $100; January 17, 1918, $200; and February 8, 1918, $50. What was due at settlement, May 11, 1918?

4. A note for $1000, dated March 1, 1913, bears interest at 7%. The indorsements are March 1, 1914, $100; September 25, 1915, $200; October 9, 1916, $150; July 4, 1917, $20; October 9, 1918, $300. What amount was due at settlement on December 1, 1918?

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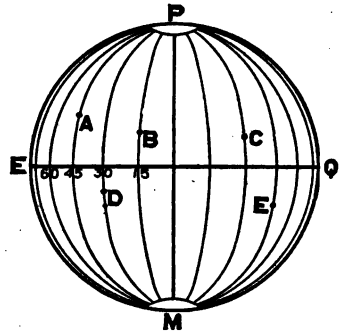
LONGITUDE AND TIME

The Equator is an imaginary line running east and west on the earth's surface equally distant from the two poles.

A Meridian is an imaginary line running north and south on the earth's surface from one pole to the other.

Longitude is the distance measured in degrees east or west from a standard meridian.

The Prime Meridian in general use is the meridian that passes through the Royal Observatory at Greenwich, England.



In the illustration there are shown the equator (*EQ*), the prime meridian (*PM*), and several other meridians. A place east of the prime meridian is in East Longitude, and a place west of the prime meridian is in West Longitude. The longitude of a place is usually stated in brief form by using " E." or " W." with the number of degrees representing the measurement from the prime meridian.

In the figure we would say that

Longitude of *A* is 45° W.,
B is 15° W.,
C is 30° E.,
D is 30° W.,
E is 45° E.

In considering longitude it is not necessary to take up the question of position north or south of the equator.

Measurements made north or south of the equator are called measurements of latitude. Each meridian may be divided into 360 degrees. In navigation and in engineering both latitude and longitude are considered.

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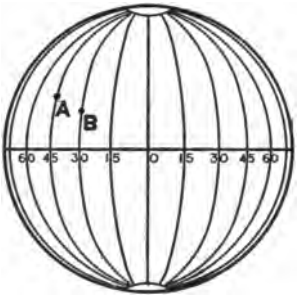
Difference in Longitude of Two Given Places

In finding the number of degrees of circular measure between two given points we must consider the location of each relative to the prime meridian. There are two possible cases.

1. When the two places considered are both east of, or both west of, the prime meridian.

2. When the two places are on opposite sides of the prime meridian.



(1)

In the figure (1)

Longitude of $A = 45°$ W.
Longitude of $B = 30°$ W.

Hence, the difference in their location, expressed in degrees, is

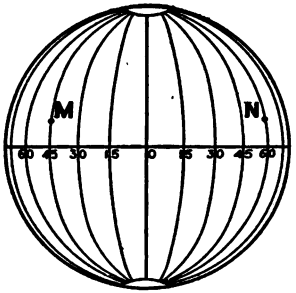
$$45° - 30° = 15°.$$

Therefore,

Difference in Longitude is 15°.

If both A and B were east of the prime meridian, the difference in their longitude would be obtained by subtraction.

If A were north of the equator and B were south of it, the difference in their longitude would be the same, 15°.



(2)

In the figure (2)

Longitude of $M = 45°$ W.
Longitude of $N = 60°$ E.

Hence, the difference in their location, expressed in degrees, is

$$45° + 60° = 105°.$$

Therefore,

Difference in Longitude is 105°.

In this case, the measurement is made up of two different portions, both measured from the same point but in opposite directions.

Here, as in the first case, we are seeking to find the number of degrees of measurement *between the two points.*

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From these illustrations we have the general rule:

When two places are both in east longitude, or both in west longitude, the difference in their longitudes is found by subtraction.

When one place is east, and the other is west, longitude, the difference in their longitudes is found by addition.

BLACKBOARD PRACTICE

Find the difference in longitude between two places whose longitudes are respectively:

1. 30° E., 45° E.	**7.** 30° E., 45° W.	**13.** 0°, 57° W.
2. 20° E., 75° E.	**8.** 30° W., 60° E.	**14.** 45° E., 90° E.
3. 45° W., 75° W.	**9.** 45° W., 45° E.	**15.** 60° W., 35° E.
4. 10° W., 90° W.	**10.** 60° E., 30° W.	**16.** 16° E., 96° W.
5. 5° E., 120° E.	**11.** 100° E., 50° W.	**17.** 35° W., 80° W.
6. 15° W., 135° W.	**12.** 90° W., 77° E.	**18.** 81° W., 37° E.

19. 10° 30′ E., 25° 15′ E.

20. 15° 30′ E., 70° 20′ E.

21. 90° 10′ W., 75° 10′ W.

22. 15° 20′ W., 85° 45′ W.

23. 45° 10′ W., 98° 25′ W.

24. 60° 45′ E., 10° 17′ W.

25. What is the difference in longitude of Boston, 71° 3′ 50″ W., and Denver, 104° 58′ 0″ W.?

26. What is the difference in longitude of San Francisco, 122° 25′ 42″ W., and Chicago, 87° 36′ 45″ W.?

27. What is the difference in longitude of Washington, 77° 3′ 6″ W., and Pittsburg, 80° 2′ 0″ W.?

28. What is the difference in longitude of London, 0° 5′ 48″ W., and New York, 74° 0′ 24″ W.?

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The Relation between Longitude and Time

Once Every Twenty-four Hours a complete revolution through a circumference of 360° is made by every point between the poles on the earth's surface. Therefore,

In 1 *hour any point passes through* $\frac{360°}{24}$, *or through* 15° *of longitude.*

Again, since 1 minute of time equals $\frac{1}{60}$ of 1 hour,

In 1 *minute any point passes through* $\frac{15}{60}°$, *or* $\frac{1}{4}°$, *or* 15′ *of longitude.*

Finally, since 1 *second of time equals* $\frac{1}{60}$ *of* 1 *minute,*

In 1 *second any point passes through* $\frac{15}{60}'$, *or* $\frac{1}{4}'$, *or* 15″ *of longitude.*

These corresponding equivalents should be memorized.

Table of Relation between Longitude and Time

360° of longitude correspond to 24 hours of time
15° of longitude correspond to 1 hour of time
15′ of longitude correspond to 1 minute of time
15″ of longitude correspond to 1 second of time

It is helpful to remember, also, that

(1) When the sun's rays are vertical on any point on a meridian, it is noon at all places on that meridian.

(2) The earth turns from west to east, hence it appears that the sun moves from east to west.

(3) When it is noon at any place it is before noon, or earlier, at all points west of that place, for the sun has not yet become vertically located over the meridians of those places.

(4) When it is noon at any place it is after noon, or later, at all points east of that place, for the sun has already been vertically located over the meridians of those places.

The navigator of a ship sometimes uses the difference in time between two places to determine his longitude; and, if he knows the difference in longitude, to determine the time.

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I. To Find the Difference in Time between Two Places, When the Difference in the Longitude of the Places Is Known.

Illustration:

When it is noon at Denver (104° 58′ 0″ W.) what time is it at Boston (71° 3′ 50″ W.)?

	104°	58′	0″
	71°	3′	50″
15)	33°	54′	10″
	2	15	$36\frac{2}{3}$

The difference in longitude is 33° 54′ 10″. We have found that 15° corresponds to 1 hr., 15′ to 1 min., and 15″ to 1 sec. Therefore, the difference in time between the two given places is as many hours, minutes, and seconds, respectively, as there are degrees, minutes, and seconds in one fifteenth of the difference in longitude. In this particular illustration Boston is *east* of the point where the time is noon; hence it is later than noon in Boston by the time obtained, that is, 2 hr. 15 min. $36\frac{2}{3}$ sec. That is, when it is noon at Denver it is 15 minutes and 37 seconds after 2 o'clock in Boston. (The time is to the nearest second.)

Longitude of Important Cities

Boston	71° 03′ 50″ W.	Bombay	72° 45′ 56″ E.
Chicago	87° 36′ 45″ W.	Canton	113° 16′ 30″ E.
Denver	104° 58′ 00″ W.	Cape Town	18° 28′ 40″ E.
New York	74° 00′ 24″ W.	London	0° 05′ 48″ W.
Pittsburgh	80° 02′ 00″ W.	Manila	120° 58′ 06″ E.
Portland, Me.	70° 15′ 40″ W.	Melbourne	144° 58′ 35″ E.
San Francisco	122° 25′ 42″ W.	Paris	2° 20′ 14″ E.
Washington	77° 03′ 06″ W.	Tokyo	139° 44′ 30″ E.

BLACKBOARD PRACTICE

Using the longitudes given in the table find the difference in time between:

1. Boston and Pittsburgh.
2. New York and London.
3. Portland and Pittsburgh.
4. New York and Chicago.
5. Washington and Denver.
6. New York and San Francisco.

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7. Chicago and San Francisco.
8. New York and Paris.
9. Canton and London.
10. Tokyo and Paris.
11. Boston and Denver.
12. Denver and New York.
13. Pittsburgh and Denver.
14. Boston and Chicago.
15. Cape Town and Melbourne.
16. Canton and Bombay.
17. Canton and Tokyo.
18. London and Paris.
19. New York and Manila.
20. London and Denver.
21. Paris and San Francisco.
22. Melbourne and New York.

When the sun's rays are vertical upon the meridian at Washington, find the time at each of the following places:

23. Chicago.
24. Denver.
25. Pittsburgh.
26. Boston.
27. Manila.
28. San Francisco.
29. London.
30. Tokyo.
31. Canton.

II. To Find the Difference in the Longitude of Two Places, When the Difference in the Time of the Places Is Known.

Illustration:

1. The difference in the time at New York and at Denver is 2 hr. 3 min. 50.4 sec. What is the difference in the longitude of the two cities?

hr.	min.	sec.	
2	3	50.4	
		15	
30	57	36	
30°	57′	36″	the difference in longitude.

We have found that 1 hour of time corresponds to 15° of longitude, and that 1 minute of time corresponds to 15′ of longitude, and that 1 second of time corresponds to 15″ of longitude. Therefore, the difference in the longitude is as many degrees as 15 times the number of hours, as many minutes as 15 times the number of minutes, and as many seconds as 15 times the number of seconds. That is, the difference in longitude of New York and Denver is 15 times (2 hr. 3 min. 50.4 sec.), or 30° 57′ 36″.

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2. Find the longitude of Paris, its time being 5 hours, 5 minutes, 22.5 seconds earlier than New York.

hr.	min.	sec.	
5	22	22.5	
		15	
76	20	38	
76°	20′	38″	the difference in longitude

76°	20′	38″	dif. in longitude
74°	00′	24″	W. (New York)
2°	20′	14″	E., the longitude of Paris.

Since 1 hr. corresponds to 15°, 1 min. to 15′, and 1 sec. to 15″, the difference in longitude between Paris and New York is as many degrees, minutes, and seconds, respectively, as there are hours, minutes, and seconds in 15 times the difference in time between the two places. This difference in longitude we find to be 76° 20′ 38″; and as this difference is greater than the longitude of New York (74° 00′ 24″) by 2° 20′ 14″, Berlin must be *east* of the prime meridian by that longitude.

BLACKBOARD PRACTICE

Find the longitude of the place whose time, when the sun is on the meridian at Washington, D. C., is

1.	2.00 P.M.	**7.**	10.00 A.M.	**13.**	$3.15\frac{1}{4}$ P.M.
2.	3.00 P.M.	**8.**	9.15 A.M.	**14.**	$1.20\frac{3}{4}$ P.M.
3.	4.30 P.M.	**9.**	7.30 A.M.	**15.**	$9.10\frac{1}{2}$ A.M.
4.	6.45 P.M.	**10.**	5.50 A.M.	**16.**	$10.15\frac{3}{4}$ P.M.
5.	8.30 P.M.	**11.**	6.20 A.M.	**17.**	$11.15\frac{1}{2}$ P.M.
6.	11.00 P.M.	**12.**	4.00 A.M.	**18.**	$12.20\frac{1}{2}$ A.M.

19. The Stock Exchange at Paris closes at 3 P.M on their meridian. What is the corresponding time in New York? the difference in longitude?

20. An accident occurred in Rome at 4 hr. 45 min. $49\frac{2}{15}$ sec. A.M. Monday. What is the longitude of Rome if the corresponding time in New York was 11 P.M. Sunday?

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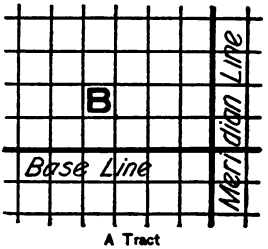
MEASUREMENT OF PUBLIC LANDS

The public lands in the Western and the Southern states have been systematically divided into townships and sections.

A Township is a square whose sides are 6 miles long. A township contains 36 square miles. The sides of a township run east and west, and north and south.

A Section is one of the square miles into which a township is divided.

A section of 1 square mile contains, therefore, 640 acres.



A Tract

Tracts are laid out as in the figure above. An east and west line is chosen for a base line, and a north and south line for a principal meridian. Lines parallel to the base line, with other lines parallel to the principal meridian, cut the tract into square townships.

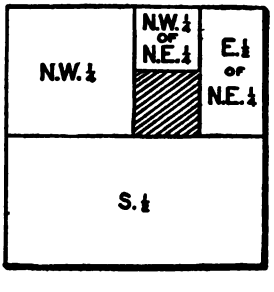
6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A Township

Townships run east and west in *tiers*, and north and south in *ranges*. Thus: The township B in the figure is in the second tier north and in the fourth range west. Hence, B is numbered "T. 2 N., R. 4 W." The sections in a township are numbered as in the figure. The drawing at the left represents the division of the township B above into 36 "sections" of one square mile each.

Each section is then divided into parts, and the drawing at the right represents section 10 from the township plan. Excepting its shaded part, each portion of the section is named. The shaded portion with reference to

(1) Its location in the section;
(2) Its township; and
(3) Its tract;



is called S. W. ¼ of N. E. ¼, Sec. 10, T. 2 N., R. 4 W.

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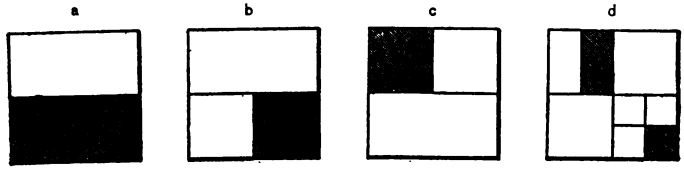
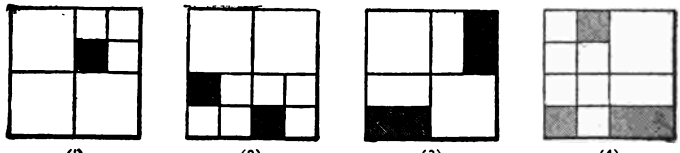
ORAL PRACTICE

If each of the drawings represents Section 10 of the Township B, give the location of each of the shaded portions.



1. How many acres are there in the shaded part of (a)?
2. How many acres are there in the shaded part of (b)?
3. How many acres are there in the shaded part of (c)?
4. How many acres in each of the shaded parts of (d)?

Give the location of each of the shaded portions in the following. Thus: The shaded part of No. 1 is the "S. W. $\frac{1}{4}$ of N. E. $\frac{1}{4}$ of Sec. 10 T. 2 N., R. 4."



WRITTEN PRACTICE

Draw a plan containing townships as in a tract.

Show the following in your plan:

1. T. 3 N., R. 2 W.
2. T. 3 N., R. 3 W.
3. T. 4 N., R. 2 E.
4. T. 1 N., R. 4 W.
5. T. 3 N., R. 2 W.
6. T. 2 N., R. 3 E.

On the plan of a township locate:

7. Section 12.
8. Section 23.
9. Section 25.

On the plan of a section indicate:

10. S. $\frac{1}{2}$ of N. E. $\frac{1}{4}$.
11. S. E. $\frac{1}{2}$ of N. W. $\frac{1}{4}$.
12. S. W. $\frac{1}{4}$ of N. E. $\frac{1}{4}$.
13. S. E. $\frac{1}{2}$ of N. W. $\frac{1}{4}$.

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THE METRIC SYSTEM

The Metric System is a decimal system of weights and measures.

The Meter is the principal unit of the metric system.

The meter was originated by the French government and was intended to be a length exactly one ten-millionth part of the distance from the equator to either pole of the earth's surface. Due to a slight error, however, the meter is not exactly the length intended, but the usefulness of this system is in no way impaired by that error.

Principles Underlying the Metric System

The standard unit of the system is the meter.

Units smaller than this standard are made by dividing the meter *decimally*.

Still smaller units are made by *further decimal divisions*.

The successive decimal divisions are named by using Latin or Greek prefixes. Thus:

deci means .1	Hence, decimeter means .1 of a meter
centi means .01	Hence, centimeter means .01 of a meter
milli means .001	Hence, millimeter means .001 of a meter

Units larger than the standard are made by multiplying the standard unit by 10 or by multiples of 10. Thus:

deka means 10 times	Hence, dekameter means 10 meters
hekto means 100 times	Hence, hektometer means 100 meters
kilo means 1000 times	Hence, kilometer means 1000 meters

The three subdivisions of the standard unit, together with the three multiples of that unit, make up the table for

Metric Measures of Length

10 millimeters (mm.)	= 1 centimeter	abbreviated cm.
10 centimeters	= 1 decimeter	" dm.
10 decimeters	= 1 meter	" m.
10 meters	= 1 dekameter	" Dm.
10 dekameters	= 1 hektometer	" Hm.
10 hektometers	= 1 kilometer	" Km.

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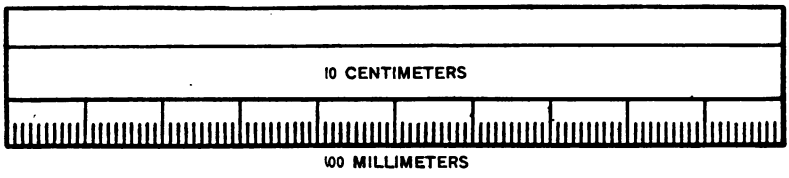
Convenient approximate equivalents in terms of English Standards.

1 meter = 39.37 inches	1 yard = .9144 meter
1 kilometer = .6214 mile	1 mile = 1.609 kilometers

The meter is used in measuring short distances, fabrics, etc.

The kilometer is used in measuring long distances just as we use the mile.

The centimeter and the millimeter are in common use in the sciences.



Changing from One Denomination to Another Denomination. Since each denomination in the table is *ten times* the preceding denomination, we may change from one denomination to the next higher denomination by moving the decimal point *one place to the left.*

Thus:	4500 mm. = 450 cm.	Also:	1256 m. = 125.6 Dm.
	450 cm. = 45 dm.		125.6 Dm. = 12.56 Hm.
	45 dm. = 4.5 m.		12.56 Hm. = 1.256 Km.

The same principle permits us to change from one denomination to the next lower denomination by moving the decimal point *one place to the right.*

Thus:	8 m. = 80 dm.	Also:	154.75 Km. = 1547.5 Hm.
	15 cm. = 150 mm.		24.125 Dm. = 241.25 M.

For a general illustration it is helpful to note that 1235672 mm. = 123567.2 cm. = 12356.72 dm. = 1235.672 m. = 123.5672 Dm. = 12.35672 Hm.

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Advantage of the Metric System. Compare the work necessary in the following transformations.

(1) The English System, by long and careful calculation, shows that 4.352 mi. = 1392.64 rd. = 7659.52 yd. = 22978.56 ft.

(2) The Metric System, *by merely moving the decimal point*, shows that 4.352 Km. = 43.52 Hm. = 435.2 Dm. = 4352 m.

In changes to higher denominations the advantage in the metric system is even greater.

BLACKBOARD PRACTICE

Change to meters, or to meters and a decimal of a meter:

1. 5 Km.	**6.** 6 Hm.	**11.** 125 dm.	**16.** 2400 mm.
2. 3.5 Km.	**7.** 45 Dm.	**12.** 12.5 dm.	**17.** 354.6 mm.
3. 15 Km.	**8.** 1.8 Hm.	**13.** 1.25 dm.	**18.** 11.75 cm.
4. 21.7 Km.	**9.** 12.3 Dm.	**14.** 175 cm.	**19.** 11.75 dm.
5. 15.08 Km.	**10.** 11.15 Dm.	**15.** 1450 cm.	**20.** 1405 cm.

Change to meters or to decimals of a meter:

21. 3 Km., 5 Hm.	**24.** 4 m., 5 dm.	**27.** 45 dm., 45 cm.
22. 5 Hm., 8 Dm.	**25.** 15 m., 8 dm.	**28.** 15 dm., 57 cm.
23. 4 Km., 4 Dm.	**26.** 35 m., 25 cm.	**29.** 25 cm., 75 dm.

Express in meters:		Express in feet:	
30. 10 ft.	**33.** 35.5 in.	**36.** 12 m.	**39.** 2500 cm.
31. 32 ft.	**34.** 110 in.	**37.** 15.75 m.	**40.** 2500 dm.
32. 5.6 ft.	**35.** 2 ft. 5.5 in.	**38.** 4.875 Dm.	**41.** 11250 mm.

Change to miles:		Change to kilometers:	
42. 4 Km.	**44.** 10.5 Km.	**46.** 100 mi.	**48.** .025 mi.
43. 12.5 Km.	**45.** 125.5 Km.	**47.** 125 mi.	**49.** .00125 mi.

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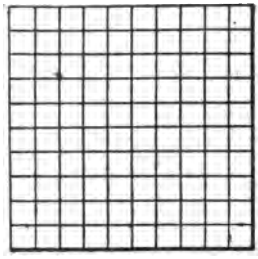
Metric Measures of Surface

The Square Meter is the standard unit of surface measure in the metric system.

The square meter is a square whose side is 1 meter long.



If the sides of the square meter are each divided into 10 equal parts, each part is 1 decimeter long. If lines are drawn connecting the points of division on opposite sides, the square meter is divided into 100 equal parts, and since the sides of each of these 100 squares are each 1 decimeter long, the square meter is equal in area to 100 square decimeters.

In like manner, each square decimeter may be divided into 100 equal squares, each of which is a square centimeter. Moreover, the square constructed on a length equal to 10 meters would give 100 square meters, or a square Dekameter, and the process could be repeated to obtain larger square units of measure.

From this principle of division or multiplication of the standard square unit of measure we have the table for

Metric Measures of Surface

100 square millimeters	= 1 square centimeter	abbreviated	sq. cm.
100 square centimeters	= 1 square decimeter	"	sq. dm.
100 square decimeters	= 1 square meter	"	sq. m.
100 square meters	= 1 square dekameter	"	sq. Dm.
100 square dekameters	= 1 square hektometer	"	sq. Hm.
100 square hektometers	= 1 square kilometer	"	sq. Km.

Approximate equivalents in terms of English Standards.

1 sq. m. = 1.196 sq. yd.	1 sq. yd. = .8361 sq. m.
1 sq. Km. = .386 sq. mi.	1 sq. mi. = 2.59 sq. Km.

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The square meter is used for measuring surfaces like floors, walls, etc.

The square kilometer is used for measuring large land areas.

For small land areas the **square dekameter** is the principal unit of measure and is called the **Are** (pronounced âr). For land areas it is customary to use the table in the following form:

100 ares = 1 centare
100 centares = 1 hektare

Changing from One Denomination to Another Denomination. The table shows that each square unit of measure is 100 times the preceding square unit of measure. Therefore, to change from one denomination to the next higher denomination we move the decimal point *two places to the left.*

Thus: 4500 sq. cm. = 45 sq. dm. = .45 sq. m.

We may also change from one denomination to the next lower denomination by moving the decimal point *two places to the right.*

Thus: 12 sq. m. = 1200 sq. dm. = 120000 sq. cm.

BLACKBOARD PRACTICE

Change to square meters:

1. 125 sq. Km.
2. 160 sq. Hm.
3. 250 sq. Dm.
4. 124.5 sq. Km.
5. 375.25 sq. Hm.
6. 1250.05 sq. Dm.
7. 15625 sq. mm.
8. 25400 sq. cm.
9. 154500 sq. dm.

Express in square yards:

10. 100 sq. m.
11. 11.5 sq. m.
12. 1.125 sq. m.
13. .125 sq. Km.

Express in square miles:

14. 12.5 sq. Km.
15. 24.1 sq. Km.
16. .0125 sq. Km.
17. 11375 sq. m.

Metric Measures of Volume

The Cubic Meter is the standard unit of volume measure in the metric system.

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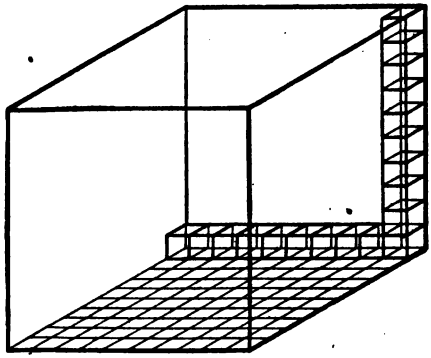
The cubic meter is a cube whose edge is 1 meter long.

If the edges of the cubic meter are each divided into 10 equal parts, each part is 1 decimeter long. If the cubic meter were then cut by planes passing through the points of division of each edge, and parallel to the three faces that meet in any point, the cubic meter would be cut into 1000 cubes, each small cube being 1 decimeter on each edge.



In like manner each **cubic decimeter** could be cut by planes into 1000 smaller cubes, each of which would be 1 cubic centimeter. The cubic centimeter could be cut, furthermore, into 1000 smaller cubes, each one a cubic millimeter. We have, therefore, the table for

Metric Measures of Volume

1000 cubic millimeters (cu. mm.)	**=1 cubic centimeter**	**abbreviated**	**cu. cm.**
1000 cubic centimeters	**=1 cubic decimeter**	**"**	**cu. dm.**
1000 cubic decimeters	**=1 cubic meter**	**"**	**cu. m.**

(Measures larger than the cubic meter have no practical use.)

Changing from One Denomination to Another Denomination. Each unit in the table of volume measure is 1000 times the preceding unit. Therefore,

To change from one denomination to the next higher denomination move the decimal point *three places to the left.*

To change from one denomination to the next lower denomination move the decimal point *three places to the right.*

Thus: 4575 cu. cm.=4.575 cu. dm.
And: 15.658 cu. m.=15658 cu. dm.=15658000 cu. cm.

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BLACKBOARD PRACTICE

Change to higher denominations:

1. 1500 cu. cm., 150000 cu. cm.
2. 35000 cu. mm., 125000 cu. dm.
3. 4570000 cu. mm., 12575000 cu. mm.

Change to lower denominations:

4. 15 cu. m., 150 cu. dm.
5. 275 cu. m., 4500 cu. dm.
6. 15750 cu. cm., 354000 cu. cm.

Metric Measures of Capacity

The Liter is the standard unit of capacity measure, either dry or liquid, in the metric system.

A standard liter is a cubical box whose inside edges are each 1 decimeter long. Therefore, the liter is *a cubic decimeter*.

Since each edge of the cubic decimeter is 10 centimeters long, *the liter contains* 1000 *cubic centimeters*.

Metric Measures of Capacity

10 milliliters (ml.)	= 1 centiliter	abbreviated	cl.
10 centiliters	= 1 deciliter	"	dl.
10 deciliters	= 1 liter	"	l.
10 liters	= 1 dekaliter	"	Dl.
10 dekaliters	= 1 hektoliter	"	Hl.

Convenient approximate equivalents in terms of English Standards.

1 liter = 1.0567 liquid quarts	1 hektoliter = 2.8375 bushels
1 liter = .908 dry quart	1 hektoliter = 26.417 gallons

The liter is used for measuring liquids or small fruits in small quantities. The hektoliter is used for measuring large quantities.

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BLACKBOARD PRACTICE

Change to higher denominations:

1. 15 l.
2. 12.5 l.
3. 500 cl.
4. 3500 ml.
5. 254.75 dl.
6. 254.75 Dl.

Change to lower denominations:

7. 35 Kl.
8. 45.8 Hl.
9. 175.25 Dl.
10. 147.5 l.
11. 35.125 l.
12. 45.25 cl.

Express in liquid quarts:

13. 10 l.
14. 12.5 l.
15. 175.2 l.
16. 15 dl.
17. 1.4 Hl.
18. 25.8 Kl.

Express in dry quarts:

19. 35 l.
20. 15.8 l.
21. 250.75 dl.
22. 175 dl.
23. 25.125 Dl.
24. 15000 cl.

Express in bushels:

25. 5.5 Hl.
26. 12.1 Hl.
27. 175 l.
28. 25000 dl.

Express in hektoliters:

29. 15 bu.
30. 75.5 bu.
31. 320 pk.
32. 1600 qt.

Metric Measures of Weight

The Gram is the standard unit of weight measure in the metric system.

The gram was established by taking **the weight of 1 cubic centimeter of water** as a standard of measure.

Therefore,

since 1 liter = 1 cubic decimeter = 1000 cubic centimeters,

and

since 1 cubic centimeter of water = 1 gram of weight,

it follows that

1 liter of water weighs 1000 grams or 1 kilogram

The advantage of this relation lies in the ease with which we may change from a standard quantity to a standard weight by merely changing the name of the unit.

Thus: 150 cu. cm. water weighs 150 grams
2.5 l. water weighs 2.5 Kg.

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Metric Measures of Weight

10 milligrams (mg.)	= 1 centigram	abbreviated	cg.
10 centigrams	= 1 decigram	"	dg.
10 decigrams	= 1 gram	"	g.
10 grams	= 1 dekagram	"	Dg.
10 dekagrams	= 1 hektogram	"	Hg.
10 hektograms	= 1 kilogram	"	Kg.

Large quantities of heavy commodities necessitate units of higher denominations than the kilogram. These are:

10 kilograms = 1 myriagram Mg.
100 kilograms = 1 metric quintal Q.
1000 kilograms = 1 metric ton T.

Convenient approximate equivalents in terms of English standards.

1 gram = 15.432 grains 1 kilogram = 2.20462 pounds
1 metric ton = 2204.621 pounds

The gram is used in the sciences, in weighing drugs, medicines, etc., the kilogram is used for weighing commodities for which the English standard uses the pound, and the metric ton is used for heavy articles in large amounts.

BLACKBOARD PRACTICE

Change to grams:

1. 12 Kg.
2. 15.5 Kg.
3. 1.75 Kg.
4. 75.8 Hg.
5. 157.1 Dg.
6. 154.25 Kg.
7. 1500 mg.
8. 1750 cg.
9. 3750 dg.
10. 17500 mg.
11. 187500 mg.
12. 187500 cg.

Change to pounds:

13. 15 Kg.
14. 1.25 Kg.
15. 12.5 Kg.
16. 12.5 Dg.
17. 157.1 Hg.
18. 157100 g.
19. 35.125 Dg.
20. 120000 cg.

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Finding the Given Weight of Any Given Solid by comparing its weight with the weight of *the same quantity of water* is possible through application of the metric system. By careful experiments scientists have determined the *relation* of the weight of common metals, liquids, etc., to the weight of the same volume of water.

Thus: Cast Iron is known to be 7.2 times as heavy as water.
Lead is known to be 11.37 times as heavy as water.
Alcohol is known to be .792 of the weight of water, etc.

These known relations, or ratios, between the weight of a substance and the weight of the same quantity of water enable us to calculate by simple multiplication the weight of any quantity of a substance.

Illustrations:

1. Find the weight of 12.5 l. of milk, if milk is 1.03 times as heavy as water.

1 l. of water weighs 1000 g. Hence, 12.5 l. water weighs 12500 g.
But milk is 1.03 times the weight of water.
Therefore, 12500 g. milk weighs (1.03×12500) g. or 12875 g. *Result.*

2. Find the weight of a bar of steel 1.5 m. long, 3 cm. wide, and 1 cm. thick, if steel is 7.8 times the weight of water.

We find first the number of cu. cm. in the steel bar.
$(1.5 \times 100) \times 3 \times 1 = 450$, the number of cu. cm. in the volume of the bar.
Now the weight of 450 cu. cm. of water = 450 g.
And the steel is 7.8 times as heavy as water.
Hence, the weight of the bar $= 7.8 \times 450$ g. $= 3510$ g. *Result.*

The ratio of the weight of a given substance to the weight of the same volume of water is called the **Specific Gravity** of the substance.

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Specific Gravity of Common Substances			
Steel	7.8	Yellow Pine	.70
Copper	8.8	White Oak	.75
Lead	11.3	Alcohol	.79
Gold	19.3	Machine Oils	.92
Platinum	21.1	Rubber	.92

BLACKBOARD PRACTICE

Find the weight of:

1. 100 cu. dm. of steel.
2. 120 cu. dm. of gold.
3. 15.125 cu. dm. of copper.
4. 100000 cu. cm. of lead.
5. 25000 cu. cm. of platinum.
6. 10000 cu. cm. of copper.
7. 156500 cu. cm. of lead.
8. 345000 cu. cm. of gold.
9. 5000 cu. cm. of yellow pine.
10. 150000 cu. dm. of white oak.
11. 2500 Kl. of alcohol.
12. 10000 Dl. of machine oil.
13. 150000 cu. dm. of rubber.
14. 200000 cu. dm. of alcohol.
15. 15000 cu. dm. of yellow pine.
16. 35000 cu. cm. of white oak.

Table of Equivalents			
Metric to Common		Common to Metric	
1 m.	=39.37 in.	1 yd.	=.9144 m.
1 Km.	=.62137 mi.	1 mi.	=1.60935 Km.
1 sq. m.	=1.196 sq. yd.	1 sq. yd.	=.836 sq. m.
1 Ha.	=2.471 A.	1 A.	=.4047 Ha.
1 cu. m.	=1.308 cu. yd.	1 cu. yd.	=.765 cu. m.
1 l.	=.908 qt. (dry)	1 qt. (dry)	=1.1012 l.
1 l.	=1.0567 qt. (liq.)	1 qt. (liq.)	=.94636 l.
1 Hl.	=2.8377 bu.	1 bu.	=.35239 Hl.
1 Kg.	=2.2046 lb. (av.)	1 lb. (av.)	=.45359 Kg.
1 M. T.	=1.1023 T.	1 T.	=.90718 M. T.

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Miscellaneous Applications of Metric System

1. The "bore," or inside diameter, of a rifle barrel is 7 mm. What is the diameter in inches?

2. The Panama Canal is approximately 40.5 miles long. What is the length of the canal in kilometers?

3. Three successive distances measured along a straight line were 5.14 Km., 275 m., and 31.25 Hm., respectively. What was the total distance?

4. 100 liters of olive oil cost a dealer $.75 per liter. Find the amount of gain he made when he sold the oil for $.90 per quart.

5. A rectangular plot is 24 meters long and 15 meters wide. What is the area of the plot in square meters and in square feet?

6. A consignment of 100 cars of coal averaging 50 tons each is shipped by an exporter. How many metric tons were there in the consignment?

7. The Great Pyramid of Egypt has a square base 764 feet on a side, and its altitude is 480 feet. Find the volume in cubic meters.

8. How many pint bottles must be provided by a dealer for the contents of a cask containing 2 kiloliters of olive oil?

9. Broad street, Philadelphia, is 100 feet wide between the curbstones. Express this width in meters, also in a decimal part of a kilometer.

10. A barrel of flour weighs 196 pounds. How many kilograms of flour are there in a consignment of 800 barrels of flour?

11. A building lot 20 meters wide and 90 meters deep is sold at the rate of $1200 per acre. What amount is received for the lot?

12. An importer places an order for 15 gallons of perfume, and the foreign dealer bills it in liters. For how many liters was the bill written?

13. If you find that the length of your average step in walking is 30 inches, how many steps will you take in going over a distance of 1 kilometer?

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14. A druggist is required to compound a quantity of capsules, each one to contain 12 cg. of a certain drug. He used 1.2 Kg. of the drug. How many capsules did he make?

15. The French cable between New York and Brest is 3305.164 miles long. What is the length of this cable in meters? In kilometers?

16. The specific gravity of a certain grade of olive oil is .915. Find the weight in kilograms of twenty-five cans of oil, each can containing 1.3 l.

17. Eight yards of silk in a dress cost the importer $2.00 per meter. If he received $3 per yard, what was the amount of his profit on the goods?

18. A liter of glue weighs 2.0 Kg. What is the weight of the glue contained in a full tank that is known to hold 240 gallons?

19. The distance from Brussels to Paris is 326 Km., and the railway fare is approximately 19.3 centimes per mile. Find the fare between the cities.

20. A French express train travels 75 Km. in 1 hour, and a New York Central express train travels 75 miles in the same time. How many more miles per hour does the faster train travel?

21. A foreign builder orders from the United States Steel Corporation a plate girder, giving its exact length as 10.125 m. Find the exact length of the girder in feet and inches to the nearest hundredth of an inch.

22. An imported plate glass mirror is 56 cm. long and 35.5 cm. wide. Find the number of feet of molding required to frame the mirror, the molding being 2 inches wide. (Allow for the loss in making the corners.)

23. When empty a tank weighs 15.75 Kg., and when filled with water the total weight of the tank and the water is 142 Kg. How many liters of water will the tank contain? How many gallons?

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24. The diameter of an automobile wheel is 28 inches, and the tire is 3.5 inches in diameter when allowance is made for flattening. How many turns will this wheel make in going over a distance of 1 kilometer?

25. Brass is composed of 60% copper and 40% zinc. How many kilograms of each element, copper and zinc, are there in a bar of brass which weighs 45,000 g.?

26. The duty on 100 l. of olive oil was $15.60, and the cost of the cartage $1.25. The oil cost $.60 per liter and was sold for $3.50 per gallon. What was the amount of profit made by the importer?

27. A tank of oil whose specific gravity is .91 weighs 145 Kg. The empty cask weighs 44.092 pounds. How many liters of oil are there in the cask, and how many quarts?

28. Scientists have determined that a liter of air weighs 1.292 g. At that rate calculate the weight of the air in a room thirty feet long, 22 feet wide, and 12 feet high, giving the weight in kilos and in pounds.

29. An automobile uses an average of one gallon of gasoline to every sixteen miles of a journey. At the same rate determine how many kilometers the automobile would run on one liter of gasoline.

30. A quantity of silk cost in Paris $2.80 per meter, and the duty was 45%. The total expense for shipping, freight, insurance, etc., amounted to $.60 per meter. At what price per yard must the importer sell the silk in order to make a profit of 30%?

31. Compare the labor of finding the number of liters in a tank 5 m. long, 3 m. wide, and 15 dm. deep, with the labor of finding the number of gallons in a tank 16 feet long, 10 feet wide, and 1 yard deep. Time yourself on the work of finding each result. Find what per cent of the time needed for the longer process is the time needed for the shorter one.

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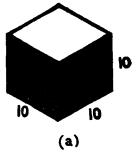
CUBE ROOT

The Formation of a Cubic Solid.

Suppose that we have in (*a*) a cube whose edge is 10 inches long.

Then any three adjacent faces have areas measuring 100 square inches each.

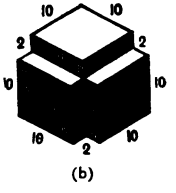


(The volume of the solid is 1000 cubic inches.)

Suppose, further, that we wish to enlarge the given cube to a cube whose edge shall be 12 inches, that is, 2 inches longer than the cube in (*a*). We can accomplish this by *building upon three square faces that are adjacent.*

In (*b*) we have built on each of three faces a new volume 2 inches thick.

These three additions are built upon square faces 10 inches by 10 inches, hence each new volume is made up of $10 \times 10 \times 2$ cubic inches.

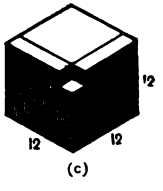


(The volume thus added is 600 cubic inches.)

The solid as it now stands has three unfilled "notches," each of which is 10 inches long, 2 inches wide, and 2 inches deep. Into these notches we may *build, therefore, three square prisms.*

In (*c*) we have built in the three solids necessary to fill up the notches.

These three prisms are each 10 inches long, 2 inches wide, and 2 inches deep.



(The volume thus added is 120 cubic inches.)

Excepting the small cubical portion where the three prisms come together, the larger cube is now complete. This small portion is 2 inches long, 2 inches wide, and 2 inches deep. In (*d*) this small remaining volume has been added, and the new cube is complete.

(The volume thus added is 8 cubic inches.)

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(d)

It is well to remember at this point that the process of increasing the original cube to a larger cube required the addition of three square prisms, three other square prisms and a cube.

The process of finding the cube root of a number depends upon the principles that governed us in building the cube just illustrated, but the steps of the process will be reversed in order.

The Cube of a Number.

The cube of a number is the product obtained by using the number three times as a factor.

In the following illustration the cube of 12 is obtained in two ways, first, by multiplying $12\times12\times12$ in the ordinary way; and, second, by finding the product of $(10+2)$ by $(10+2)$ by $(10+2)$.

$$
\begin{array}{rr}
12 = & 10+2 \\
\underline{12} = & \underline{10+2} \\
24 = & (10\times2)+2^2 \\
\underline{120} = & \underline{10^2+(10\times2)} \\
144 = & 10^2+2(10\times2)+2^2 \\
\underline{12} = & \underline{10+2} \\
288 = & (10^2\times2)+2(10\times2^2)+2^2 \\
\underline{1440} = & \underline{10^3+2(10^2\times2)+(10\times2^2)} \\
1728 = & 10^3+3(10^2\times2)+3(10\times2^2)+2^3
\end{array}
$$

It will be observed that the result is made up of four parts, and that they are:

$$
\begin{array}{rr}
10^3 & =1000 \\
3(10^2\times2) & = 600 \\
3(10\times2^2) & = 120 \\
2^3 & = 8
\end{array}
\qquad 1000+600+120+8=1728.
$$

Observe that

$10^3 = 1000$ corresponds to figure (*a*)
$3(10^2\times2) = 600$ corresponds to the additions that give (*b*)
$3(10\times2^2) = 120$ corresponds to the additions that give (*c*)
$2^3 = 8$ corresponds to the additions that give (*d*)

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The student should go through the same process of reasoning with a cube of different dimensions. For example, take 2744, and make the drawings that result when the cube first removed is 10 units on each edge, and the solid additions are each 4 units thick.

With a knowledge of the relation of a cubic solid to a numerical cube we are ready to learn the process of extracting cube root.

The General Method for Finding Cube Root

The Relation between the Number of Places in a cube and the number in the corresponding cube root will be seen from the following.

The largest number having one place is 9. $9^2 = 729$, *three* places.

The largest number having two places is 99. $99^2 = 970299$, *six* places.

The largest number having three places is 999. $999^2 = 997002999$, *nine* places.

That is, a number having either one, two, or three places may have in its cube as many as three, six, or nine places, respectively.

If, therefore, a number is considered as " separated into periods of three places each," the number of such periods obtained will equal the number of places in the cube root that is sought.

For example: 112678587 written thus, 112 678 587, gives *three periods*, and its cube root, 483, has *three places*.

The period at the left need not have three places.

For example, the cube root of 15625, or 15 625, is 25.

And the cube root of 22188041, or 22 188 041, is 281.

As in square root (p. 155), the cube root of a decimal number is obtained by the process used for the cube root of integers, care being taken to " point off " properly in the root.

The cube of a decimal of one place has three decimal places; the cube of a decimal of two places has six decimal places, etc.

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Finding the Cube Root of a Number.

Illustration:

Find the cube root of 39304.

		39 304 | 30+4
I. 30^3	=	27 000
II. $3(30)^2$	=2700	12 304
III. $3\times30\times4$	= 360	
IV. 4^2	= 16	
	3076	
V. 3076×4	=	12 304

Explanation:

Beginning at the decimal point, separate the given cube into periods of three figures each.

The first period is 39000.

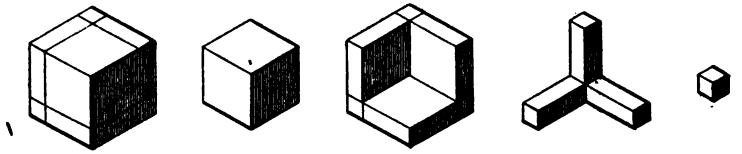
The greatest cube in 39000 is 27000.

The cube root of 27000 is 30.

Write 30 in the root.

Subtract 2700 from 39304.

Reference to the figures will help you as you follow the steps from this point on.



(a) (b) (c) (d)

When we subtract 27000 we have removed the largest possible cube that we know by inspection lies in the given solid.

Suppose that (*a*) represents the cube 27000 that we have removed. (See line I.)

Then the remaining volume, 12304 cubic units, is represented by (*b*).

We must now find the *approximate thickness* of the volume represented by our 12304 cubic units, and we note that the area of any one inside face in (*b*) is 30^2 units, or 900 square units.

We, therefore, divide the entire volume, 12304 square units, by the known area of all three faces, or by (3×900) square units, or 2700 square units. (See line II.) The quotient is apparently 4, which we write in the root as a *trial*.

Having considered the volume of the three faces, there remains to be considered the three prisms that are left, as in (*c*).

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Each prism is 30 units long, and we have assumed them to be 4 units thick; hence the total area of these inside faces is apparently $3(30\times4)$ square units, or 360 square units. (See line III.)

With the three prisms disposed of there remains the small solid (d) the area of whose face is (4×4) square units since we have assumed its thickness in two dimensions as 4 units. This completes the assumed areas and we write 4^2. (See line IV.)

Adding the three areas, lines II, III, and IV, we have a total area of 3076 square units.

Multiplying this area by our assumed thickness we have a volume of (4×3076) cubic units. This volume, subtracted from 12304 cubic units, gives a remainder of 0.

The following examples will further illustrate the process, and the student will familiarize himself with the method by going carefully through each step in each illustration.

It will be observed that (1) a cube whose root consists of three figures is obtained by repeating the process to obtain root figures after the second; and (2) the cube root of a decimal number is obtained by working as in whole numbers, pointing off at the end of the process the proper decimal places in the root.

Illustrations:

1. Find the cube root of 14706125.

			14 706 125 | 245 *Result.*
20^3	=		8 000
3×20^2	=	1200	6 706
$3\times20\times4$	=	240	
4^2	=	16	
		1456	
4×1456	=		5 824
3×240^2	=	172800	882 125
$3\times240\times5$	=	3600	
5^2	=	25	
		176425	
5×176425	=		882 125

Note that in each use of the trial divisor we consider it to be tens in the first division, hundreds in the second division, etc.

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2. Find the cube root of 1175.45 to two decimal places.

Pointing off from the decimal point, and annexing four ciphers for the necessary decimal, we have

		1 175.450 000 | 1052
10^3 =		1 000
3×10^2 =	300	175
The trial divisor being too large, bring down another period, and place a "0" in the root.		
		175 450
3×100^2 =	30000	
$3\times100\times5$ =	1500	
5^2 =	25	
5×31525 =		157 625
3×1500^2 =	6750000	17 825 000
$3\times1500\times2$ =	9000	
2^2 =	4	
2×6759004 =		13 518 008

The figures in the required root are 1052.

Pointing off *two* places for the two periods in the decimal, we have

10.52+ *Result.*

From the foregoing illustrations we may state the general

Rule: *Beginning at the decimal point, separate the given number into periods of three figures each.*

Find the greatest cube in the first period at the left, subtract this cube from the first period, and write its cube root for the first figure of the result. Annex the next period to the remainder for a dividend.

Take three times the square of the root already found, considered as tens, and divide the dividend by this trial divisor.

The quotient will be the second figure of the root.

To this partial divisor add three times the product of the first part of the root, considered as tens, by the second part; and add also the square of the second part. Their sum will be the complete divisor.

Multiply the complete divisor by the second part of the root, and subtract the product from the dividend.

Repeat this process in the same order until all the figures of the root have been found.

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BLACKBOARD PRACTICE

Find the cube root of:

1. 3375.	**6.** 68921.	**11.** 658503.	**16.** 7.529536.
2. 9261.	**7.** 79507.	**12.** 941192.	**17.** .004913.
3. 29791.	**8.** 85184.	**13.** 8242408.	**18.** 141.420761.
4. 54872.	**9.** 157464.	**14.** 7880599.	**19.** .065939264.
5. 46656.	**10.** 314432.	**15.** 2248.091.	**20.** 124251.499.

Find, to two decimal places, the cube root of:

21. 1500.	**23.** 15000.	**25.** .00675.	**27.** $\frac{3}{5}$.	**29.** $\frac{15}{19}$.
22. 28000.	**24.** 11.198.	**26.** .0008.	**28.** $\frac{10}{18}$.	**30.** $\frac{115}{131}$.

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ANSWERS

Page 8. — 1. $11.90. 2. $40.15.

Page 9. — 3. $2.15. 4. $.74.

Page 10. — 1. $3.51. 2. $3.95. 3. $3.49. 4. $49.85.

Page 11. — 1. $150.50. 2. $1909.10.

Page 12. — 3. $639. 4. $9522.50. 5. $4163.25. 6. $202.50.

Page 14. — 1. 12. 2. 14. 3. 10. 4. $11\frac{1}{2}$. 5. 9. 6. $11\frac{1}{2}$. 7. 8. 8. $10\frac{1}{2}$. 9. $6\frac{1}{2}$. 10. $2\frac{1}{2}$. 11. 3. 12. 4. 13. 6. 14. $3\frac{1}{2}$. 15. 7. 16. $9\frac{1}{2}$. 17. 10. 18. $10\frac{1}{2}$. 19. 10. 20. $12\frac{1}{2}$. 21. $13\frac{1}{2}$. 22. 15. 23. 15. 24. 18. 25. 20. 26. 15. 27. 25. 28. 36. 29. 15. 30. 20. 31. 20. 32. 24. 33. 14. 34. 21. 35. $9\frac{1}{2}$. 36. $17\frac{1}{2}$. 37. $166\frac{2}{3}$. 38. 315. 39. 500. 40. 384. 41. $261\frac{1}{2}$. 42. 400. 43. 810. 44. $72.48.

Page 15. — 45. $135.98. 46. $92.05.

Page 16. — 1. 210. 2. 225. 3. 320. 4. $234\frac{3}{4}$. 5. $253\frac{1}{2}$. 6. 290. 7. $161\frac{1}{4}$. 8. $255\frac{1}{4}$. 9. $307\frac{7}{12}$. 10. $12.96. 11. $16.13. 12. $12.96. 13. $8.10. 14. $12.67. 15. $16.51. 16. $9.45. 17. $13.02. 18. $18.12.

Page 17. — 1. 26 bunches. 2. 33 bunches. 3. 41 bunches. 4. 47 bunches. 5. 51 bunches. 6. 47 bunches. 7. 33 bunches. 8. 47 bunches. 9. 81 bunches. 10. $15.46. 11. $27.42. 12. $34.54. 13. $166.50.

Page 18. — 14. $150.11.

Page 19. — 1. 9 bundles. 2. $6.30. 3. $9.80.

Page 20. — 1. 16 sq. yd. 2. $23\frac{1}{3}$ sq. yd. 3. $21\frac{1}{3}$ sq. yd. 4. $24\frac{8}{9}$ sq. yd. 5. $28\frac{1}{3}$ sq. yd. 6. 34 sq. yd. 7. 14 sq. yd. 8. $20\frac{4}{9}$ sq. yd. 9. $22\frac{2}{17}$ sq. yd. 10. $31\frac{7}{18}$ sq. yd. 11. $40\frac{1}{18}$ sq. yd. 12. $49\frac{1}{17}$ sq. yd. 13. $14\frac{4}{9}$ sq. yd. 14. $13\frac{1}{17}$ sq. yd. 15. $20\frac{8}{11}$ sq. yd. 16. $28\frac{7}{8}$ sq. yd. 17. $39\frac{7}{11}$ sq. yd. 18. $56\frac{3}{17}$ sq. yd. 19. $19.25. 20. $26.95. 21. $26.25. 22. $28. 23. $30.80. 24. $35.35. 25. $44.10. 26. $50.40. 27. $59.85. 28. $71.75. 29. $16.80. 30. $41.65. 31. $52.80.

Page 21. — 1. .72 gal. 2. .896 gal. 3. 1.08 gal. 4. 1.28 gal. 5. 1.584 gal. 6. 1.748 gal. 7. 2.56 gal. 8. 5.4 gal. 9. 8.4 gal. 10. $56.80. 11. $14.40. 12. $18.67. 13. $28.80. 14. $59.12. 15. $100. 16. $25.56. 17. $31.67. 18. $38. 19. $48.67. 20. $60.78. 21. $110.22. 22. $1.08. 23. $1.35 24. $1.52. 25. $1.62. 26. $1.80. 27. $1.92. 28. $2.38. 29. $2.70. 30. $3.17.

Page 22. — 31. $23.04. 32. $23.04. 33. $25.60. 34. $36.87. 35. $72.54. 36. $249.60. 38. $61.61. 39. $199.22.

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Page 23. — **1.** 10 rolls. **2.** 10 rolls. **3.** Walls, 12 rolls. Ceiling, 8 rolls.

Page 24. — **4.** $4.50. **5.** $5.25. **6.** $14.40.

Page 25. — **1.** 20 yd. **2.** 30 yd. **3.** 24 yd. **4.** 21½ yd. **5.** 28½ yd. **6.** 40 yd. **7.** 20⅝ yd. **8.** 36⅛ yd. **9.** 38⅜ yd. **10.** 28 yd. **11.** 40 yd. **12.** 32 yd. **13.** 32 yd. **14.** 49 yd. **15.** 55 yd. **16.** 29⅝ yd. **17.** 53¼ yd. **18.** 65⅓ yd. **19.** $44.80. **20.** $42.67. **21.** $43.63. **22.** $57.63. **23.** $51.20. **24.** $69.34. **25.** $181.13. **26.** $90.75. **27.** $131.25. **28.** $176. **29.** $22.80. **30.** 31 yd. **31.** Lengthwise, $85.69.

Page 26. — **1.** $56. **2.** $105. **3.** $117.60. **4.** $252. **5.** $315. **6.** $560. **7.** $63. **8.** $141.75. **9.** $189. **10.** $52.50. **11.** $448. **12.** $412.42. **13.** $183.75. **14.** $436.80. **15.** $268.80. **16.** $793.80. **17.** $1066.67. **18.** $1280. **19.** $3333.34. **20.** $4000. **21.** $8400. **22.** $9000. **23.** $13,333.34. **24.** $24,000. **25.** $42,240. **26.** $140,800. **27.** $1600. **28.** $3110.40.

Page 27. — **29.** $768. **30.** $1228.80. **31.** $2048. **32.** $5120. **33.** $10,240. **34.** $67,584. **35.** $16,896. **36.** $15,840. **37.** $38,016. **38.** $42,240. **39.** $76,032. **40.** $84,480. **41.** $214.63. **42.** $1.75. **43.** 76 sq. yd. **44.** 325 sq. yd. **45.** $288.

Page 28. — **1.** 2 cords. **2.** 1½ cords. **3.** 3 cords. **4.** 2½ cords. **5.** 2½ cords. **6.** 3¾ cords. **7.** 3⅝ cords. **8.** 5⅛ cords. **9.** 7½ cords. **10.** 9$\frac{11}{16}$ cords. **11.** 10 cords. **12.** 18 cords. **13.** ½ cord. **14.** ⅜ cord. **15.** 1⅛ cords. **16.** 1$\frac{15}{16}$ cords. **17.** 1$\frac{11}{24}$ cords. **18.** 1½ cords. **19.** 1⅝ cords. **20.** 2$\frac{1}{12}$ cords. **21.** 3 cords. **22.** 4⅛ cords. **23.** 6¼ cords. **24.** 5 cords.

Page 29. — **25.** $12.66. **26.** 5¼ cords.
1. 28⅜ bu. **2.** 96 bu. **3.** 128 bu. **4.** 288 bu. **5.** 864 bu. **6.** 1728 bu. **7.** 140⅘ bu. **8.** 70⅕ bu. **9.** 105$\frac{7}{45}$ bu. **10.** 303$\frac{1}{17}$ bu. **11.** 360$\frac{8}{27}$ bu. **12.** 1159$\frac{2}{3}$ bu.

Page 30. — **13.** 312½ cu. ft. **14.** 375 cu. ft. **15.** 562½ cu. ft. **16.** 650 cu. ft. **17.** 812½ cu. ft. **18.** 1125 cu. ft. **19.** 1250 cu. ft. **20.** 1562½ cu. ft. **21.** 1875 cu. ft. **22.** 3125 cu. ft. **23.** 3750 cu. ft. **24.** 9000 cu. ft.

1. $57.60. **2.** $66.67. **3.** $84. **4.** $112.50. **5.** $108. **6.** $145.35. **7.** $190. **8.** $456. **9.** $162.56. **10.** $187.47. **11.** $201.88. **12.** $469.24. **13.** $7.50. **14.** $6.40. **15.** $13. **16.** $15. **17.** $1056. **18.** $1848. **19.** $352. **20.** $34.67. **21.** $52.74.

Page 32. — **1.** 7920 bricks. **2.** 7700 bricks. **3.** 11,880 bricks. **4.** 14,850 bricks. **5.** 17,600 bricks. **6.** 9900 bricks. **7.** 8800 bricks. **8.** 16,500 bricks. **9.** 17,600 bricks. **10.** 29,700 bricks. **11.** 2520 bricks. **12.** 3600 bricks. **13.** 3510 bricks. **14.** 4200 bricks. **15.** 6000 bricks. **16.** 7020 bricks. **17.** 8700 bricks. **18.** 9450 bricks. **19.** 19,947 bricks. **20.** 10,080 bricks. **21.** $68.64.

Page 33. — **1.** $300. **2.** $397.50. **3.** $406.67. **4.** $667.50. **5.** $229.17.

Page 34. — **6.** $205.56. **7.** $103.68. **8.** $14.89. **9.** $78.75. **10.** 66⅔ cu. yd. **11.** $211.86. **12.** 200; 400; 800. **13.** $325.37.

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Page 35. — 1. 8 perches. 2. 27 perches. 3. 30 perches. 4. 66 perches. 5. 44 perches. 6. 104 perches. 7. 67 perches. 8. 94 perches. 9. 99 perches. 10. 142 perches. 11. 97 perches. 12. 117 perches. 13. 10.24 perches. 14. $20.67.

Page 37. — 1. $20.85. 2. $100.32. 3. $84. 4. $316.50. 5. $16.40. 6. $47.40. 7. $63; $12.

Page 43. — 1. $630. 2. 45¢. 3. ½. 4. 40 yd. 5. 88 yd. 6. $233.50. 7. $6.75. 8. ½. 9. $20.

Page 44. — 10. $60. 11. $1800. 12. 13 lb. 13. $7.50. 14. $3.30. 15. $4. 16. $26.60. 17. $1.96 each; $4.92. 18. $22 gain. 19. $624 gain. 20. $\frac{5}{16}$, $\frac{5}{48}$.

Page 45. — 21. $1.50. 22. 2 doz. 23. 32¢. 24. 48½ lb. 25. 76½¢. 26. $17.50. 27. $3.84. 28. 15$\frac{1}{10}$¢. 29. $4.50. 30. $11.10. 31. $4.75.

Page 46. — 32. 10¢. 33. $680. 34. $9 cheaper to rent. 35. $293.60. 36. 56$\frac{2}{11}$ qt. 37. 24 cows. 38. 900 bu. 39. 25 A. 40. 1st, 50¢; 2d, 60¢. 41. $6800; $100 per A.

Page 47. — 42. 4 bbl. 43. 100 cu. ft. cement; 200 cu. ft. sand; 400 cu. ft. gravel. 44. 60 da. 45. $384. 46. 94 bu. 47. 40 cu. ft. cement; 80 cu. ft. sand; 160 cu. ft. gravel. 48. $99. 49. $31.95.

Page 51. — 1. 4. 2. 3. 3. 1. 4. 4. 5. 4. 6. 10. 7. 12. 8. 11. 9. 17. 10. 5. 11. 8. 12. 5. 13. 6. 14. 3. 15. 7. 16. ½. 17. ½. 18. $\frac{1}{8}$. 19. 3. 20. 2. 21. 3. 22. 5. 23. 4. 24. 2. 25. $\frac{2}{3}$. 26. $\frac{1}{5}$. 27. 3. 28. $\frac{1}{5}$.

Page 53. — 1. 16. 2. 16. 3. 12. 4. $25. 5. 30 cars. 6. $18. 7. 330 ft.

Page 54. — 8. 85. 9. 108. 10. 114. 11. 149. 12. 48¢. 13. 36¢. 14. $850. 15. 85 cans. 16. 16 yr. 17. 145 lb. 18. $56; $80.

Page 55. — 19. 25, 75. 20. 10, 50. 21. 6 yr., 18 yr. 22. 12 yr., 36 yr. 23. 60 lb., 180 lb. 24. 20, 40. 25. $3; $12. 26. Wagon, $50; Horse, $200.

Page 56. — 27. Lot, $500; House, $2500. 28. 5. 29. 15. 30. 15. 31. 10. 32. 7. 33. 12. 34. 10, 40. 35. $3. 36. 5 boys. 37. $40. 38. $5.

Page 57. — 39. $1200. 40. Harry, 20¢; Tom, 35¢; Jack, 45¢. 41. Jerry, 57½ lb.; his father, 192½ lb. 42. Kate, 4; Emily, 12; Mary, 36. 43. Dick, $10; Earl, $18. 44. Truck, 2450 lb.; coal, 4900 lb., or 2$\frac{3}{5}$ T. 45. By boat, 40 mi.; by rail, 120 mi. 46. Repairs, $25; gasoline, $150. 47. Food, $1000; rent, $500. 48. Sister, 6 yr.; boy, 12 yr.; father, 36 yr.

Page 62. — 1. 4, 5.6, 4.5, 11.25. 2. 4.8, 7.6, 11, 19.2. 3. 5.4, 9.81, 15, 27. 4. 6.3, 13, 21, 41.25. 5. $41.25. 6. $45. 7. $57.60. 8. $68. 9. $31.38. 10. $91.21. 11. $183.15. 12. $234.76. 13. $500. 14. $900. 15. $1612.50. 16. $3000. 17. $100, $200, $400, $800. 18. $187.50, $375, $562.50, $750. 19. $800, $1600, $2096. 20. $150, $200, $400, $2000. 21. $550, $625, $900. 22. $5, $250. 23. .1. 24. .15. 25. .05. 26. .1. 27. .1. 28. .9. 29. 4.5. 30. 2.5. 31. 1.6. 32. 1.5. 33. 4. 34. 4.

1. 6 boys. 2. $400. 3. $437.50. 4. $760.

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Page 63. — 5. \$1.12½. 6. \$3600. 7. \$24. 8. 1020 gal. 9. 180 men, 270 women, 90 children. 10. \$2800. 11. \$4884.08. 12. Real estate, \$8100; bonds, \$18,000; business, \$21,600; savings bank, \$6300. 13. \$480. 14. \$4500.

Page 65. — 1. 20 %, 33⅓ %, 25 %, 20 %. 2. 12½ %, 20 %, 50 %, 20 %. 3. 20 %, 26⅔ %, 40 %, 55⅝ %. 4. 40 %, 60 %, 80 %, 100 %. 5. 6¼ %, 8⅓ %, 5 %, 50 %. 6. 20 %, 20 %, 20 %, 20 %. 7. 33⅓ %, 6¼ %, 25 %, 1 %. 8. 5 %, 2 %, 2 %, .2 %. 9. 40 %, 60 %, 75 %, 30 %. 10. 48.1 %, 40 %, 28.6 %, 29.6 %. 11. 42.9 %, 41.3 %, 35.2 %, 46.3 %. 12. 28.1 %, 34.8 %, 25.7 %, 48.8 %. 13. 22.8 %, 30.5 %, 33.6 %, 36.7 %. 14. 28.2 %, 29.2 %, 36.6 %, 35 %. 15. 44 %, 42.7 %, 18.9 %, 19 %. 16. 31.9 %, 28.1 %, 21.5 %. 17. 6.5 %, 43.3 %, 7.9 %.

1. 60 %. 2. 83⅓ %. 3. 20 %. 4. 25 %.

Page 66. — 5. \$40; 25 %. 6. 20 %. 7. 75 %. 8. 85⅞ %. 9. 16⅔ %. 10. 14⅞ %. 11. House, \$5200; lot, \$2800. 12. 3.94 %. 13. 166⅔ %. 14. 5 %. 15. 65 %; 65 cents on the dollar. 16. 30 %.

Page 67. — 17. 360 mi. 18. 41⅔ % in 5 days; 25 % in 3 days; 58⅓ % in 7 days; 91⅔ % in 11 days. 19. 66⅔ %. 20. 16⅔ % to individuals; 33⅓ % to hotels; 50 % to grocers. 21. Lot, 25 % of house; house, 80 % of total. 22. 60 %. 23. 4.58⅓ %. 24. Crescents, 34, 70.58 %; Stars, 36, 69.44 %; Imperials, 33, 60.6 %; Orioles, 34, 76.47 %.

Page 69. — 1. 200, 150, 250, 250. 2. 300, 200, 166⅔, 200. 3. 200, 160, 280, 350. 4. 150, 300, 300, 384. 5. 125, 120, 180, 288. 6. 400, 225, 200, 160. 7. 200, 148 4/7, 200, 200. 8. 225, 200, 300, 208⅓. 9. 120, 208⅓, 200, 125. 10. 80, 200, 175, 137½. 11. 200, 200, 200, 175.

1. 60 boys. 2. 125 lb. 3. \$4000. 4. 300 miles. 5. 6. 6. 1400 bu.; 980 bu. 7. Cost. \$166.67; Profit, \$33.33; 20 %.

Page 70. — 8. \$1536; 37½ %. 9. 128; 80. 10. \$1500. 11. \$4000.

Page 71. — 1. 1.56. 2. 2.19. 3. 2.81. 4. 3.44. 5. 4.06. 6. 4.69. 7. 4.17. 8. 6.25. 9. 7.5. 10. 8.75. 11. 9.58. 12. 10.42. 13. 9.38. 14. 11.25. 15. 13.13. 16. 14.38. 17. 16.25. 18. 18.13. 19. 8.33. 20. 12.5. 21. 14.17. 22. 15. 23. 15.83. 24. 17.5. 25. 16.67. 26. 23.33. 27. 26.67. 28. 31.67. 29. 36.67. 30. 38.33. 31. 46.88. 32. 56.25. 33. 65.63. 34. 71.88. 35. 81.25. 36. 90.63. 37. 50. 38. 54.17. 39. 58.33. 40. 62.5. 41. 66.67. 42. 75. 43. 79.17. 44. 87.5. 45. 91.88. 46. 96.25. 47. 105. 48. 109.38. 49. 113.75. 50. 118.13. 51. 122.5. 52. \$52.50. 53. \$82.50. 54. \$103.50. 55. \$120. 56. \$48.60. 57. \$70.19. 58. \$78.13. 59. \$26.56.

Page 72. — 60. \$137.50. 61. \$156.25. 62. \$16.28. 63. \$22.28. 64. \$40.37. 65. \$39.94. 66. \$57.09. 67. \$172.56. 68. \$223.44. 69. \$264.06. 70. \$.25. 71. \$.37½. 72. \$.50. 73. \$2.00. 74. \$3.00. 75. \$3.77. 76. \$11.25. 77. \$22.50. 78. \$75.00. 79. \$200. 80. 10 %. 81. 11⅓ %. 82. 20 %. 83. 20 %. 84. 20 %. 85. 20 %. 86. 20.2+ %. 87. 20 %. 88. 20 %. 89. 30 %. 90. 25 %. 91. 13.8+ %. 92. 12½ %. 93. 10 %. 94. 10.8+ %. 95. 10.7+ %. 96. 12.2+ %. 97. 13 %. 98. 12½ %. 99. 15.6+ %. 100. 25 %. 101. 20 %. 102. 20 %. 103. 18 %. 104. 12½ %. 105. 10 %.

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106. 16⅔ %. **107.** 42.8+ %. **108.** 33⅓ %. **109.** 45.4+ %. **110.** 280. **111.** 325. **112.** 500. **113.** 480. **114.** 125. **115.** 62.4. **116.** 419. **117.** 563.12½. **118.** .6. **119.** .02. **120.** .312. **121.** .004. **122.** $15.625. **123.** $27.50. **124.** $28. **125.** $22.50. **126.** $96. **127.** $80. **128.** $93.75. **129.** $1350. **130.** $1125. **131.** $1000.

Page 73. — **1.** $20,000. **2.** 2 %. **3.** 46⅔ %. **4.** $322. **5.** 5 %. **6.** 25 % of cost price; 20 % of selling price. **7.** Marked price, $375; Selling price, $300; Cost price, $250. **8.** $2000.

Page 74. — **9.** $47.25; $.33⅓. **10.** 1250 A; 325 A. **11.** $.04½; $.05. **12.** 33⅓ %. **13.** $1687.50; 93¾ %. **14.** $3125. **15.** $57,000. **16.** $3570; $570 profit. **17.** 15$\frac{10}{17}$ %; 2$\frac{11}{17}$ %.

Page 75. — **18.** $889. **19.** 2100 bu. **20.** 20⅝ %. **21.** 18¢. **22.** 120 %. **23.** $142.95. **24.** 30.9+ %. **25.** 60 %, 25 %, 15 %. **26.** 120 lb., 50 lb., 30 lb.

Page 76. — **27.** $16. **28.** 60 lb. **29.** 148 lb. **30.** The latter. **31.** 94.5 lb. **32.** 39.6 lb. **33.** 30.41 lb. **34.** $173.25.

Page 79. — **1.** $4. **2.** $6. **3.** $7.80. **4.** $11.25. **5.** $18. **6.** $19. **7.** $12.50. **8.** $20⅝. **9.** $22.50. **10.** $40. **11.** $41.25. **12.** $87.50. **13.** $6.27½. **14.** $19.22. **15.** $25.25. **16.** $562.50. **17.** $3200. **18.** $8750. **19.** $115. **20.** $150. **21.** $150.80. **22.** $200. **23.** $228. **24.** $230. **25.** $220. **26.** $287.50. **27.** $337.50. **28.** $466.67. **29.** $540. **30.** $937.50. **31.** $274.96. **32.** $431.48. **33.** $523.19. **34.** $696.73. **35.** $9000. **36.** $17,000. **37.** $50. **38.** $90. **39.** $112.50. **40.** $180. **41.** $231.25. **42.** $292.50. **43.** $127.55. **44.** $306.34. **45.** $595.88. **46.** $685.13. **47.** $8583.33. **48.** $10,342.50. **49.** $1020. **50.** $1312.50. **51.** $1487.50. **52.** $2125. **53.** $3187.50. **54.** $3400. **55.** $1125.45. **56.** $1728.56. **57.** $1881.95. **58.** $3123.92. **59.** $4666.93. **60.** $5210,63.

Page 80. — **1.** $750. **2.** $1041⅔. **3.** $1666⅔. **4.** $4000. **5.** $3750. **6.** $600. **7.** $600. **8.** $1000. **9.** $1000. **10.** $1500.

Page 82. — **1.** 33⅓ %. **2.** 20 %. **3.** 10 %. **4.** 15 %. **5.** 33⅓ %. **6.** 33⅓ %. **7.** 10 %. **8.** 20 %. **9.** 11⅓ %. **10.** 16⅔ %. **11.** 25 %. **12.** 16⅔ %.

1. rul. **2.** rel. **3.** ril. **4.** rml. **5.** ram. **6.** dum. **7.** dml. **8.** rmlx. **9.** rtnm. **10.** uxml. **11.** umtx. **12.** dulx. **13.** muml. **14.** etnm.

Page 83. — **1.** $360. **2.** 11½ %. **3.** $2400. **4.** $.60. **5.** $700. **6.** $43.70. **7.** $8 loss; 4 %. **8.** $27.20. **9.** $4.80. **10.** 27⅔ %. **11.** $.54. **12.** $60.

Page 84. — **13.** $5. **14.** 8⅓ %. **15.** $86.25. **16.** $5 loss. **17.** 100 %, $2.25. **18.** $2100 gain; 20 %. **19.** $3750, $5625, $375 loss.

Page 86. — **1.** $128.21. **2.** $138.89. **3.** $198.41. **4.** $375. **5.** $1000. **6.** $1855.65. **7.** $2977.62. **8.** $3525.64. **9.** $14,486.98. **10.** $37,692.31. **11.** $2131.94. **12.** $4369.09. **13.** $5769.23.

Page 88. — $15. **2.** $12.50. **3.** $30. **4.** $37.50. **5.** $78. **6.** $150. **7.** $27.55. **8.** $44.28. **9.** $62.54. **10.** $155.30. **11.** $157.73. **12.** $258.47. **13.** $24.75. **14.** $39.01. **15.** $84.66. **16.** $92.08.

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17. $131.90. 18. $203.03. 19. $47.69. 20. $64.51. 21. $88.40.
22. $187.87. 23. $235.56. 24. $375.27.
1. $4.80. 2. $19.20. 3. 20%. 4. 20%. 5. $405. 6. $152.

Page 89. — 7. $135. 8. 10%. 9. $2.08. 10. $446.39. 11. $28.
12. $80. 13. $918.75. 14. $266.67. 15. $1460. 16. No profit.

Page 90. — 17. $.05 per cake, 100% gain. 18. $2.75. 19. $40.
20. $800. 21. $112.50. 22. $3375, $9618.75. 23. $140. 24. $2460.

Page 91. — 1. $64.13. 2. $76.95. 3. $72. 4. $85. 5. $90.
6. $254.36. 7. $314.64. 8. $402.04. 9. $575.91. 10. $513.

Page 92. — 1. $432. 2. $146.88. 3. 38⅛%. 4. $4.61. 5. 1st $.22½ better. 6. $69.90. 7. The same. 8. 25%. 9. 2d $6.39 better.
10. $81. 11. Agent's gain, $26.60; Amount paid, $216.60.

Page 94. — 1. $477.94. 2. $746.90. 3. $3420. 4. $686.35.
5. $692.52.

Page 96. — 1. $200. 2. $216. 3. $393.75. 4. $18.83. 5. $45.15.
6. $98.77. 7. $105. 8. $136.50. 9. $406. 10. $22.63. 11. $413.50.
12. $149.17. 13. $30.50.

Page 97. — 14. $83.63. 15. $142.20. 16. $88.09. 17. $248.95.
18. $252.50. 19. $547.92.

Page 98. — 1. $22.05. 2. $37.80. 3. $65.07. 4. $36. 5. $74.41.
6. $80.96. 7. $136.86. 8. $189.99. 9. $241.53. 10. $239.41.
11. $810.94. 12. $90.68. 13. $17.50. 14. $50.63. 15. $65.50.
16. $18.73. 17. $62.49. 18. $136.05. 19. $289.85.

Page 99. — 20. $170. 21. $144.50. 22. $1087.20.

Page 100. — 1. $137.30. 2. $171.08. 3. $227.24. 4. $282.14.
5. $300.40. 6. $361.40. 7. $428.08. 8. $497.46. 9. $272.75.
10. $285.97. 11. $383.42. 12. $478.92. 13. $373.52. 14. $448.49.
15. $499.46. 16. $632.56. 17. $48.72. 18. $54.36. 19. $51.57.
20. $63.90. 21. $110.55. 22. $227.70. 23. $399.74. 24. $468.59.
25. $99.17. 26. $333.11. 27. $306.14. 28. $459.15. 29. $679.38.
30. $561.47. 31. $695.61. 32. $816.93.

Page 101. — 1. $1.32. 2. $1.56. 3. $3.05. 4. $3.25. 5. $4.02.
6. $5.72. 7. $6.01. 8. $6.70. 9. $12.43. 10. $23.59. 11. $2.55.
12. $7.51. 13. $5.26. 14. $3.32. 15. $7.11. 16. $9.32. 17. $6.56.
18. $8.02. 19. $8.79. 20. $10.91. 21. $17.92. 22. $28.62.
23. $37.83. 24. $12.33. 25. $20.68. 26. $45.04. 27. $32.36.
28. $61.29.

Page 102. — 1. $2.20. 2. $3.27. 3. $8.00. 4. $13.05. 5. $19.79.
6. $16.13. 7. $42.24. 8. $39.05. 9. $39.50. 10. $47.58.
11. $77.81. 12. $13.93. 13. $15.63. 14. $54.08. 15. $103.42.

Page 104. — 1. $18. 2. $49.50. 3. $72. 4. $110. 5. $210.
6. $69.75. 7. $106.77. 8. $166.25. 9. $169.13. 10. $181.25.
11. $149.72. 12. $494.40. 13. $729.53. 14. $784.74. 15. $1247.38.
16. $18. 17. $26.33. 18. $251.67. 19. $15. 20. $1048.06.

Page 106. — 1. $208.08. 2. $364.20. 3. $406.04. 4. $624.18.

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Page 108. — 1. $35. 2. $148.75. 3. $32. 4. $817.50. 5. $106,232,939.75. 6. $674.49. 7. $474,536,463.63.

Page 111. — 1. $458.40. 2. $604.42. 3. $1096.20. 4. $1072.17. 5. $2436.90. 6. $6374.97. 7. $1180.30.

Page 113. — 1. $442.75. 2. $1296.26.

Page 114. — 3. $3600.33. 4. $8018.30. 5. Mon. $2670, Tues. $3236.03, Wed. $1790.93, Thurs. $2885.26. 6. $1328.71. 7. $1273.87.

Page 115. — 1. $1.50, $298.50. 2. $3.50, $346.50. 3. $4.50, $445.50. 4. $7.19, $567.81. 5. $7.50, $742.50. 6. $5, $995. 7. $12.50, $1487.50. 8. $33.75, $2216.25. 9. $31.25, $2468.75. 10. $30, $2970.

Page 116. — 5. $2479.17. 6. $2975. 7. $4458.75.

Page 117.* — 1. April 30, 60 days, $9.86, $990.14. 2. June 19, 60 days, $9.04, $1090.96. 3. June 8, 60 days, $.83, $100.65. 4. March 2, 60 days, $8.29, $999.93.

Page 118.* — 5. April 9, 24 days, $3.96, $1200.97. 6. 60 days, $14.98, $1503.51. 7. 60 days, $16.68, $2012.91. 8. July 31, 46 days, $7.62, $1202.24. 9. $250.91. 10. $452.55.

Page 119. — 1. $300. 2. $1500. 3. $500. 4. $651.64. 5. $2030.46. 6. $2016.81. 7. $1224.49. 8. $20,227.56. 9. $3553.29.

Page 121. — 1. $76.41. 2. $78.81. 3. $101.92. 4. $338.23. 5. $393.71. 6. $47.19. 7. $70.77. 8. $102.93. 9. $81.68. 10. $155.36. 11. $92.73. 12. $465.34.

Page 123. — 1. $272.10. 2. $345.27. 3. $462.28. 4. $349.15. 5. $435.94.

Page 124. — 6. $333.09. 7. $529.75. 8. $504.03. 9. $638.76. 10. $50.07. 11. $87.72. 12. $170.63. 13. $114.75.

Page 125. — 1. $533.50. 2. $831.44. 3. $764.70.

Page 127. — 1. $48,000,000. 2. 40,000 shares. 3. $1,320,000 of stock.

Page 128. — 4. $4,584,000. 5. $200,000. 6. $6,894,994; 58+ %.

Page 130. — 1. $.06 difference. 2. $.65. 3. $.08 difference. 4. $1.52, $.24.

Page 136. — 1. $2000.40. 2. $1999.60. 3. $3000.90. 4. $4997.50. 5. $4003. 6. $1005. 7. $1488.75. 8. $1995. 9. $3496.50. 10. $5991. 11. $4488.75.

Page 137. — 12. $3022.50. 13. $4003.50. 14. $3998. 15. $4477.50. 16. $1.50. 17. $10,005. 18. $9514.68. 19. $10,630.25. 20. $8,258.81, $41.19.

Page 139. — 1. $\frac{1}{2}$, 2, $\frac{1}{2}$, 2, $\frac{1}{2}$. 2. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{2}{3}$. 3. 2, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$. 4. 4, $\frac{10}{3}$, 4, $\frac{5}{3}$, $\frac{2}{3}$. 5. $\frac{2}{3}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{11}{12}$, $\frac{1}{3}$. 6. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{16}{11}$. 7. $\frac{4}{5}$, $\frac{4}{5}$, $\frac{12}{15}$, $\frac{4}{5}$, $\frac{4}{5}$. 8. $\frac{4}{7}$, $\frac{4}{7}$, $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{5}$. 9. $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{10}$, 20, $\frac{1}{10}$.

* Interest calculated by the Exact Method (p. 100).

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Page 140. — **1.** 9, 27. **2.** 20, 25. **3.** 35, 40. **4.** 40, 70. **5.** 42, 98 **6.** $51\frac{9}{16}$, $113\frac{7}{16}$. **7.** 36, 114. **8.** 70, 130. **9.** 64, 144. **10.** $93\frac{8}{17}$, $170\frac{1}{17}$. **11.** $111\frac{2}{3}$, $212\frac{8}{9}$. **12.** $146\frac{14}{17}$, $353\frac{3}{17}$.
1. 3 to 2. **2.** 5 to 1. **3.** 7 to 27.

Page 141. — **4.** 525 lb. **5.** $1562\frac{1}{2}$ lb. **6.** \$18; \$42. **7.** $131\frac{1}{11}$ lb. **8.** \$2000, \$2800. **9.** Child, \$10,714.29; widow, \$26,785.71. **10.** \$18, \$12.

Page 143. — **1.** 3. **2.** 2. **3.** 15. **4.** 27. **5.** 8. **6.** $13\frac{1}{3}$. **7.** 10. **8.** 1. **9.** $\frac{15}{16}$. **10.** 7. **11.** $\frac{1}{2}$. **12.** .04. **13.** .02. **14.** 3.

Page 145. — **1.** \$27. **2.** \$35. **3.** \$60. **4.** \$160. **5.** \$38.40. **6.** \$135. **7.** \$159.50. **8.** \$140. **9.** \$157.50. **10.** \$637.50. **11.** \$419.75. **12.** \$680.40. **13.** \$27.30. **14.** \$528. **15.** 40 mi. **16.** \$10.56. **17.** \$6.57. **18.** 38 lb.

Page 146. — **19.** 11,000. **20.** \$3.20. **21.** \$86.40. **22.** $5\frac{1}{3}$ days. **23.** $37\frac{1}{2}$ lb., 27 lb. **24.** 8 days. **25.** 6 days. **26.** 39 days. **27.** 48 women. **28.** 108 days.

Page 147. — **29.** $6\frac{2}{3}$ days. **30.** $27\frac{7}{9}$ days. **31.** 15 days. **32.** 60 more. **33.** $117\frac{5}{9}$ days. **34.** 80 days. **35.** 3 men. **36.** 38 men. **37.** 54 men.

Page 150. — **1.** 676. **2.** 1225. **3.** 1764. **4.** 2916. **5.** 4225. **6.** 225. **7.** 441. **8.** 625. **9.** 900. **10.** 2025. **11.** $\frac{1}{4}$. **12.** $\frac{4}{9}$. **13.** $\frac{81}{64}$. **14.** $\frac{25}{144}$. **15.** $\frac{289}{576}$. **16.** 6.25. **17.** 9.9225. **18.** 64.481201. **19.** 4.62225. **20.** 34.328125. **21.** 225 sq. in. **22.** 576 sq. in. **23.** 900 sq. ft. **24.** 1225 sq. ft. **25.** 1764 sq. ft. **26.** 2500 sq. ft. **27.** 2500 sq. yd. **28.** 2500 sq. rd. **29.** 4225 sq. yd. **30.** 5625 sq. rd. **31.** 110.25 sq. ft. **32.** 232.5625 sq. ft. **33.** $348\frac{4}{9}$ sq. ft. **34.** 420.25 sq. ft. **35.** 588.0625 sq. ft. **36.** $106\frac{7}{9}$ sq. yd. **37.** $160\frac{5}{9}$ sq. yd. **38.** $235\frac{1}{9}$ sq. yd. **39.** $348\frac{4}{9}$ sq. yd. **40.** $427\frac{1}{9}$ sq. yd. **41.** 216 cu. in. **42.** 1000 cu. in. **43.** 4096 cu. in. **44.** 729 cu. ft. **45.** 1728 cu. ft. **46.** 8000 cu. ft. **47.** $3\frac{3}{8}$ cu. ft. **48.** 1.953125 cu. ft. **49.** $15\frac{5}{8}$ cu. ft. **50.** $42\frac{7}{8}$ cu. ft. **51.** $190\frac{7}{64}$ cu. ft. **52.** $614\frac{1}{8}$ cu. ft.

Page 151. — **1.** 15. **2.** 18. **3.** 24. **4.** 27. **5.** 28. **6.** 31. **7.** 32. **8.** 33. **9.** 35. **10.** 36. **11.** 42. **12.** 45. **13.** 48. **14.** 50. **15.** 56. **16.** 64. **17.** 22 ft. **18.** 26 ft. **19.** 30 ft. **20.** 31 ft. **21.** 81 ft. **22.** 84 ft. **23.** $907\frac{1}{2}$ ft. **24.** 1056 ft.

Page 154. — **1.** 34. **2.** 35. **3.** 43. **4.** 39. **5.** 44. **6.** 42. **7.** 45. **8.** 51. **9.** 54. **10.** 41. **11.** 56. **12.** 59. **13.** 57. **14.** 66. **15.** 64. **16.** 69. **17.** 74. **18.** 72. **19.** 79. **20.** 88. **21.** 86. **22.** 93. **23.** 97. **24.** 96. **25.** 98.

Page 155. — **26.** 121. **27.** 135. **28.** 127. **29.** 124. **30.** 138. **31.** 151. **32.** 147. **33.** 158. **34.** 175. **35.** 172. **36.** 181. **37.** 196. **38.** 203. **39.** 208. **40.** 302. **41.** 305. **42.** 309. **43.** 321. **44.** 353. **45.** 388. **46.** 486. **47.** 577. **48.** 647. **49.** 773. **50.** 888. **51.** 899. **52.** 979. **53.** 5210. **54.** 5970. **55.** 5121. **56.** 8005. **57.** 9015.

Page 157. — **1.** 5.14. **2.** 51.4. **3.** .514. **4.** 54.7. **5.** .596. **6.** .036. **7.** .027. **8.** 54.8 **9.** .0847. **10.** .0181. **11.** 70.31. **12.** 9.807. **13.** .0177. **14.** .0057. **15.** 500.3. **16.** 80.02. **17.** 12.247+. **18.** 18.027+. **19.** 23.874+. **20.** 28.124+. **21.** 34.641+. **22.** 132.287+. **23.** 188.175+. **24.** 255.180+. **25.** 352.641+. **26.** 456.146+. **27.** 12.414+. **28.** 58.794+.

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29. .433+. **30.** .013+. **31.** .715+. **32.** 25.988+. **33.** .816+. **34.** .774+. **35.** .845+. **36.** .935+. **37.** .957+. **38.** .888+. **39.** .887+. **40.** .850+.

Page 158. — **1.** 68 ft. **2.** 24 in. **3.** 208.710+ ft. **4.** 1669.68+ ft. **5.** 60 ft. **6.** ½ hr. **7.** $625. **8.** 240 rd. **9.** $4800. **10.** 122.474+ ft. **11.** 31 boys.

Page 160. — **1.** 20 in. **2.** 25 in. **3.** 30 ft. **4.** 40 ft. **5.** 80 ft. **6.** 13 ft. **7.** 26 rd. **8.** 18.439+ ft. **9.** 33.54+ ft. **10.** 39.051+ ft. **11.** 90.138+ ft. **12.** 109.658+ ft. **13.** 134.5+ ft. **14.** 178.044+ ft.

Page 162. — **1.** 14.142+ ft. **2.** 21.213+ ft. **3.** 42.426+ ft. **4.** 70.71+ ft. **5.** 56.568+ rd. **6.** 91.923+ rd. **7.** 127.278+ yd. **8.** 141.421+ yd. **9.** 18.027+ ft. **10.** 23.323+ ft. **11.** 32.311+ yd. **12.** 42.720+ yd. **13.** 58.309+ rd. **14.** 119.268+ rd.

1. 7.93+ ft. **2.** 13.22+ ft. **3.** 33.54+ ft. **4.** 11.18+ yd. **5.** 13.266+ rd. **6.** 27 yd. **7.** 45 ft. **8.** 41.23 ft. **9.** 80 rd.

Page 163. — **1.** 15 ft., 10.816+ ft. **3.** Boy, 112.5 rd.; father, 157.5 rd. **4.** $528. **5.** 196.977+ ft. **6.** 31.622+ ft.

Page 164. — **7.** 12 ft. **8.** 240.865+ miles. **9.** 17 ft. 2 in., approx. **10.** 15 ft. **11.** 101.935+ ft.

Page 166. — **1.** 47.124 in. **2.** 62.832 in. **3.** 131.9472 in. **4.** 106.8144 ft. **5.** 84.8232 rd. **6.** 8.48232 in. **7.** 11.62392 ft. **8.** 18.22128 ft. **9.** 28.90272 yd. **10.** 26.7036 rd. **11.** 67.23024 ft. **12.** 42.4116 yd. **13.** 64.4028 yd. **14.** 98.64624 rd. **15.** 16.4934 rd. **16.** 16.084992 yd. **17.** 9.676128 rd. **18.** 5.4978 yd. **19.** 159.90744 rd. **20.** 472.8108 yd. **21.** 31.416 ft. **22.** 47.124 ft. **23.** 58.1196 ft. **24.** 65.9736 ft. **25.** 21.9912 ft. **26.** 30.3688 ft. **27.** 23.0384 yd. **28.** 34.8432 rd. **29.** 98.9604 yd. **30.** 790.1124 yd. **31.** 823.41336 rd. **32.** 1573.9416 ft. **33.** 44 in. **34.** 88 in. **35.** 110 in. **36.** 133.5⅓ in. **37.** 78.8⅓ ft. **38.** 118.8 yd. **39.** 446⅔ ft. **40.** 795⅓ ft. **41.** 143 yd. **42.** 1.364 ft. **43.** 3.26⅔ yd. **44.** 35.89⅓ rd.

Page 168. — **1.** 154 sq. in. **2.** 314⅔ sq. in. **3.** 616 sq. in. **4.** 1386 sq. in. **5.** 2828⅔ sq. in. **6.** 186.34 sq. in. **7.** 98.56 sq. in. **8.** 58.11⅓ sq. in. **9.** 51,492⅔ sq. in. **10.** 69,774⅔ sq. in. **11.** .0616 sq. ft. **12.** 24.64 sq. ft. **13.** 260.26 sq. ft. **14.** 679.14 sq. in. **15.** 138,600 sq. in. **16.** 19.64⅔ sq. ft. **17.** 88.28⅔ sq. ft. **18.** 116.94⅔ sq. ft. **19.** 211.32⅔ sq. ft. **20.** 301.84 sq. ft. **21.** 452.3904 sq. in. **22.** 804.2496 sq. in. **23.** 2463.0144 sq. in. **24.** 3216.9984 sq. in. **25.** 3019.0776 sq. ft. **26.** 55.417824 sq. ft. **27.** 81.713016 sq. ft. **28.** 167.415864 sq. ft. **29.** 113.0976 sq. in. **30.** 254.4696 sq. in. **31.** 153.9384 sq. ft. **32.** 283.5294 sq. ft. **33.** 47.783736 sq. ft. **34.** 25.517646 sq. ft. **35.** 2922.4734 sq. yd. **36.** 55.417824 sq. yd. **37.** 176.715 sq. in. **38.** 78.54 sq. in. **39.** 13.854456 sq. ft. **40.** 1590.435 sq. ft. **41.** 2.18 sq. ft. **42.** 245.0448 sq. ft.

Page 169. — **1.** $26.45. **2.** $55,000. **3.** $44,077.13. **4.** $377.73. **5.** $1441.98.

Page 171. — **1.** 80 cu. in. **2.** 120 cu. in. **3.** 144 cu. in. **4.** 128 cu. in. **5.** 720 cu. in. **6.** 1680 cu. in. **7.** 1584 cu. in. **8.** 450 cu. in. **9.** 168 cu. in. **10.** 465.75 cu. in. **11.** 33⅓ cu. ft. **12.** 49 1/12 cu. ft. **13.** 69⅓ cu. ft.

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14. 113.72 cu. ft. 15. 1099.56 cu. in. 16. 4071.5136 cu. in. 17. 1206.3744 cu. in. 18. 150.7968 cu. ft. 19. 565.488 cu. ft. 20. 622.0368 cu. it. 21. 19.5477 cu. ft. 22. 67.593 cu. ft. 23. 141.4883 cu. ft. 24. 224.002625 cu. ft. 25. 530.145 cu. ft. 26. 1090.626075 cu. ft.

Page 172. — 1. 1570.8 cu. ft. 2. 1884.96 cu. ft. 3. 2714.3424 cu. ft. 4. 3392.9280 cu. ft. 5. 4241.16 cu. ft. 6. 6031.872 cu. ft.

Page 173. — 7. 72 T. 8. 90 T. 9. 120 T. 10. 150 T. 11. 210 T. 12. 270 T. 13. 300 T. 14. 390 T. 15. 166+ days. 16. 150+ days. 17. 176+ days. 18. 147+ days. 19. 129+ days. 20. 149.4 T.

Page 174. — 1. 2272 cu. in. 2. 4389 cu. in. 3. 8085 cu. in. 4. 53,760.5 cu. in. 5. 68,813.4 cu. in. 6. 96,768.9 cu. in. 7. 189 cu. ft. 8. 268.8 cu. ft. 9. 504 cu. ft. 10. 144 cu. ft. 11. 22 cu. ft. 12. 28 cu. ft. 13. 337.5 gal. 14. 525 gal. 15. 682.5 gal. 16. 5 gal. 17. 6 gal. 18. 70 gal. 19. 937.5 lb. 20. 125 lb. 21. 3150 lb. 22. 128.5 bu. 23. 120.5 bu. 24. 241.06 bu. 25. 462.8 bu. 26. 674.9 bu. 27. 1542.7 bu. 28. 900 gal. 29. 1800 gal. 30. 4320 gal. 31. 2880 gal. 32. 10,080 gal. 33. 13,200 gal. 34. 45.7 bbl. 35. 83.3 bbl. 36. 120 bbl. 37. 160 bbl. 38. 411.4 bbl. 39. 1097.1 bbl. 40. 14.93+ cu. ft. 41. 34.84+ cu. ft. 42. 56.0+ cu. ft. 43. 80.88 cu. ft. 44. 89.60+ cu. ft. 45. 124.44+ cu. ft. 46. 149.33+ cu. ft. 47. 311.11+ cu. ft. 48. 448.0+ cu. ft. 49. 653.33+ cu. ft. 50. 6972.6 lb. 51. 14849.681+ lb. 52. 70,000 lb. 53. 140,683.59 lb. 54. 318,087 gal. 55. 659.736 gal. 56. 1178.1 gal. 57. 848.23 gal. 58. 7068.6 gal. 59. 11,875.248 gal. 60. 28,627.83 gal. 61. 37,699.2 gal.

Page 178. — 2. $75.96.

Page 179. — 3. $18.96, $29.44, $9.44.

Page 181. — 1. $5.60. 2. $8.95. 3. $11.20. 4. $9.24. 5. $141.75. 6. $19.59. 7. $3.70. 8. $109.55. 9. $125.35. 10. $157.51. 12. $340.95.

Page 184. — 1. $.14. 2. $.55. 3. $.16 less. 4. $2.78.

Page 186. — 1. $16. 2. $27. 3. $26.67. 4. $31.25. 5. $78.75. 6. $168.75. 7. $125. 8. $300. 9. $500. 10. $962.50. 11. $1000. 12. $980. 13. $67.50. 14. $208.25. 15. $465. 16. $500. 17. $480. 18. $875. 19. $32. 20. $54. 21. $70.50. 22. $86.40. 23. $110. 24. $140. 25. $300. 26. $324. 27. $220.50. 28. $275. 29. $360. 30. $843.75.

Page 187. — 1. $18. 2. $15. 3. $11.25. 4. 33⅓%. 5. $12,000. 6. $12. 7. $20. 8. $13,500. 9. $517.50.

Page 190. — 1. $40.28. 2. $144.09. 3. $114.25. 4. $240.56. 5. $487.10. 6. $180.85. 7. $95.08. 8. $92.82. 9. $68.55. 10. $146.13. 11. $456.90. 12. $605.10. 13. $464.10. 14. $536.40.

Page 191. — 15. $640.38. 16. $1275.20. 17. $102.40, $2452.40.

Page 193. — 1. $3.60. 2. $6.30. 3. $9.60. 4. $14. 5. $12. 6. $6.25. 7. $11.25. 8. $30. 9. $48.75. 10. $90.

Page 194. — 1. 1½ mills. 2. 2 mills. 3. 2¼ mills. 4. 3 mills. 5. 3 mills. 6. 2½ mills. 7. 3¾ mills. 8. 4⅛ mills. 9. 4 mills. 10. 3$\frac{A}{10}$ mills.

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1. $4000. 2. $5000. 3. $10,000. 4. $7500. 5. $4500.
6. $50,000. 7. $50,000. 8. $45,800. 9. $42,763.16. 10. $53,450.

Page 195. — 1. $1.24. 2. $1.37. 3. $1.42. 4. $1.56. 5. $1.63.
6. $1.98. 7. $2.40. 8. $3.12. 9. $3.47. 10. $3.96. 11. $5.78.
12. $6.44. 13. $7.02. 14. $7.59. 15. $7.97. 16. $9.24.
17. $10.32. 18. $12.38. 19. $14.40. 20. $15.60.

Page 196. — 1. 2½ mills. 2. 3½ mills.

Page 197. — 3. 4½ mills. 4. 4 mills. 5. $60, $69.75. 6. 13.21 mills, $165.13 7. $115.87. 9. $94.75.

Page 198. — 10. $4750.

Page 199. — 1. $78. 2. $115 gain.

Page 204. — 1. $4515. 2. $2415. 3. $19,030. 4. $30,040.
5. $16,530. 6. $29,537.50. 7. $5965. 8. $22,480. 9. $17,370.
10. $43,893.75. 11. $32,910. 12. $30,495. 13. $7350. 14. $45,885.
15. $103,635. 16. $16,808.75. 17. $21,096.25. 18. $84,780.
19. $29,936.25. 20. $34,012.50. 21. $63,840. 22. $44,775.
23. $181,575. 24. $142,050.

Page 205. — 1. $8085. 2. $16,680. 3. $1835. 4. $18,480.
5. $6145. 6. $7020. 7. $10,252.50. 8. $9270. 9. $5868.75.
10. $30,060. 11. $24,480. 12. $25,322.50. 13. $56,020. 14. $6290.
15. $6550. 16. $19,057.50. 17. $43,917.50. 18. $60,660.
19. $74,445. 20. $47,380. 21. $52,100. 22. $160,200. 23. $348,300.
24. $375,437.50.

Page 206. — 1. $241.50. 2. $1728. 3. $4513.50. 4. $1382.50.
5. $7848.75. 6. $5662. 7. $14,570. 8. $11,265. 9. $14,370.
10. $10,802.50. 11. $1440. 12. $22,511.25. 13. $30,440. 14. $857.
15. $3619. 16. $7502.50. 17. $8388.75. 18. $267.50. 19. $10,435.
20. $9047.50. 21. $5470. 22. $44,902.50. 23. $45,237.50.
24. $70,475. 25. $80,600. 26. $76,650.

Page 207. — 1. $660 gain. 2. $728.75 gain. 3. $6747.50 loss.
4. $145 gain. 5. $1320 gain. 6. $6335 loss. 7. $1830 loss.
8. $4405 loss. 9. $3397.50 gain. 10. $885 gain. 11. $3925 gain.
12. $2840 loss. 13. $13,117.50 gain. 14. $14,700 gain. 15. $8575 gain.
16. $38,950 gain.

Page 209. — 1. 49, $119.77 uninvested. 2. 2207, $13.70 uninvested.
3. 99, $60.27 uninvested. 4. 199, $80.65 uninvested. 5. 400, $40 uninvested. 6. 281, $15.35 uninvested. 7. 84, $97.20 uninvested.
8. 174, $75.40 uninvested. 9. 232, $15.20 uninvested. 10. 176, $85.60 uninvested. 11. 201, $40.72 uninvested. 12. 199, $73.52 uninvested.
13. 101, $9.72 uninvested.

Page 210. — 1. 6¼ %. 2. 5 %. 3. 4 %. 4. 4⅘ %. 5. 5⅓ %.
6. 5⅓ %. 7. 5 %. 8. 5 %. 9. 4.1 %. 10. 4.93+ %. 11. 5 %.
12. 5⅛ %. 13. 4.73+ %. 14. 4.32+ %. 15. 3.69+ %.

Page 211. — 16. First, .24 %. 17. The same. 18. $36 gain.
19. $395 gain. 20. $1850; 1st; 1st, 3.07+ % more. 21. $400, 7.97+ %.
22. $138.75, .3 %. 23. 1st, $170.

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Page 215.

	Rent	Food	Clothing	Misc.	Saving
1.	$240	$420	$180	$300	$ 60
2.	300	525	225	375	75
3.	320	560	240	400	80
4	360	540	270	450	180
5.	260	455	195	325	65
6.	280	490	210	350	70
7.	320	560	240	400	80
8.	340	595	255	425	85
9.	400	600	300	500	200
10.	440	660	330	550	220
11.	480	720	360	480	360
12.	500	750	375	500	375
13.	540	810	405	540	405
14.	560	840	420	560	420
15.	600	750	600	450	600
16.	700	875	700	525	700

Page 216. — 1. $310. 2. $295.

Page 217. — 3. $80. 4. $50. 5. $221.
1. $240. 2. $9.60. 3. $12 (walls only).

Page 218. — 4. $33.60. 5. $55.51. 6. $175.30. 7. $247.26. 8. $110.70.

Page 220. — 13. $2.31. 14. $.21. 15. $2.96, $2.08, $1.76, $.72, $1.04, $8.56.

Page 221. — 1. 75 watts. 2. 120 watts. 3. 450 watts. 4. 1125 watts. 5. 980 watts. 6. 1095 watts. 7. $1.08. 8. $3.29. 9. $2.17.

Page 222. — 1. $9.90. 2. $33\frac{1}{3}$ lb. 3. $25\frac{1}{2}$ bu. 4. 3 : 20. 5. $46.80. 6. Gas stove. 7. $16.39; $27\frac{81}{100}¢$ (cost of can included).

Page 223. — 1. $.90. 2. $1.75. 3. $4.68. 4. $.30. 5. $.33\frac{1}{3}$. 6. $8.25. 7. $.20.

Page 224. — 8. $3.04.
1. $4\frac{1}{2}¢$. 2. $.20. 3. Second. 4. $4.65, $.58\frac{1}{3}$.

Page 225. — 5. $.45\frac{1}{2}$, $.08. 6. $1.70. 7. $2.35.

Page 233. — 1. 35 sq. in. 2. 120 sq. in. 3. 180 sq. in. 4. 320 sq. in. 5. 378 sq. in. 6. $8\frac{1}{4}$ sq. ft. 7. 14 sq. ft. 8. $18\frac{1}{3}$ sq. ft. 9. $36\frac{1}{8}$ sq. ft. 10. $75\frac{3}{4}$ sq. ft. 11. 75 sq. ft. 12. 92 sq. ft. 13. 84 sq. ft. 14. $106\frac{1}{8}$ sq. ft. 15. $202\frac{3}{16}$ sq. ft. 16. 280 sq. in. 17. 540 sq. in. 18. 864 sq. in. 19. 1200 sq. in. 20. $11\frac{1}{4}$ sq. ft. 21. $19\frac{1}{4}$ sq. ft. 22. 54 sq. ft. 23. $181\frac{1}{2}$ sq. ft. 24. 248 sq. ft. 25. 452 sq. ft. 26. 630 sq. ft. 27. $1166\frac{2}{3}$ sq. ft. 28. $10\frac{1}{4}$ sq. ft. 29. $16\frac{1}{4}$ sq. ft. 30. 16 sq. ft. 31. $56\frac{3}{8}$ sq. ft. 32. 109 sq. ft. 33. $72\frac{1}{8}$ sq. ft. 34. 97 sq. ft. 35. $61\frac{7}{8}$ sq. ft. 36. $90\frac{1}{16}$ sq. ft. 37. $94\frac{17}{24}$ sq. ft. 38. $262\frac{1}{6}$ sq. ft. 39. $315\frac{13}{72}$ sq. ft. 40. $380\frac{1}{24}$ sq. ft. 41. $354\frac{13}{72}$ sq. ft.

Page 234. — 1. 50 cu. in. 2. 105 cu. in. 3. 144 cu. in. 4. 240 cu. in. 5. 450 cu. in. 6. 1152 cu. in. 7. 1944 cu. in. 8. 9072 cu. in.

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9. 160 cu. ft. **10.** 216 cu. ft. **11.** 300 cu. ft. **12.** 432 cu. ft.
13. 1200 cu. in. **14.** 4050 cu. in. **15.** 6144 cu. in. **16.** 8100 cu. in.
17. 20 cu. ft. **18.** 31½ cu. ft. **19.** 33¾ cu. ft. **20.** 96 cu. ft. **21.** 85¾ cu. ft.
22. 385⅓ cu. ft. **23.** 601⅜ cu. ft. **24.** 1560⅝ cu. ft. **25.** 360 cu. in.
26. 540 cu. in. **27.** 800 cu. in. **28.** 1470 cu. in. **29.** 120 cu. ft.
30. 150 cu. ft. **31.** 144 cu. ft. **32.** 150 cu. ft. **33.** 21 cu. ft. **34.** 22 cu. ft.
35. 29¼ cu. ft. **36.** 47½ cu. ft.

Page 236.—**1.** 150 sq. in. **2.** 432 sq. in. **3.** 600 sq. in. **4.** 840 sq. in.
5. 270 sq. in. **6.** 540 sq. in. **7.** 972 sq. in. **8.** 11½ sq. ft. **9.** 12.5664 sq. ft.
10. 37.6992 sq. ft. **11.** 25.556 sq. ft. **12.** 56.5488 sq. ft. **13.** 791.6832 sq. in.
14. 1306.9056 sq. in. **15.** 2199.12 sq. in. **16.** 3166.7328 sq. in.
17. 49.0875 sq. ft. **18.** 133.518 sq. ft.

1. 150 cu. in. **2.** 216 cu. in. **3.** 300 cu. in. **4.** 320 cu. in.
5. 432 cu. in. **6.** 6 cu. ft. **7.** 10 cu. ft. **8.** 12 cu. ft. **9.** 20 cu. ft.
10. 30 cu. ft. **11.** 5.25 cu. ft. **12.** 10.2 cu. ft. **13.** 17.5666 cu. ft.
14. 20.25 cu. ft. **15.** 49.4666 cu. ft.

Page 237.—**16.** 502.656 cu. in. **17.** 424.116 cu. in. **18.** 942.48 cu. in.
19. 1696.464 cu. in. **20.** 4580.4528 cu. in. **21.** 43.9824 cu. ft.
22. 11.5192 cu. ft. **23.** 53.4072 cu. ft. **24.** 9.8175 cu. ft. **25.** 35.343 cu. ft.
26. 17.180625 cu. ft. **27.** 25.9182 cu. ft. **28.** 43.295175 cu. ft.
29. 115.42471+ cu. ft. **30.** 262.9781 cu. ft.

Page 238.—**1.** 75 sq. in. **2.** 96 sq. in. **3.** 135 sq. in. **4.** 170 sq. in.
5. 216 sq. in. **6.** 1½ sq. ft. **7.** 3 sq. ft. **8.** 5 sq. ft. **9.** 6½ sq. ft.
10. 9 sq. ft. **11.** 4⅜ sq. ft. **12.** 2⅝ sq. ft. **13.** $6\frac{5}{144}$ sq. ft. **14.** 12¾ sq. ft.
15. 17½ sq. ft. **16.** 240 sq. in. **17.** 288 sq. in. **18.** 360 sq. in. **19.** 480 sq. in.
20. 864 sq. in. **21.** 20 sq. ft.

Page 239.—**22.** 9 sq. ft. **23.** 4⅛ sq. ft. **24.** 11⅞ sq. ft. **25.** 12 sq. ft.
26. 6¾ sq. ft. **27.** 9⅛ sq. ft. **28.** 18½ sq. ft. **29.** 35$\frac{17}{24}$ sq. ft. **30.** 46⅜ sq. ft.

Page 240.—**1.** 24 sq. in. **2.** 28 sq. in. **3.** 22½ sq. in. **4.** 27 sq. in.
5. 40 sq. in. **6.** ⅝ sq. ft. **7.** 1½ sq. ft. **8.** $1\frac{7}{12}$ sq. ft. **9.** 1½ sq. ft.
10. 2⅜ sq. ft. **11.** $2\frac{1}{12}$ sq. ft. **12.** $4\frac{7}{12}$ sq. ft. **13.** 8¼ sq. ft. **14.** 6.69 sq. ft.
15. 14.29 sq. ft. **16.** 18.8496 sq. in. **17.** 37.6992 sq. in. **18.** 56.5488 sq. in.
19. 141.372 sq. in. **20.** 179.071 sq. in. **21.** 791.6832 sq. in. **22.** 942.48 sq. in.
23. 1244.0736 sq. in. **24.** 1130.976 sq. in. **25.** 3392.928 sq. in.
26. 1099.56 sq. in. **27.** 2387.616 sq. in. **28.** 4241.16 sq. in.
29. 8372.364 sq. in. **30.** 10555.776 sq. in.

Page 241.—**1.** 180 cu. in. **2.** 540 cu. in. **3.** 756 cu. in. **4.** ⅜ cu. ft.
5. 10⅜ cu. ft. **6.** 18.75 cu. ft. **7.** 3.125 cu. ft. **8.** 6.9375 cu. ft.
9. 16.245+ cu. ft. **10.** 565.488 cu. in. **11.** 1340.416 cu. ft.
12. 2544.696 cu. in. **13.** 2.618 cu. ft. **14.** 7.65765 cu. ft. **15.** 5.30145 cu. ft.
16. 1.84078125 cu. ft. **17.** 3.337+ cu. ft. **18.** 2.6761 cu. ft. **19.** 1.959+ cu. ft.
20. 8.062+ cu. ft. **21.** 6.4141+ cu. ft.

Page 243.—**1.** 50.2656 sq. in. **2.** 113.0976 sq. in. **3.** 1.3962 sq. ft.
4. 2.1816 sq. ft. **5.** 3.1416 sq. ft. **6.** 5.585 sq. ft. **7.** 12.5664 sq. ft.
8. 15.2053 sq. ft. **9.** 28.2744 sq. ft. **10.** 32.1699 sq. ft. **11.** 40.7151 sq. ft.
12. 19.635 sq. ft. **13.** 2.1816 sq. ft. **14.** 4.276 sq. ft. **15.** 4.9087 sq. ft.
16. 7.0686 sq. ft. **17.** 8.7266 sq. ft. **18.** 11.541 sq. ft. **19.** 4.276 sq. ft.
20. 4.9087 sq. ft. **21.** 5.585 sq. ft. **22.** 7.8758 sq. ft. **23.** 9.6211 sq. ft.

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24. 10.5592 sq. ft. **25.** 904.7808 sq. ft. **26.** Earth, 201,062,400 sq. mi.; Jupiter, 24,328,550,400 sq. mi.

Page 244. — **1.** 4.1888 cu. in. **2.** 33.5104 cu. in. **3.** 113.0976 cu. in. **4.** 268.0832 cu. in. **5.** 523.6 cu. in. **6.** 4.1888 cu. ft. **7.** 7.2382 cu. ft. **8.** 14.1372 cu. ft. **9.** 24.429 cu. ft. **10.** 8.18125 cu. ft. **11.** 65.45 cu. in. **12.** 268.0832 cu. in. **13.** 381.7044 cu. in. **14.** 696.9116 cu. in. **15.** 1150.3492 cu. in. **16.** 1.0226 cu. ft. **17.** 1.2411 cu. ft. **18.** 2.424 cu. ft. **19.** 5.9641 cu. ft. **20.** 8.181 cu. ft.

Page 250. — **1.** $314.67. **2.** $79.85.

Page 251. — **3.** $145.31. **4.** $1437.96. **5.** $290.04. **6.** $57.35.

Page 252. — **1.** $227.87. **2.** $94.80. **3.** $105.54. **4.** $527.16.

Page 255. — **1.** 15°. **2.** 55°. **3.** 30°. **4.** 80°. **5.** 115°. **6.** 120°. **7.** 75°. **8.** 90°. **9.** 90°. **10.** 90°. **11.** 150°. **12.** 167°. **13.** 57°. **14.** 45°. **15.** 95°. **16.** 112°. **17.** 45°. **18.** 118°. **19.** 14° 45′. **20.** 54° 50′. **21.** 15°. **22.** 70° 25′. **23.** 53° 15′. **24.** 71° 2′. **25.** 33° 54′ 10″. **26.** 34° 48′ 57″. **27.** 2° 58′ 54″. **28.** 73° 54′ 36″.

Page 257. — **1.** 35 min., 52½ sec. **2.** 4 hr., 55 min., 38⅔ sec. **3.** 39 min., 5⅓ sec. **4.** 54 min., 25⅔ sec. **5.** 1 hr., 51 min., 39⅗ sec. **6.** 3 hr., 13 min., 41⅕ sec.

Page 258. — **7.** 2 hr., 19 min., 15⅘ sec. **8.** 5 hr., 5 min., $22\frac{8}{15}$ sec. **9.** 7 hr., 33 min., 29⅕ sec. **10.** 9 hr., 9 min., $37\frac{1}{15}$ sec. **11.** 2 hr., 15 min., 36⅔ sec. **12.** 2 hr., 3 min., 50⅔ sec. **13.** 1 hr., 39 min., 44 sec. **14.** 1 hr., 6 min., 11⅔ sec. **15.** 8 hr., 25 min., 59⅔ sec. **16.** 2 hr., 42 min., $2\frac{4}{15}$ sec. **17.** 1 hr., 45 min., 52 sec. **18.** 9 min., $44\frac{2}{15}$ sec. **19.** 12 hr., 59 min., 54 sec. **20.** 6 hr., 59 min., 28⅘ sec. **21.** 8 hr., 19 min., $3\frac{11}{15}$ sec. **22.** 14 hr., 35 min., $55\frac{14}{15}$ sec. **23.** 42 min. 14⅗ sec. before 12, noon. **24.** 8 min. 20⅗ sec. after 10, A.M. **25.** 11 min. 55⅗ sec. before 12, noon. **26.** 23 min. $57\frac{1}{15}$ sec. after 12, noon. **27.** 12 min. 4⅘ sec. before 11, P.M. **28.** 1 min. 30⅘ sec. before 9, A.M. **29.** 5 hr. 7 min. 49⅕ sec. after 12, noon. **30.** 27 min. $10\frac{2}{15}$ sec. before 10 P. M. **31.** 41 min. 18⅔ sec. before 12, midnight.

Page 259. — **1.** 47° 3′ 6″ W. **2.** 32° 3′ 6″ W. **3.** 9° 33′ 6″ W. **4.** 24° 11′ 54″ E. **5.** 50° 26′ 54″ E. **6.** 87° 56′ 54″ E. **7.** 107° 3′ 6″ W. **8.** 118° 18′ 6″ W. **9.** 144° 33′ 6″ W. **10.** 169° 33′ 6″ W. **11.** 162° 3′ 6″ W. **12.** 162° 56′ 54″ E. **13.** 28° 14′ 21″ W. **14.** 56° 51′ 51″ W. **15.** 119° 25′ 36″ W. **16.** 76° 53′ 9″ E. **17.** 91° 49′ 24″ E. **18.** 97° 49′ 24″ E. **19.** 5 min. $22\frac{8}{15}$ sec. before 10 A.M.; 76° 20′ 38″. **20.** 12° 26′ 53″ E.

Page 264. — **1.** 5000 m. **2.** 3500 m. **3.** 15,000 m. **4.** 21,700 m. **5.** 15,080 m. **6.** 600 m. **7.** 450 m. **8.** 180 m. **9.** 123 m. **10.** 111.5 m. **11.** 12.5 m. **12.** 1.25 m. **13.** .125 m. **14.** 1.75 m. **15.** 14.5 m. **16.** 2.4 m. **17.** .3546 m. **18.** .1175 m. **19.** 1.175 m. **20.** 14.05 m. **21.** 3500 m. **22.** 580 m. **23.** 4040 m. **24.** 4.5 m. **25.** 15.8 m. **26.** 35.25 m. **27.** 4.95 m. **28.** 2.07 m. **29.** 7.75 m. **30.** 3.048+ m. **31.** 9.753+ m. **32.** 1.706+ m. **33.** .901+ m. **34.** 2.794+ m. **35.** .749+ m. **36.** 39.37+ ft. **37.** 51.67+ ft. **38.** 159.94+ ft. **39.** 82.02+ ft. **40.** 820.20+ ft. **41.** 36.9093+ ft. **42.** 2.4856 mi. **43.** 7.7675 mi. **44.** 6.5247 mi. **45.** 77.9857 mi. **46.** 160.926 Km. **47.** 201.125 Km. **48.** .0402 Km. **49.** .00201 Km.

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Page 266. — **1.** 125,000,000 sq. m. **2.** 1,600,000 sq. m. **3.** 25,000 sq. m. **4.** 124,500,000 sq. m. **5.** 3,752,500 sq. m. **6.** 125,005 sq. m. **7.** .015625 sq. m. **8.** 2.54 sq. m. **9.** 1545 sq. m. **10.** 119.6 sq. yd. **11.** 13,754 sq. yd. **12.** 1.3455 sq. yd. **13.** 149,500 sq. yd. **14.** 4.825 sq. mi. **15.** 9.3026 sq. mi. **16.** .004825 sq. mi. **17.** .00439075 sq. mi.

Page 268. — **1.** 1.5 cu. dm., 150 cu. dm. or .15 cu. m. **2.** 35 cu. cm., 125 cu. m. **3.** 4570 cu. cm. or 4.57 cu. dm., 12,575 cu. cm. or 12.575 cu. dm. **4.** 15,000 cu. dm. or 15,000,000 cu. cm., 150,000 cu. cm. **5.** 275,000 cu. dm. or 275,000,000 cu. cm., 4,500,000 cu. cm. **6.** 15,750,000 cu. mm., 354,000,000 cu. mm.

Page 269. — **1.** 1.5 Dl. **2.** 1.25 Dl. **3.** 50 dl. or 5 l. **4.** 350 cl., 35 dl., 3.5 l. **5.** 25.475 l. or 2.5475 Dl. **6.** 25.475 Hl. or 2.5475 Kl. **7** 350 Hl. **8.** 458 Dl. **9.** 1752.5 l. **10.** 1475 dl. **11.** 351.25 dl. or 3512.5 cl. or 35,125 ml. **12.** 452.5 ml. **13.** 10.567 qt. **14.** 13.2087 qt. **15.** 185.1338 qt. **16.** 1.585+ qt. **17.** 147,938 qt. **18.** 27,262.86 qt. **19.** 31.78 qt. **20.** 14.3464 qt. **21.** 22.7681 qt. **22.** 15.89 qt. **23.** 228.135 qt. **24.** 136.2 qt. **25.** 15.60625 bu. **26.** 34.333 bu. **27.** 4.9656 bu. **28.** 70.9375 bu. **29.** 5.286 Hl. **30.** 26.607 Hl. **31.** 28.193 Hl. **32.** 17.621 Hl.

Page 270. — **1.** 12,000 g. **2.** 15,500 g. **3.** 1750 g. **4.** 7580 g. **5.** 1571 g. **6.** 154,250 g. **7.** 1.5 g. **8.** 17.5 g. **9.** 375 g. **10.** 17.5 g. **11.** 187.5 g. **12.** 1875 g. **13.** 33.0693 lb. **14.** 2.75577 lb. **15.** 27.5577 lb. **16.** .2755 lb. **17.** 34.6345 lb. **18.** 346.3458 lb. **19.** .7744 lb. **20.** 2.6455 lb.

Page 272. — **1.** 780 Kg. **2.** 2316 Kg. **3.** 133.1 Kg. **4.** 1,130,000 g. **5.** 527,500 g. **6.** 88,000 g. **7.** 1,768,450 g. **8.** 6,658,500 g. **9.** 3500 g. **10.** 112,500,000 g. **11.** 1,975,000 Kg. **12.** 92,000 Kg. **13.** 138,000 Kg. **14.** 158,000 Kg. **15.** 10,500,000 g. **16.** 26,250 g.

Page 273. — **1.** .27559 in. **2.** 65.178675 Km. **3.** 8540 m. **4.** $20.10+. **5.** 360 sq. m., 3875.04 sq. ft. **6.** 4535.9 metric tons. **7.** 2,644,448.9+ cu. m. **8.** 8454 bottles. **9.** 30.48 m., .03048 Km. **10.** 71,122.912+ Kg. **11.** $533.64. **12.** 56.7816 l. **13.** 1312+ steps.

Page 274. — **14.** 10,000 capsules. **15.** 5319.1656834 Km., 5319165.6834 m. **16.** 29.7375 Kg. **17.** $9.37 approx. **18.** 1817.0112 Kg. **19.** 39.095 francs, or 3909.5+ centimes. **20.** 28.39725 miles. **21.** 33 ft. 2.62 in. **22.** 6.67 ft. **23.** 126.25 l., 33.352+ gal.

Page 275. — **24.** 358+ turns. **25.** 27 Kg. Copper, 18 Kg. Zinc. **26.** $15.59. **27.** 137.362+ l., 145.150+ qt. **28.** 289.9248 Kg., 639.168+ lb. **29.** 6.8+ Km. **30.** $5.54.

Page 282. — **1.** 15. **2.** 21. **3.** 31. **4.** 38. **5.** 36. **6.** 41. **7.** 43. **8.** 44. **9.** 54. **10.** 68. **11.** 87. **12.** 98. **13.** 202. **14.** 199. **15.** 13.1. **16.** 1.96. **17.** .17. **18.** 5.21. **19.** .404. **20.** 49.9. **21.** 11.44+. **22.** 30.36+. **23.** 24.66+. **24.** 2.23+. **25.** .18+. **26.** .09+. **27.** .84+. **28.** .82+. **29.** .92+. **30.** .95+.

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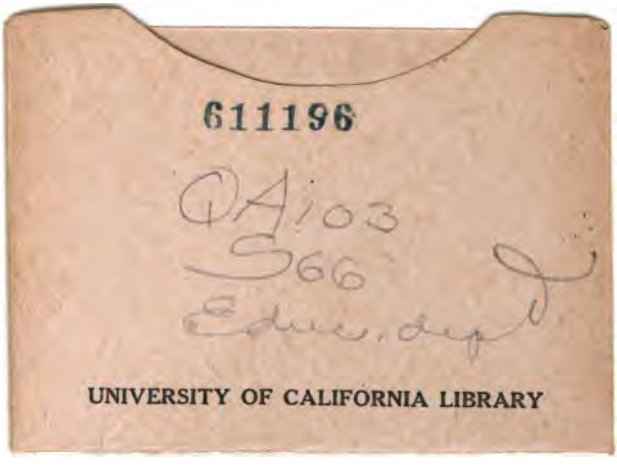
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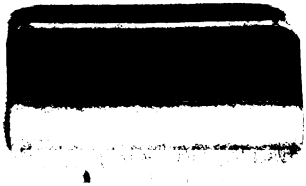

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